Electrical Engineering

June
1936

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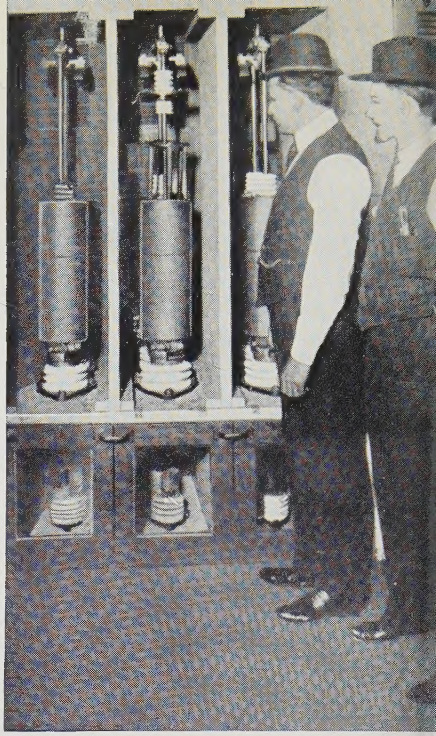
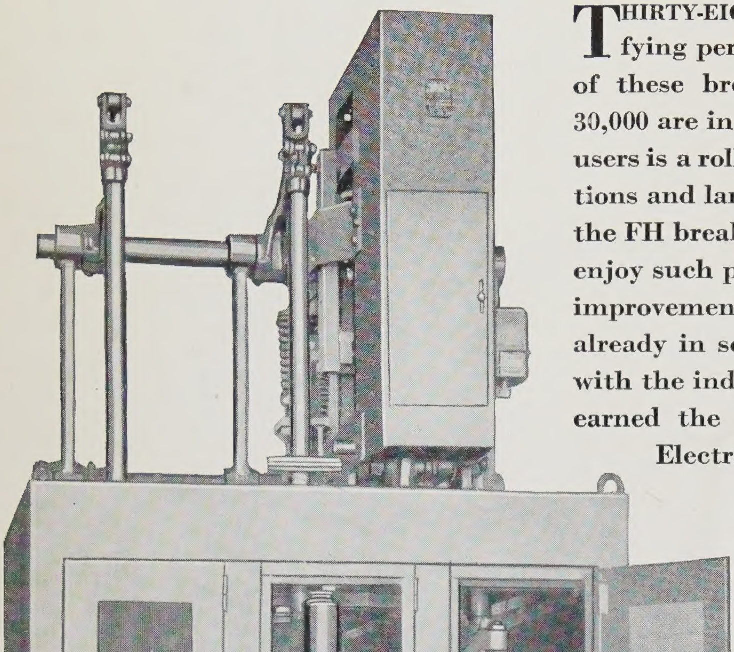
Since 1898

Type FH Breakers Have Been Preferred for Station Service



An early installation in the Fisk Str
Station, Commonwealth Edison



THIRTY-EIGHT YEARS of satisfying performance are back of these breakers. More than 30,000 are in service. The list of users is a roll call of central stations and large industrials. But the FH breaker, even with its fundamentally correct design, would n enjoy such preference had it not been continually improved, and t improvements so made that they could be applied to FH breake already in service. This G-E breaker set its own standard and gre with the industry's demands. It is a modern protective device that h earned the unqualified confidence of operating engineers. Gener Electric Co., Schenectady, N. Y.

HERE'S WHY THEY ARE PREFERRED TODAY

1903 Explosion-chamber principle applied

1913 Tanks were made easily removable

1919 Separating chamber and oil-tight features added

1930 Trip-free motor operating mechanism introduced

1931 OIL-BLAST added

AND NOW

Silver current-carrying contacts — reduced maintenance

Mechanism improvements — reduced operating time

Increased insulation strength — greater electrical safety

860-22

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Front Cover

Airplane view of the Huntington Hotel, Pasadena, Calif., headquarters for the Institute's forthcoming summer convention.

Special Articles

A.I.E.E. Papers

—Turn to next page

In This Issue—

ADOPTION of 275 kv (287.5 kv at sending end) for the Boulder Dam-Los Angeles transmission lines, together with other requirements imposed by desired operating characteristics, has necessitated the designing and building of special equipment. Much of this equipment has been described in previous issues of ELECTRICAL ENGINEERING. In this issue, some of the switching apparatus is described. One paper describes the huge disconnecting switches and outlines the considerations entering into their design (*pages 582–9*). A second paper describes one type of oil circuit breaker designed to interrupt a short circuit on these lines in 3 cycles (*pages 626–35*). A third paper describes the circuit arrangement designed for testing oil circuit breakers of another type which also will interrupt a short circuit on these lines in 3 cycles, and which were described in a previous issue (*pages 710–17*).

DISTANCE RELAYING is featured in 2 papers in this issue. One paper shows that the interposition of a grounded neutral delta-star or star-delta power transformer bank between relay and fault affects the relay performance, and presents formulas for predetermining the performance under operating conditions (*pages 660–72*). A second paper describes a new distance ground relay that is said to avoid certain difficulties involved in the application of conventional distance relays to this type of protection (*pages 697–703*).

MODERN power transformers are superior to their predecessors in so many respects that it is quite impossible to express the degree of over-all improvement by a single quantity or statement. According to one authority, this improvement has been achieved through correlated progress in 5 basic fields (*pages 649–59*). These improved units, together with the growing use of complicated circuits, have necessitated the introduction of new methods for calculating the short circuit impedances (*pages 717–30*).

SYNCHRONOUS machines operating out of synchronism are subject to high pulsating torques that may cause mechanical damage. In order to design machines properly to withstand these abnormal forces, and to determine the starting and synchronizing ability of machines under all conditions, it is necessary to be able to predetermine the instantaneous torque and current relationships under these conditions. Methods and examples are given (*pages 636–49*).

VISIBILITY on the highways at night must be improved if there is to be any marked reduction in fatalities resulting from automobile accidents at night, which are said to comprise more than half the total highway fatalities with only a quarter of the daytime traffic volume. One of the suggested logical ways of providing such improvement is by installing scientifically designed systems of fixed lights along the highways (*pages 614–18; 735–46*).

BY MEANS OF simultaneous directional measurements at 2 points as much as 900 miles apart, thunderstorms at distances of several hundred miles from one or both of these points have been located with sufficient accuracy to permit conclusive correlation of storm locations thus indicated with locations of recorded thunderstorms (*pages 575–82*).

MINIATURE methods have been developed for making laboratory specimens of oil-impregnated paper insulation, subjecting them to high voltage tests, observing by electrical measurements any changes during life, and finally examining them minutely for physical and chemical changes that might be indicative of the causes of failure (*pages 590–9*).

RECOGNITION of all factors contributing to the mechanical characteristics of porcelain is considered to be necessary in order to make most efficient use of such material for electrical insulation. Some factors that have been either unknown or neglected have been found to be of major importance (*pages 618–25*).

SECTION AND BRANCH activities annual report for 1935–36 shows that 540 Section meetings were held during the year ending April 30, 1936—the largest number ever reported; during the same period 1,045 Branch meetings were reported (*pages 752–4*).

AN engineering view of Steinmetz, as presented in the tenth Steinmetz Memorial Lecture at a recent meeting of the Institute's Schenectady, N. Y., Section, gives electrical engineers food for thought (*pages 572–4*).

CURRENT VIEWS of problems concerning electrical engineering curricula in educational institutions based upon broad concepts of the social purposes that engineering serves are summarized (*pages 730–4*).

HYDROGEN COOLING, which was applied to large synchronous condensers several years ago, has been extended during recent years to frequency converters, and most recently to turbine generators (*pages 703–709*).

VIBRATION of transmission line conductors caused by wind, and means of reducing vibration are discussed and analyzed in 2 papers in this issue (*pages 600–14; 673–88*).

ALL is in readiness for the Institute's 1936 summer convention to be held June 22–26, at Pasadena, Calif. The program for the Student sessions has been announced (*pages 747–8*).

CARRIER pilot relaying has been subjected to further study with the result that a faster and simpler system has been developed (*pages 688–97*).

THE recent meeting held by the Institute's North Eastern District at New Haven, Conn., is reported in the news section of this issue (*pages 748–50*).

PRIZE awards for papers presented during the calendar year 1935 have been announced by the committee on award of Institute prizes (*pages 754–5*).

Associates, Members, and Fellows

—A Message From the President

EDUCATIONAL ADVANTAGES and the fortunate graces bestowed by birth and breeding may serve to ornament individual resourcefulness, but they cannot take its place, nor can they, within themselves, justify discriminations in a common experience. Within every man is the latent power to increase his material possessions and to raise his standing among men. However, the attainment of these ideals is contingent, first, upon the desire; second, upon the constant application of energy; and third, upon the vigilance to take advantage of every available opportunity.

Thus the fundamental reason behind the establishment of the several grades of membership in our American Institute of Electrical Engineers is to provide recognition of the achievement of a higher standing in the electrical engineering profession.

Primarily, these several grades of membership are designed for personal advantage. However, they also convey the member's duty or obligation to the Institute and to the engineering profession as a whole.

There are 2 advantages to the member himself. The first of these is the material advantage. While it is true that no direct material advantage can be promised by the Institute to any engineer as a result of his becoming a Member or Fellow, nevertheless, material advantages do come indirectly to the great majority of those who advance to the higher grades.

Admission to Membership or Fellowship in the Institute is a recognition of experience and standing in the profession, and certainly if a position of greater responsibility is to be filled, the preference, other things being equal, most likely would be given to a candidate who had attained to one of the higher grades of membership.

The second advantage to the member himself is based upon a foundation of self-respect and pride in accomplishment. This serves in some degree as compensation for hard work and attainment; however, in a broader sense it is a reward for achievement. The higher grades of membership are, in reality, marks of distinction.

The only way that now exists to indicate to the public a higher professional standing of individual engineers is by granting them promotion to the higher grades of membership in the national engineering societies.

In addition to the advantages accruing to the individual by advancement to the higher grades of membership, there is a duty or obligation resting on engineers to take as high a position in the profession as their qualifications and experience permit, as this is a most important way in which the engineering profession can advance its standing in the eyes of the public.

There is, however, another obligation. To maintain its position of eminence in the engineering field, the Institute needs the best available material from which to select its officers and the men for key positions. For this reason, every electrical engineer should place himself in the position of availability for election to office as soon as he is so qualified.

The privilege of holding office in the Institute, as in all the national engineering societies, usually is reserved for those holding the higher grades of membership. A certain amount of prestige comes from holding such an office, but the real advantages to the individual are the larger acquaintanceship, the new experiences, and the broader viewpoint gained.

Many members do not realize that the Institute was formed primarily for (1) the advancement of the theory and practice of electrical engineering and of the allied arts and sciences, and (2) the maintenance of a high professional standing among members, and that its strength and prestige are due largely to the constant adherence by its leaders to these objectives.

Similarly, many members do not appreciate the full significance of the several grades of membership, in that they were established not only as a means of recognition of achievement, but also as encouragement to participation in the development of individual engineers as well as in technical developments. A thorough study of the requirements of each grade will make this apparent.

During the past several years there has been a great deal of discussion in committee meetings and among individuals regarding the small proportion of the Institute membership in the grades of Member and Fellow. As a result, many suggestions have been made to increase the membership in the higher grades.

From the standpoint of endeavoring to interest properly qualified Associates and Members in transferring to the higher grades, there appear to be 3 possible methods by which this can be accomplished. The first is by personal appeal. The second possible method is by recommendations of local committees which presumably are in a position to be familiar with the candidate's experience and professional record. The third method might be called the general broadcasting method, and involves talks on the subject by officers and prominent engineers at Section and national meetings, and also the distribution of special printed material by headquarters.

Fundamentally, the aspiration to a higher professional standing must originate with the individual engineer. He must be actuated by a desire to progress in his chosen field. He must be conscious of the fact that as a token of the realization and acceptance of the ideals and responsibilities of the engineering profession, of which he is a part, he owes it to himself as well as to his profession to strive for the highest possible position.

In thus striving the individual engineer stimulates the continued healthy growth of the Institute and enhances the standing of the engineering profession as a whole.

An Engineering View of and From Steinmetz

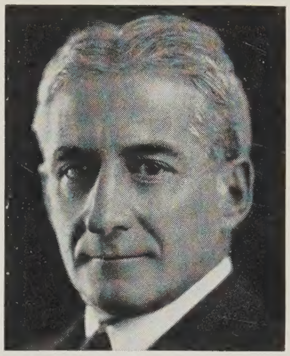
By GERARD SWOPE, FELLOW A.I.E.E.
President, General Electric Co., Schenectady, N. Y.

Despite his handicaps, Steinmetz not only made the most of his native ability, but at the same time felt warm and active sympathy for his fellow men. His life and achievements are a genuine tribute to the conception and tradition of open-minded sympathy and tolerance of ability and ideas in America, no matter in what form or shape they may come. Upon recommendation of the A.I.E.E. committee on education, full text of the Tenth Steinmetz Memorial Lecture delivered by Doctor Swope under the auspices of the Steinmetz Memorial Foundation before the Institute's Schenectady, N. Y., Section, April 20, 1936, is presented herewith.

STEINMETZ, endowed with a great brain, physically weak with a misshapen body, not able to play or take part in sports or social activities, driven in upon himself in his early life, gave entire devotion to mental development. Educated in one of the best secondary schools and then in one of the best universities in Germany, in the latter half of the nineteenth century, his training in science and mathematics was thorough, but theoretical. The German university did not specialize in technical work or in any particular kind of engineering, but gave comprehensive training in fundamentals, which prepared the student to use the tools that were placed in his hands in any direction that his scientific or engineering work might carry him.

Emphasis on fundamentals, rather than on specialization in a particular branch of engineering, is still the great difference between engineering training in America and in Europe. I think it is a question of serious moment for engineers to consider, whether the training in our technical schools and colleges should not be conceived on a broader base, rather than laying emphasis on proficiency in narrow and specialized fields, as at present. Such specialized training, beginning too early and without a proper foundation of a broad character, warps the viewpoint and outlook of the engineer, does not train him to think comprehensively, limits his ability and his opportunity throughout the major portion of his life and especially during the years of greatest creative development and accomplishment. Some of our engineering schools are beginning to place more emphasis on the broad studies of pure mathematics and science, rather than on special applications, and many industries are looking for men with such training, deferring specialization until the men have chosen the field of their life's work.

Although Steinmetz did not mingle with his fellow students, his great mental endowment, his preeminent ability in mathematics, soon brought him their admiration. This admiration was the basis of his meeting some of these students in extracurricular activities along the lines of social discontent, which was very prevalent in Germany at that time. In Steinmetz's association at home and with his fellow students, he soon was brought to a realization of the economic limitations under which most people lived, to which, having been so immersed in his studies, he previously had not given much thought. His naturally sympathetic and generous viewpoint was affected, possibly to even a larger extent than it would have been in a normal healthy young man, and he became the center of this small group of students, writing articles on the social and economic conditions that prevailed during that period. It was these meetings of discussion and consideration of social theories and his articles on social problems that finally compelled him to leave Germany and eventually to find a shelter in America. Here, fortunately, he found work in a sympathetic atmosphere of the kind he could do best, finding mathematical solutions of the early problems of alternating current, for which at that time there was a crying need. But all through his life he never forgot his experience in Germany, nor did he ever lose his feeling of sympathy and consideration for others, especially those less fortunate than himself in the conditions under which they lived. It might have been so easy for a man, absorbed as he was in the abstract and abstruse nature of his mathematical work, involving highly complicated problems, to forget the human side; and it was characteristic of him that as he became more prosperous he never lost touch with human beings, as such, nor with their needs. It was this human aspect of social problems and the human interest he took in people that endeared him to children and to anyone who got beyond the threshold and learned to know the inner man.

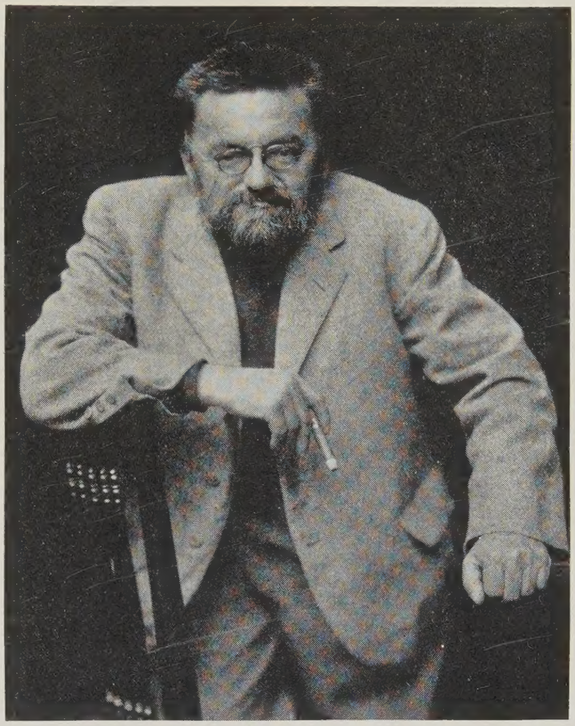


Doctor Swope

It was his particular province, as experiments in electricity unfolded the law that underlay the phenomena, to develop and apply his mathematical formulas so that in the design of machinery the results could be predicted. It was his contribution and the application of his work that made the process

of designing one of real engineering, rather than an accomplishment by trial and error.

As better electrical apparatus was designed and built, he believed that this scientific development, as carried out in engineering products, eventually would serve to ameliorate the conditions under which people lived, to reduce the toil and drudgery of life and the number of hours people would have to work to earn a livelihood, and so give them greater leisure to pursue their individual tastes. He recognized, however, that the achievement of the application of science, with the aid of mathematics and engineering work, would take time, that it could not be done at once or by fiat. A shorter workday, or workweek, could come only through the increased mechanization of industry, which while not lessening the workers' earnings would lower costs, thus lowering the selling prices and making the articles available to a larger portion of the community, thereby increasing the demand and increasing employment. Furthermore, a shorter workday must be accompanied by education and training of men and women for the better and well considered use of the leisure that ensued.



Charles Proteus Steinmetz

As I said, his abstract mathematical computations permitted engineers to design better, more efficient, and more enduring apparatus than before. He had the joy of finding a solution for seemingly insolvable problems, and the even greater joy of knowing that his professional attainments would bring greater comfort to millions of people. These considerations of the sphere and opportunity of a scientist, mathematician, inventor, or engineer are ennobling and give to him not only great joy, but also keen satisfaction in the material help that his work will bring to many.

The second aspect is what Steinmetz's life and accomplishment meant to others. His life and career seem to me, to use a mathematical term, to illustrate to the nth degree how unpredictable a man's attainments may be. In colleges, in industry, in the professions, and possibly to a lesser degree in other fields of activity, we try to devise methods of selecting young men and women of promise, who have character, personality, and ability, who may go far in accomplishment and leadership, so much needed in industry and in a democratic society such as ours. One wonders if the best methods we have been able yet to devise for selecting the promising young people, gauging their ability by examinations either in college or in the civil service, judging their characters by people who have known them and their personalities by the way they impress people they meet, would ever have selected Steinmetz for the career he carved out for himself. This does not mean that we must leave such selections to the haphazard processes of probability and chance. This is the rule in nature, where time is not a factor and the units are countless. With human beings, however, time is a very definite limitation; furthermore, we must try to use the consciousness and ability with which men are endowed, which alone distinguish them among the earth's inhabitants, and we must continue our efforts to develop methods for the best selection of men and women of promise. It always will be true that human effort will fall short when confronted with such an exceptional case as Steinmetz presents, but this should not discourage us in trying to find better methods for the selection of the leaders of tomorrow.

One cannot help but feel stimulated by the thought that the open door and the freedom of opportunity that in this case was offered to an exceptionally gifted man characterizes the United States and its institutions. This country long has had a conception and a tradition of freedom and has offered an equality of opportunity to men and women for their development, which on the whole has brought about a fairer distribution of material wealth and livelihood than in any other country in the history of the world. This breadth of view and tolerance, sympathy, and admiration for a man's genius, irrespective of his nationality or his class or his creed, is illustrated magnificently by Steinmetz's entrance into this country as a penniless immigrant, finding work and recognition leading to a full realization of his capabilities. He was so impressed with the breadth of opportunity here that he became enthusiastic and soon applied for citizenship. Throughout his life he had a deep feeling of devotion for the country of his adoption, which so widely and so generously had opened its doors to him. Who would have imagined that a man fleeing from persecution because of his ideas for the betterment of society, regardless of whether the ideas were well or ill conceived, could come to foreign shores, poor and without friends, with great physical handicaps, find work in his field which was abstract and apparently far removed from this workaday world, rise to the highest pinnacle of his chosen profession, and receive the highest honor from the hands of his brother engineers

in being elected president of the American Institute of Electrical Engineers? His life and achievements are a genuine tribute to America that should make us all cling to this conception and tradition of open-minded sympathy and tolerance of ability and ideas, no matter in what form or shape they come.

He was fortunate that his first employer* in America recognized this talent and that his second and only other employer, the General Electric Company, also recognized his capacity and gave him an untrammeled opportunity, allowing him to work under conditions congenial to him and to develop his specialty to the greatest extent. This attitude toward ability, in whatever form it may come, is characteristic of industry in general in this country. Great industrial organizations must be made up of an aggregation of many people, of many viewpoints, and of varying degrees of mentality, each of whom can contribute something in small or large measure to the completed result that the industrial organization has set before it as its goal. That is why large industrial organizations can find a place for geniuses, allowing them either to work alone or with others and find an opportunity for their special talents, whether they be research workers in pure science, inventors, engineers, mathematicians as Steinmetz was, or organizers and leaders of men. In many industrial organizations, research work in pure science is conducted without any thought of immediate application, on the theory that if knowledge be broadened all the human race will benefit. Possibly the industry in which this work is done may reap the first reward in the application of such new discovery or invention; this is not true in every case, however, but even when it is so, the beneficiary of such research work is eventually the public. The benefits to the public are many times the cost of such work or the amount of profit that any particular industrial organization may secure. I do not mean to imply that large organizations can do it all, for there always will be smaller organizations that will make valuable contributions by specializing in particular fields or in particular localities, and such small organizations may be more congenial for some men to work in than larger ones. Both large and small organizations are necessary for the development of industry, individuals, and society.

Steinmetz had a pride in the institution of which he was a part, and he was never so happy as when problems were brought to him that he could aid in solving. His also was the pride of accomplishment and assistance to others in an organization of which he was proud to be a member.

From both these viewpoints, of looking at Steinmetz's work from his attitude toward life and at his accomplishment from the viewpoint of the engineering profession, what lessons can we learn to improve engineering education, in order that the young man may acquire thorough scientific training and the breadth of view engineers should have as to their responsibilities outside their purely professional work?

From the standpoint of engineering education in America, as I have said before, possibly we have gone too far in specialization and should place more emphasis upon the basic conception of science and on the study of fundamentals. It is highly desirable that we keep up the high standard of the average of our graduates of engineering courses; but while maintaining this high standard of the average, we also should seek to bring out the specially gifted man of greater ability than the average. A beginning is being made in America, in a limited way, by the introduction of what has long been the practice in England, of the honors course, in giving to the highest portion of the class an opportunity to work for honors, not holding them so rigidly to the usual courses and discipline but insisting that their work be of a standard and of a character to justify their greater liberty of action and choice of studies.

Such men should be encouraged to interest themselves in, and to study the history of, industry before and since the introduction of the factory system, which came in, as a matter of fact, as a result of the work of engineers and of applications of power to aid or replace human toil. To specialize early and too narrowly is the easiest way to immediate results. It is natural for engineers, whose work is so largely the solution of a particular problem using exact methods, to look neither to the right nor to the left. It is recognized by all to be essential in designing machinery, or in any large engineering project, that the engineer must calculate with a nicety how his engineering designs will work out; furthermore, he must take into consideration not only whether they will work or can be done, but also at what cost. This is the economic factor that engineers introduce into a laboratory experiment, not only whether the work can be done, but how much it will cost to do it; whether it be economically justified and if so whether the best way has been found. But over and beyond all this is the still broader aspect of the engineer's work. He must consider the social reactions of his profession, which means taking into consideration not only the economic and efficient design of apparatus or engineering structures and what they will accomplish, but also the condition under which the work is to be done, of stimulating and developing workers. The engineer too long has considered that his problems are all bound up with inert material. As a matter of fact, an even larger consideration must be the human beings who are to work out his conception, not only the people who are directly working on the project, but also the public which will be served by the completed product.

I commend then to engineers these 2 thoughts: first, the value of a broad conception of the engineer's responsibility to himself, to his fellow workers, and to the community; second, the richness of a life that has made the most of native ability and also has felt warm and active sympathy for fellow men—which in the case of Steinmetz brought him the recognition, the honor, and the respect of his profession, as well as of the larger community in which he lived.

* The first job Steinmetz obtained in America was as draftsman at the Osterheld and Eickemeyer electrical factory in Yonkers, N. Y.; soon he was given charge of all new and experimental work in the establishment, and later he was put in charge of the research laboratory. In 1892, the firm merged with the General Electric Company and he was sent to Lynn, Mass.; the next year he was transferred to Schenectady, N. Y., as chief consulting engineer, remaining in that position until his death on October 26, 1923.

Fields Caused by Remote Thunderstorms

The object of the studies described in this paper was to verify the supposition that certain types of short-duration longitudinal voltages appearing in communication circuits are caused by remote thunderstorms. By means of simultaneous directional measurements made in the frequency range below 40 kilocycles at 2 points as much as 900 miles apart, thunderstorms at distances of several hundred miles from one or both of these points have been located with a degree of accuracy great enough to permit conclusive correlation of the storm locations indicated by the directional measurements with the locations of recorded thunderstorms. Methods, equipment, and results are discussed.

By
K. E. GOULD
ASSOCIATE A.I.E.E.

Bell Tel. Labs., Inc.,
New York, N. Y.

IN the summer of 1931, an investigation was begun to determine the magnitudes, frequency of occurrence, and wave shapes of short-duration longitudinal voltages appearing in communication circuits in certain sections of New Jersey. While these disturbances were of about the same durations as those caused by local thunderstorms, and of somewhat similar wave shapes, they were not limited to times of local thunderstorms and it was at first thought that the voltages must be the result of transient induction from a nearby electrified railway or from adjacent power lines. The maximum horizontal electric intensity* during such non-storm periods occasionally would exceed 400 volts per mile, a voltage gradient comparable to that produced in severe exposures by induction from power line short circuit currents. Several oscillograms illustrating characteristic wave shapes of the horizontal electric intensity associated with these disturbances are shown in figure 1. These oscillograms were taken in New Jersey in 1933 with a portable cathode ray oscillograph of the glass-tube sealed-off type, with external photographic recording.

From the studies carried on in the summers of 1931, 1932, and 1933, it seemed likely that these electromagnetic disturbances were caused by remote thunderstorms. In the summer of 1934, simultaneous measurements were made of the direction of propagation of these transient fields at various pairs of points separated by distances of several hundred miles. Whippany, N. J., was used throughout as one of the test locations, and for the second location certain points were chosen at which, in view of the data obtained incidental to certain other measurements, it was thought that the fields produced by remote thunderstorms would be particularly large. The locations used for the second test point were as follows: Cadillac, Michigan; Eau Claire, Wisconsin; Atlanta, Georgia; and Hagerstown, Maryland. The base line lengths with these points were, respectively, 607, 905, 730, and 190 miles. With these locations, the base line for the triangulation measurements had several substantially different directions, thus providing favorable conditions for the location of sources in several different areas.

Experimental Procedure

One characteristic feature of the directional measurements for the location of thunderstorms by triangulation, as made in the present study, consisted of the use of "probes," or ground return circuits, mutually perpendicular from a common point and 500 feet or 2,000 feet in length. The arrangement employed is shown schematically in figure 2. With this arrangement the direction of propagation will be indicated by a straight** line on the screen of the oscillograph, if the following conditions are satisfied:

1. Plane-polarized radiation.
2. Quasi-tilt angle,[1] at any given frequency, the same for all directions of propagation.
3. Probes short enough and grounded through sufficiently low resistances at the far ends, that the voltage appearing across each pair of plates of the oscillograph is practically the integral of the electric intensity along the length of the respective probe.

The significance of the directional measurements with the probes at Whippany was verified by making simultaneous directional measurements at that point with probes and with untuned vertical loops mutually perpendicular. These measurements showed substantial agreement between the directions indicated by the 2 methods for the records in which the directions were sharply defined. Some of the directional records consisted of open figures (see figures 3 and 4), whether probes or loops were employed.

In other investigations, direction of propagation, either of atmospheric disturbances[2,3,4] or of the field produced by a transmitting aerial, usually has been determined with a receiving system that responded principally to the vertical component of the electric

A paper recommended for publication by the A.I.E.E. committee on electrophysics, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Apr. 4, 1936; released for publication Apr. 17, 1936.

*Throughout the paper, "horizontal electric intensity" refers to the horizontal component ordinarily considered in the case of the low frequency wave antenna and does not include the transverse horizontal component, which generally is of importance only at the higher frequencies.

**Nearly straight, strictly speaking. Slight curvatures result from lack of proportionality between voltage applied to the deflection plates of the oscillograph and deflection of the electron beam.

1. For all numbered references, see list at end of paper.

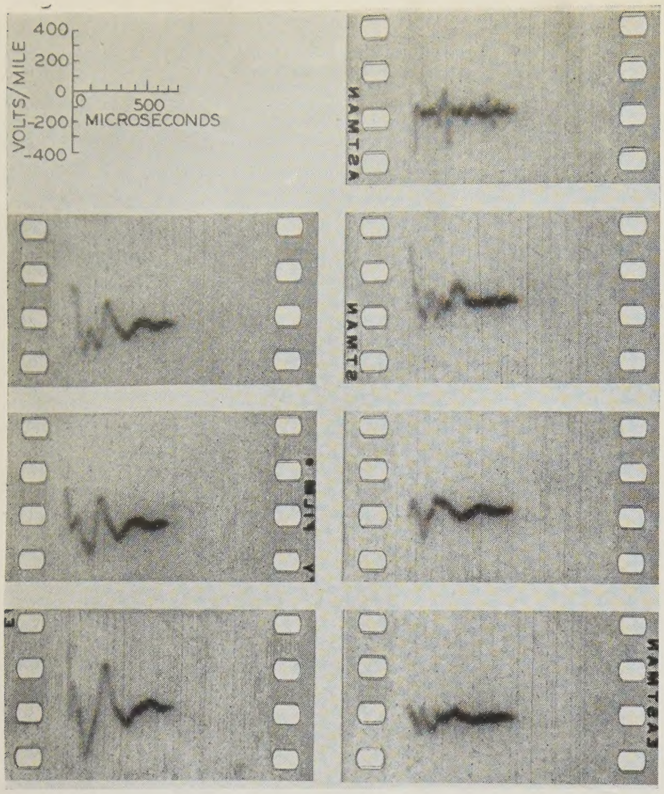


Fig. 1. Characteristic wave shapes

intensity, although it has been suggested that the spaced-aerial system might provide accurate directional measurements if arranged to respond principally to the horizontal component of electric intensity.[5] With the receiving system used in the present study, it was, of course, principally the horizontal component of the electric intensity that was effective.

For some rough comparisons of the relative magnitudes of the horizontal and vertical electric intensities a third loop in a horizontal plane was used in conjunction with the 2 vertical loops mentioned above. In particular, this horizontal loop was used to verify the supposition that the horizontal electric intensity ordinarily was small compared with the vertical electric intensity. With each of these loops, a resistance-capacity circuit was inserted between the loop and the measuring equipment so that, for a given field strength, the oscillograph deflection would be approximately constant for frequencies from 1 to 40 kilocycles.

A second characteristic feature of these directional measurements was the use of measuring equipment (probes, amplifiers, and cathode ray oscillographs) having little frequency discrimination for all frequencies of any importance below 40 kilocycles. Portable cathode ray oscillographs of the glass-tube sealed-off type were used for the directional measurements, both with the probes and with the loops. Three amplifiers, each with an adjustable voltage gain of from 1 to 50,000 (0 to 94 decibels), were used in conjunction with the 3 loops in order to provide sufficient sensitivity to obtain satisfactory deflections on the oscillograph screens. Two amplifiers with a maximum voltage gain of 60 (36 decibels) were used in conjunction with 2,000 foot probes at Whippany in order to obtain satisfactory oscillograph deflections therefrom, 2 of the high gain amplifiers being used with one of the cathode ray oscillographs for directional measurements with 500 foot probes at the second test location during the triangulation measurements. For all the amplifiers used, the gain was practically independent of the frequency from 1 to 25 kilocycles and varied but little from 25 to 40 kilocycles. Although the phase shift changed considerably with frequency, the characteristics of the 3 high gain amplifiers were practically identical as regards phase shift, as were those of the 2 low gain amplifiers.

External photographic recording of the directional patterns shown on the oscillograph screen was employed in the location of storms by triangulation, the 35 millimeter film being moved continuously and at nearly the same rate, about 1 foot per minute, in the 2 instruments. Simultaneous timing marks were put on both films occasionally, and these marks, together with the distribution characteristics of the records along the films, made possible the identification of many records unmistakably simultaneous on the 2 films. On almost all the films the frequent occurrence of smaller figures produced by fields of intensities lower than the maximum for which the measuring equipment was adjusted made it impossible to identify as simultaneous or to measure accu-

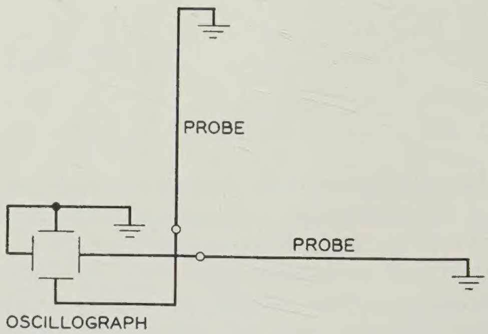


Fig. 2. Schematic diagram of circuit arrangement for directional measurements

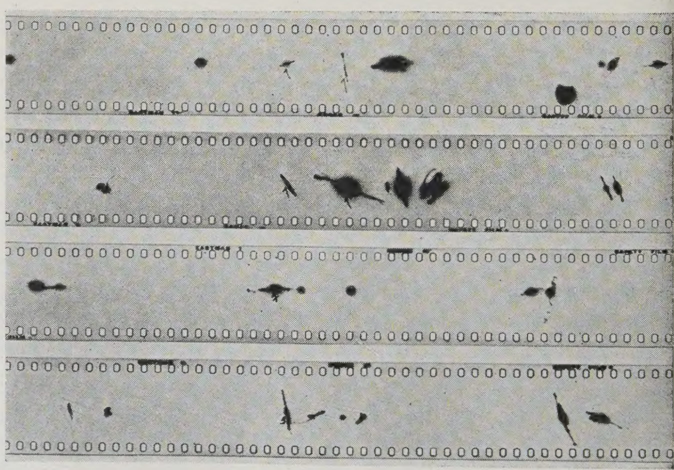


Fig. 3. Some oscillographic triangulation records from tests made at Cadillac, Mich.

rately more than a very small percentage of the total records.

In figure 3 are shown a few triangulation records from the Cadillac tests. The upper film shows directional measurements at Whippany made simultaneously with the directional measurements at Cadillac shown on the second film. The third and fourth films are similar to the first and second, respectively. Several simultaneous records are indicated with check marks to facilitate comparison of the films from the 2 locations. In figure 4 are shown triangulation records from the Atlanta tests during a storm fairly near Atlanta (location 18 of figure 5), the films being arranged as in figure 3. The complicated character of some of the directional records is illustrated in figures 3 and 4.

Method of Analyzing Triangulation Records

The directional records that showed distinct directions, and that unmistakably were simultaneous on the films taken at the 2 test locations, were projected on enlarged polar calibration sheets and the magnitudes and angles recorded (the angles in degrees east or west of north, with an ambiguity of ± 180 degrees). Usually it was possible to read the angle to within ± 2 degrees. For each pair of angles from the directional measurements made at Cadillac, Eau Claire, and Atlanta, in each case together with Whippany, the latitude and longitude of the corresponding location were read from a gnomonic chart, at the intersection of the 2 radial lines representing these angles. For the gnomonic charts, the Hydrographic Office Chart No. 1280 was used, with the longitude scale shifted westward 45 degrees and with compass roses[6] drawn from the 2 test points (Cadillac and Whippany, etc.).

The locations determined as described were plotted on an auxiliary chart simply to facilitate grouping the records that showed nearly the same angle at both test locations. The angles for these groups were averaged and the locations corresponding to these average angles read from the gnomonic charts,

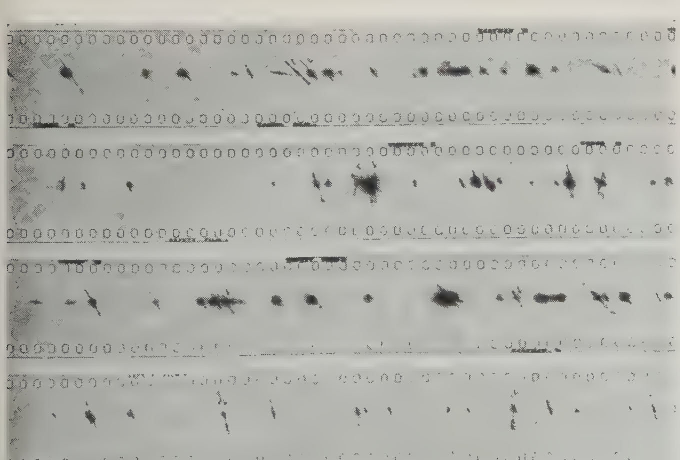


Fig. 4. Some oscillographic triangulation records from tests made at Atlanta, Ga., during a nearby storm

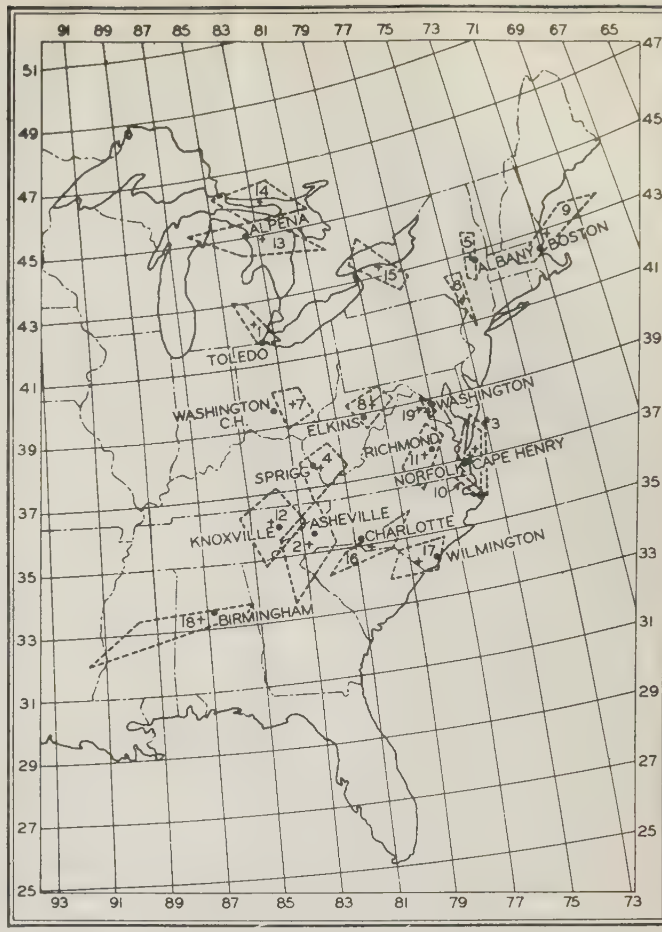


Fig. 5. Locations of areas of recorded storms as determined by triangulation

these latter locations representing, it was believed, the best determination of the locations of the sources.

In the case of the Hagerstown tests, the distance between the test points being too short to permit the use of the available gnomonic chart, the Mercator Position Plotting Charts of the Hydrographic Office were used, the half-convergency correction being read, for successive approximations, from a nomograph.[7] As a matter of fact, this correction was found to be substantially unchanged after the first approximation.

Table I gives the number of records that could be grouped as mentioned above and the spread in direction at the 2 test points, in degrees from the average direction. The angular spread generally is small, it may be noted, and appears generally to be larger for the cases in which the storm was relatively close to the measuring apparatus. This is particularly true of location 6 as regards the measurements at Whippany, 1 at Cadillac, and 16, 17, and 18 at Atlanta. (See the following section.) The size of the storm area alone might account for the larger angular spread for the closer storms, but it would seem possible that the directional measurements were less accurate for the closer storms, because in such cases more open figures were obtained, whereas for the more remote storms the directional records usually were straight lines.

In addition to the measurements that could be

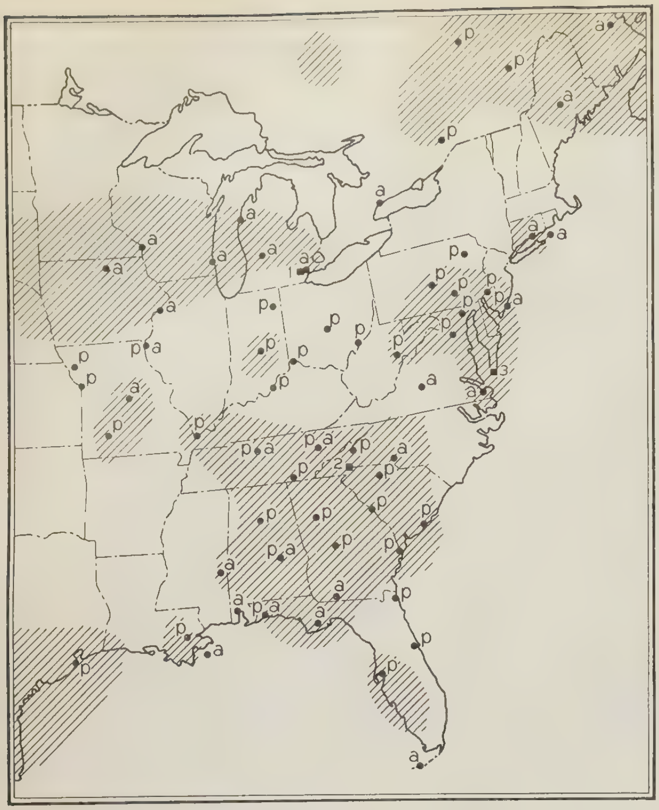


Fig. 6. Daily weather map for July 26, 1934

grouped so that the angular spread was fairly small, there were, of course, scattered measurements that could not be so grouped. The number of such measurements was not large, being perhaps 20 per cent of the total number of records. Apparently there was no way of ascertaining whether these scattered records represented sources from which only one isolated record was obtained, or erroneous apparent directions. In view of the fact that, for several of the locations, only a few records were obtained that showed accurately measurable directions and that were unquestionably simultaneous on the 2 films, it seems likely that many cases would occur of a single record from one location. In analyzing the data, only those locations indicated by 3 or more concurring records were considered, with the single exception of location 19, which was retained although indicated by only 2 records, because it was the only case of satisfactory triangulation data at Hagerstown.

Location of Storms by Triangulation

In figure 5 are shown, as crosses inside the quadrilaterals and numbered 1 to 11 inclusive, the locations resulting from the triangulation measurements at Cadillac and Whippany. These represent all the inland locations for which several records indicating substantially the same location were obtained during a test run, and for which the intersections of the 2 directions occurred at angles that provided reasonably accurate locations of the sources. Several indefinite locations resulting from intersections at small acute or large obtuse angles have been omitted, as well as some locations off the eastern coast, for which it was not possible to obtain information from weather reports.

To indicate the areas over which it seemed advisable to look for thunderstorms to correlate with the respective locations shown in figure 5, the areas bounded by the directions ± 5 degrees from the average directions indicated by the directional measurements are shown by the dotted quadrilaterals in figure 5. It is believed that the accuracy of the measurements, excluding possible false directions resulting from the type of polarization of the propagated radiation or from geological irregularities that would cause

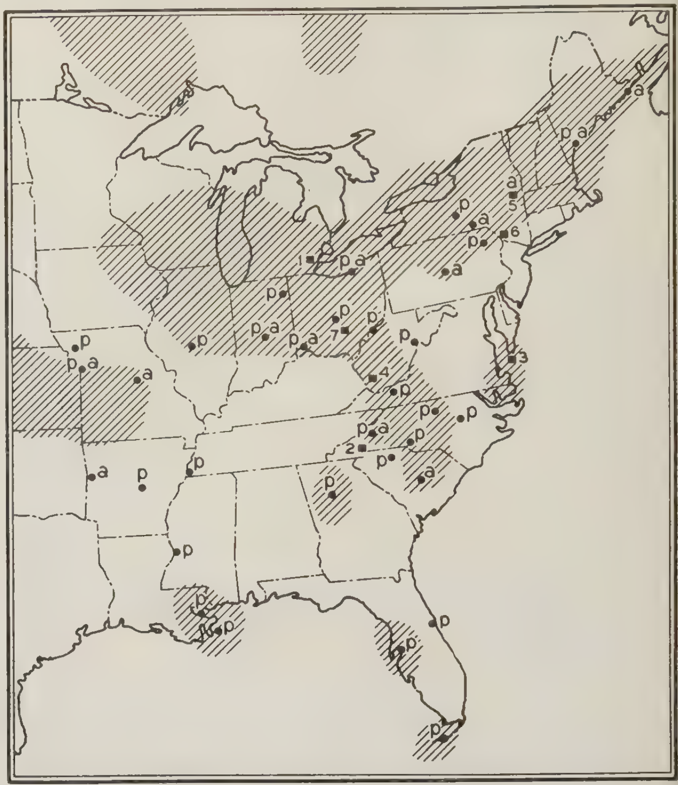


Fig. 7. Daily weather map for July 27, 1934

the tilt angle to be different in different directions, is within ± 5 degrees.

The locations shown as 12, 13, and 14 in figure 5 are those obtained from the triangulation measurements at Eau Claire and Whippany.

Figure 5 shows, as 15 to 18 inclusive, all the satisfactory locations resulting from the Atlanta tests. It may be noted that locations 16 and 17 were determined from the same test run, the 2 thunderstorms apparently occurring simultaneously. Results of the tests at Atlanta were of particular interest because, in the case of location 18, occasional flashes of lightning were visible on the horizon and in the general direction indicated by the directional measurements from the test location at Atlanta.

For the short period of the tests at Hagerstown, only one storm location could be determined satisfactorily from the triangulation records, this location being shown as 19 in figure 5. The short distance between the 2 test locations in this case made satis-

factory triangulation measurements impossible for storms several hundred miles from the test points. In other words, most of the simultaneous records obtained during the Hagerstown tests showed nearly the same direction of propagation at Whippany and at Hagerstown.

In attempting to correlate the locations from the triangulation measurements with reported thunderstorms, it was, of course, impossible to obtain complete thunderstorm reports from stations spaced closely enough to permit every thunderstorm to be reported and its location at any given time made known. Hence it was to be expected that only in fortuitous cases would it be possible to tell the exact location of the thunderstorm producing the observed disturbances. In fact, it was anticipated that the available thunderstorm reports might be insufficient, in at least some cases, to establish definitely whether or not any thunderstorm existed, at the time of the measurements, in the region from which the disturbances apparently came.

As a matter of fact, it was found that of the 19 locations resulting from the triangulation measurements, all but 1 of these locations were found to correspond at least fairly well with reported thunderstorms. Most of these cases of correlation are quite conclusive, although in a few cases the locations and the times do not agree well enough that the correlations appear unquestionable. For the single case in which no definite correlation was found to exist (location 15) it cannot be said that in all likelihood no thunderstorm occurred near the indicated location at the time of the tests, because (1) a considerable part of the area shown for this location extends over Lake Ontario, thus making weather reports for this location necessarily incomplete, and (2) the daily weather map shows that thunderstorms had occurred generally in the vicinity of this location in the 12 hour period preceding 8 a.m. of the day of the test which resulted in this location.

The correlation obtained between the results of the

Table I—Records From Triangulation Measurements

Location No. (Figure 5)	Number of Records	Maximum Difference Between Observed Directions and Average of Observed Directions, Degrees: Oscillograph at Whippany, Plus	Oscillograph at Whippany, Minus	Portable Oscillograph, Plus	Portable Oscillograph, Minus
1	4	2	2	8	5
2	3	2	3	3	2
3	8	4	5	4	3
4	4	3	2	3	6
5	3	1	2	0	1
6	4	13	5	1	3
7	4	1	0	4	6
8	5	3	3	1	2
9	7	2	5	2	1
10	8	5	2	2	3
11	6	4	3	6	5
12	9	3	2	9	7
13	7	4	4	4	4
14	4	2	1	2	4
15	4	2	2	2	2
16	7	2	3	9	12
17	11	3	5	10	7
18	33	8	4	21	29
19	2	5	5	2	3

Table II—Correlation of Results of Triangulation Measurements With Reported Thunderstorms

Location of Portable Equipment	Location From Triangulation Measurements (See Fig. 5)	Time of Measurements: Date	Time of Measurements: Hour[2] (E.S.T.)	Reported Thunderstorm Locations: Station	Time of Reported Thunderstorm: Date	Time of Reported Thunderstorm: Hour (E.S.T.)
Cadillac, Mich.	1	7/26	8:50	N. of Toledo, O.	7/26	8:42
	2	7/26	8:50–**12:42**	Asheville, N. C.[1]	7/26	**4–8**
	3	7/26	8:50–**12:42**	Cape Henry, Va.	7/26	12 N.
	4	7/27	12:04–2:03	Sprigg, W. Va.	7/26	**8:18–9:30**
	5	7/27	4:05	E. of Albany, N. Y.	7/27	4:02
	6	7/27	6:06	Albany, N. Y.	7/27	6:02
	7	7/27	6:06	Washington C. H., O.	7/27	5:45–5 55
	8	7/27	**10:05**	Elkins, W. Va.	7/27	**5–8**
	9	7/28	**2:10**	Boston, Mass.	7/28	**4–6**
				Portland, Me.	7/28	**2–3**
	10	7/29	**2:02**	Norfolk, Va.	7/29	**6**
	11	7/29	**2:02**	Richmond, Va.	7/29	**5**
Eau Claire, Wis.	12	8/1	9:33–12	Knoxville, Tenn.	8/1	11:45
	13	8/2	9:05	Alpena, Mich.	8/2	7–9
	14	8/2	9:05	Alpena, Mich.	8/2	7–9
Atlanta, Ga.	15	8/14	**12:37**			
	16	8/18	**4:28**	Charlotte, N. C.	8/18	**2–6**
	17	8/18	**4:28**	Wilmington, N. C.	8/18	**3–7**
	18	8/20	**6:52**	Birmingham, Ala.	8/20	**5**
Hagerstown, Md.	19	8/24	**2:32**	Washington, D. C.	8/24	**3**

1. Thunderheads N. W. of Anderson, S. C., at 1:42 p.m.
2. Light faced numerals indicate forenoon; bold faced numerals indicate afternoon.

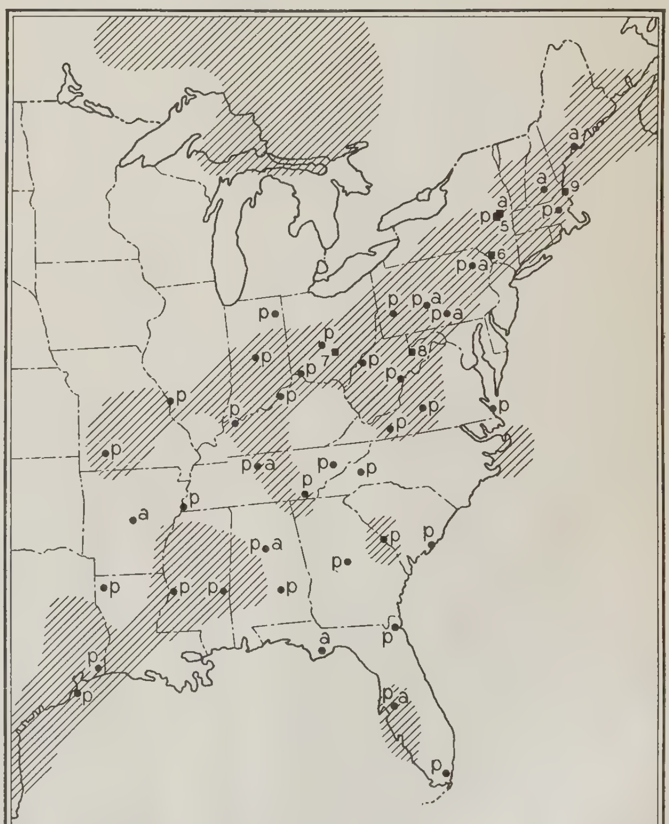


Fig. 8. Daily weather map for July 28, 1934

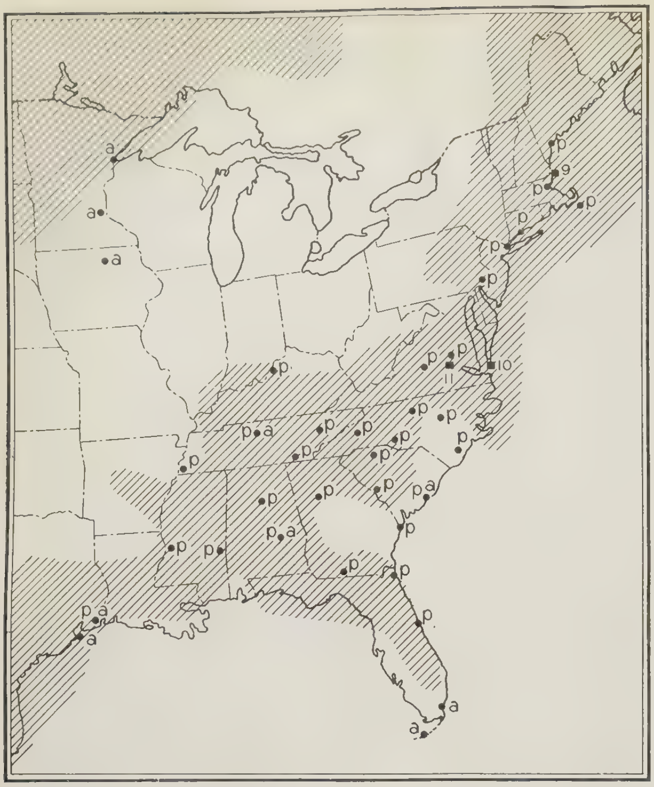


Fig. 9. Daily weather map for July 29, 1934

triangulation measurements and reported thunderstorms is summarized in table II. The reported storms of table II are from detailed data obtained from the U.S. Weather Bureau except in the case of Sprigg, W. Va. (location 4) which was from a power company trouble report. The reported thunderstorm locations are indicated in figure 5 by circles, and table II shows the extent to which agreement obtained between the times of the tests* and the times of the observed thunderstorms.

In the case of the Cadillac tests, a rather intensive study was conducted from the morning of July 26 to the afternoon of July 29, measurements being made for approximately 30 minutes at intervals of 2 hours for the first 2 days, after which the interval was increased to about 8 hours. In spite of the fact that favorable testing periods occurred only occasionally, and that even under the most favorable circumstances the only storms that can be located by such a testing procedure are the few which produce the larger disturbances at both locations, the locations shown by these triangulation measurements could be correlated, in a general but definite way, with the changes in thunderstorm areas from day to day.

This correlation is shown in figures 6 to 10 inclusive, in which the locations from the triangulation measurements have been indicated by small numbered squares on the appropriate daily weather maps for the area generally east of the Mississippi River. The Weather Bureau stations that reported thunderstorms during the period covered by a given map are shown by small circles, the accompanying designation *a* or *p*, or both, indicating whether the thunderstorm reported occurred before or after the noon of the preceding day. These weather maps cover the 24 hour period preceding 8 a.m. of the date of the map, and although the weather conditions a few hours after 8 a.m. of a certain day are usually best determined from the map for the following day, the map for the same day may be of considerable significance in such a case. Similarly, significant data regarding the weather conditions a few hours before 8 a.m. of a certain day may be shown on the map for the following day. Thus, the locations from the triangulation measurements have been shown not only on the map that would seem most applicable, but in certain cases on 2 successive maps. It may be noted that the locations shown follow the rainfall areas, which are, as the maps show, generally the thunderstorm areas.

Although the measurements with the portable equipment made elsewhere than at Cadillac were not extensive enough to correlate with movements of storm areas, the locations resulting from the triangulation measurements lay consistently in storm areas. In the case of the Atlanta tests, in particular, the correspondence between the locations obtained by triangulation and the storm areas indicated by the weather maps was the more marked because of the fewer reported thunderstorms.

Field Strengths

Obviously the horizontal electric intensities measured at the various test locations are not significant as regards the maximum intensities to be expected at these locations. The measurements made do not even provide a reliable conclusion regarding maximum intensities that occurred during the test periods, because the amplifier gains were adjusted to give frequent records. However, the intensities measured

Table III—Maximum Horizontal Electric Intensities

Location of Portable Equipment	Storm Location (Fig. 5)	Maximum Horizontal Electric Intensity, Volts per Mile		Great-Circle Distance, in Miles From Storm to	
		At Whippany	At Location of Portable Equipment	Whippany	Location of Portable Equipment
Cadillac	1	17.4	33.4	491	159
	2	10.5	13.8	583	620
	3	18.0	33.4	262	705
	4	53	31	457	499
	5	53	24	143	579
	6	53	30	62	559
	7	39	34	444	340
	8	46	34	288	470
	9	42	33	242	752
	10	23	32	269	658
	11	15.6	30	292	622
Eau Claire	12	26	20	615	709
	13	17.1	14.7	510	421
	14	19	11.8	545	439
Atlanta	15	23	76	253	748
	16	20	98	522	244
	17	21	98	510	322
	18	24.2	187	897	177
Hagerstown	19	8.8	275	200	69

*Where the test period during which the records indicating one of the storm locations were obtained was less than one hour, table II gives as the time of the measurements the beginning of this period.

during the tests may be of some general interest.

In table III are given the maximum horizontal electric intensities measured at the 2 test points for each storm location. Great-circle distances from the storm locations to the measuring points also are given in this table.

In the case of the Cadillac tests alone, the data are sufficiently extensive to indicate a marked diminution of the field strength as the distance from the storm becomes greater. No consistent relation between field strength and distance would seem likely to exist, in view of the different wave shapes of successive disturbances. Moreover inasmuch as thunderstorms differ greatly in violence, any inferences regarding propagation characteristics of the fields produced by the lightning strokes must be drawn from a comparison of the relative magnitudes of the voltages measured at the 2 test points, without regard to the absolute magnitudes, and taking account of the fact that if the earth resistivity is not the same at the 2 test points, the horizontal intensities at these points will be different even with a storm at equal distances from these 2 points.

Table IV—Ratio of Maximum Vertical Electric Intensity to Maximum Horizontal Electric Intensity, for 57 Records

Range of Ratio	Number of Records Showing Ratio Within This Range
100–150	8
150–200	11
200–250	14
250–300	12
300–350	7
350–400	4
400–450	1

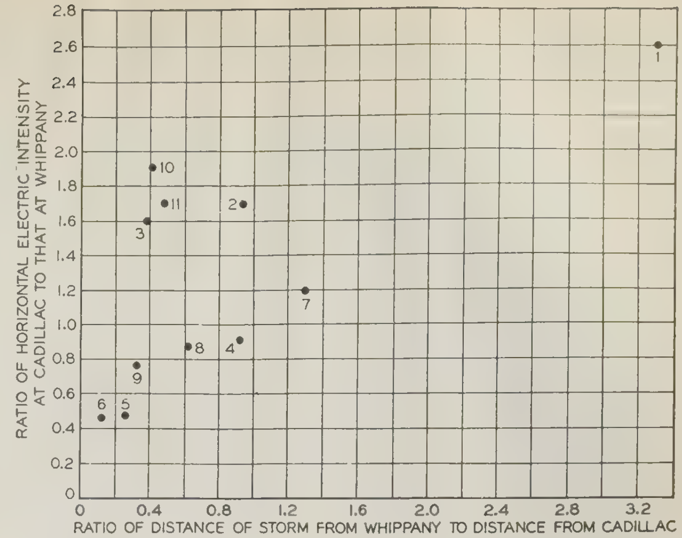


Fig. 11. Relation between field strengths and distances—Cadillac tests

For each of the locations resulting from the triangulation measurements of the Cadillac tests, the average value of the ratio of maximum horizontal electric intensity at Cadillac to that at Whippany is plotted in figure 11, against the ratios of the distances of the storms from Whippany and from Cadillac, respectively. Although obviously it is not possible to infer propagation characteristics from these data, the trend of increased ratio of intensities with increased ratio of distances, as shown in figure 11, shows that the measured field strengths are, in a general way, in accordance with the locations determined for the storms. It may be noted that this same general relationship between distance and field strength exists for the tests other than those at Cadillac.

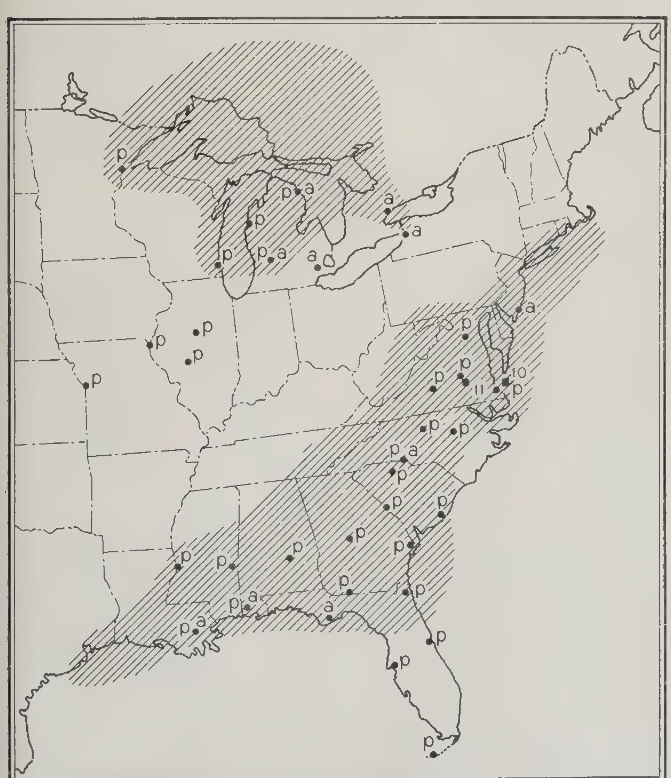


Fig. 10. Daily weather map for July 30, 1934

Quasi-Tilt Angle

As mentioned previously, the measurements at Whippany in which the voltages produced in the 3 loops were compared indicated that the maximum vertical electric intensity generally was large compared with the maximum horizontal electric intensity, but these measurements did not provide accurate quantitative data regarding the ratio of these 2 components of electric intensity.

The ratio of the maximum vertical electric intensity to the maximum horizontal electric intensity could be determined with a fair degree of accuracy from the simultaneous measurements at Whippany with the vertical loops and with the probes; in table IV are summarized values for this ratio from one set of such tests. The pair of films from which the data in this table were derived showed 57 records in which both components of electric intensity were clearly indicated. Table IV gives the number of records, out of the total of 57, that showed ratios between 100 and 150, 150 and 200, etc. The range for this ratio was from 118 to 410, the average ratio being

231. This average value probably is representative of the ratio that obtains generally at this location, although, because of the widely differing wave shapes of individual disturbances, this ratio necessarily is different for different disturbances.

References

1. The Receiving System for Long-Wave Transatlantic Radio Telephony, Austin Bailey, S. W. Dean, and W. T. Wintringham. *Bell System Tech. Jl.*, v. 8, Apr. 1929, p. 309–67.

2. Report of the Radio Research Board (Great Britain) for the year 1931.

3. Report of the Radio Research Board (Great Britain) for the period January 1, 1932, to September 30, 1933.

4. The Cathode Ray Oscillograph in Radio Research, H. M. Stationery Office, 1933.

5. Report of Radio Research Board (Great Britain), January 1, 1932, to September 30, 1933, p. 66.

6. Radio Compass Bearings, Dept. of Commerce Publication, p. 37.

7. Wireless Direction Finding and Directional Reception (a book), R. Keen. *The Wireless World*, London, Eng., 2d edition, 1927, p. 185.

287 Kv Boulder Dam Disconnecting Switches

The severity of desert climatic conditions, the necessity for absolute dependability and for minimum maintenance, and the high voltage to be handled, required departures from established practice in the design of the enormous disconnecting switches for the 287.5 kv transmission lines from Boulder Dam to Los Angeles. This equipment and the considerations entering into its design are described here.

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DURING the engineering development of the transmission line from Boulder Dam to Los Angeles, it was realized that considerable attention should be given to the specification details for the electrical equipment. Characteristics of the switching station and terminal equipment had to be coordinated with those of the transmission line. Physical strength and mechanical operation of the various parts involved had to be considered in connection with the climatic conditions encountered in the desert.

A certain amount of research work was necessary in this connection to establish a basis for the specification. A large part of the data obtained from this work by the Bureau of Power and Light was accepted by the United States Bureau of Reclamation as a basis of the specification for the disconnecting switches at Boulder Dam. The disconnecting switches for this project are of particular interest because of their importance, size, and space requirements.

Many different disconnecting switches previously manufactured, and also several new designs proposed by various manufacturers, were considered. Each design was studied from the standpoint of cost, reliability, electrical characteristics and space requirements. Types deficient in some of these requirements were eliminated until 2 designs were left: the conventional vertical break and the horizontal rotating double break.

During this investigation it became apparent that there was need for the development of an insulator stack that would have sufficient mechanical strength and proper electrical characteristics to meet the requirements. It further developed that more data were necessary on the subject of flashover values, as they affect spacing of vertical and horizontal break disconnecting switches.

Insulation—Physical Properties

Several different combinations of insulator assemblies were proposed, but all were lacking in some particular detail. A tripod arrangement was suggested, consisting of 3 stacks of small units bolted together to form a pyramid. This combination afforded considerable strength in bending, but was considered deficient in torsional strength, and required excessive space at the base as well as numerous special fittings for adaptations to the switch and its base. In addition, tripod stacks were considered very difficult to protect against cascading during flashover. Other arrangements of parallel stacks and suspension insulator strings were suggested, all of which offered similar difficulties.

The largest pillar insulator available was a unit having a bolt circle of 7 inches and a height of $14^1/_2$ inches. Eight of these units, when bolted together, offered a stack height of 116 inches. This height was sufficient to give the required electrical properties, but the combination, although ample in torsional strength, did not offer a sufficient factor of safety in bending. A calculation of the operating, wind, and earthquake stresses indicated the need for an insulator column that would have a cantilever strength of at least 140,000 inch-pounds at the base. To obtain this combination, it was believed that a stack of 8 insulators having a 10 inch bolt circle would be required, and therefore the insulator manufacturers were requested to develop a unit having the required characteristics. A combination

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was developed consisting of 5 $14^1/_2$ inch (high) pillar insulators at the top of the column having a 7 inch bolt circle; one intermediate $14^1/_2$ inch pillar insulator having a 7 inch bolt circle at the top and a 10 inch bolt circle at the bottom, and 2 $14^1/_2$ inch heavy duty units at the bottom having 10 inch bolt circles. This combination when assembled in the stack tested to an ultimate bending strength in excess of 140,000 inch-pounds, in torsion in excess of 90,000 inch-pounds, in compression 75,000 pounds, and in tension 20,000 pounds.

Sixty cycle wet and dry flashover and impulse tests were made on this stack at the laboratories of the Ohio Brass Company. Inasmuch as cascading is more apt to occur when flashover takes place with impulse waves having a short time lag as a result of excessive overvoltage, tests were made with a $1^1/_2$x40 microsecond wave with the voltage so adjusted that flashover would take place in 3 microseconds.

The first tests were made on a stack equipped with no arcing or shielding devices whatever. The arc cascaded the stack in all cases, principally over the lower units, as shown in figure 1. Following this, rings of various diameters were attached to the top and bottom of the stack in order to determine the size and position required to prevent cascading without appreciably reducing the impulse flashover value of the stack. Tests were not made with rings having a diameter greater than 5 feet. It was believed that a limit of 4 feet should be placed upon the diameter of the arcing rings in order to conserve space in the final assembly of the disconnecting switch. Tests using rings 4 feet in diameter located level with the top and bottom skirts of the respective top and bottom units of the stack assembly showed all flashovers clearing the units. Electrical characteristics almost as efficient as those obtained with this combination were obtained on the stack without the rings. (Figure 2.)

60 Cycle Requirements

It was necessary to know the relative spacing and flashover values of the 2 most favorable types of disconnecting switches selected; namely, the conventional vertical break and horizontal rotating double break. To insure protection to life and property on the open side of the disconnecting switch, it was decided that the flashover voltage across the open switch should be at least 10 per cent greater than the flashover voltage of the supporting insulator stacks. To give ample protection, this distance between energized parts or grading rings was tentatively set at 11 feet. The performance of single air gaps is relatively well known for various weather conditions. There was some doubt, however, as to the action of the double break switch when the center stack was contaminated and wet with fog or mist, as can occur when least expected.

To determine these things more definitely before a final specification could be released, it was necessary to conduct certain tests. As the Ryan high voltage laboratory at Stanford University offered the highest 60 cycle voltage obtainable on the Pacific Coast, it was there, with the co-operation of Dr. Carroll of Stanford, that the 60 cycle tests were made. The adequacy of the 11 foot separation between the line and grounded stacks of the vertical break disconnecting switch was first determined. An insulator assembly of 8 units equipped with 4 foot arcing rings at top and bottom having a separation of 109 inches was first tested and the 60 cycle flashover voltage of the

Fig. 1. Flashover of combination insulator stack without arcing rings

1 x 40 microsecond wave used, with flashover taking place in 3 microseconds; cascading occurred over the bottom units

Fig. 2. Flashover of combination insulator stack with 4 foot arcing rings

1 x 40 microsecond wave used, with flashover occurring in 3 microseconds; all flashovers cleared the units

stack was determined. Two of these insulator stacks equipped with arcing rings then were set on an appropriate base, separated approximately 14 feet, one stack serving as the line side of the disconnecting switch and the other as the grounded side. Sufficient additional insulator units were added to the stack on the line side to prevent flashover at 10 per cent overvoltage. The voltage then was increased by 10 per cent above that which flashed the 8-unit stack, as was considered requisite, and the separation between rings on the line side and the grounded side of the switch was decreased until the 110 per cent voltage was just held by the interposed gap. As the distance necessary was found to be 10 feet, the 11 feet tentatively established for this separation had a slight factor of safety.

Each pole of the horizontal double break switch under consideration consisted of 2 stationary stacks, one at each end of the base, connected by a blade carried on a center rotating stack. To determine the effect of flashover when the center rotating stack was contaminated, a $1^1/_2$ megohm shunt

resistance was placed across the center stack to ground.

The separation between the arcing ring of the line stack and arcing ring of the center stack was in-

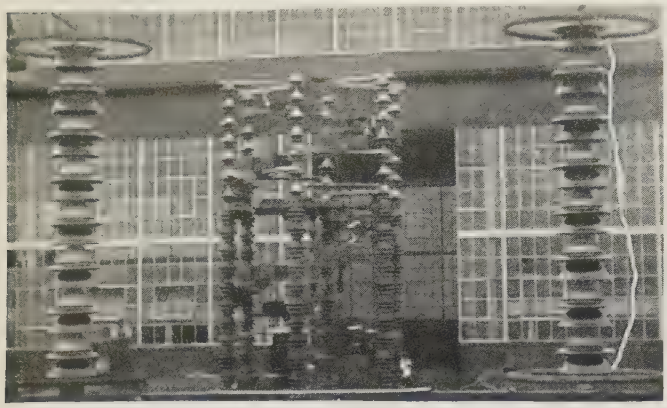


Fig. 3. Flashover with 2 insulator stacks

One ring energized and the other ring grounded; stacks placed sufficient distance apart to insure flashover to ground; 1 x 40 microsecond impulse wave used, with flashover in 3 microseconds

creased until the 110 per cent of the voltage required to flash the line stack would just flash to the center stack. This distance was 9 feet 9 inches, or 3 inches less than that required for the single vertical break switch. This voltage generally was sufficient also to flash the center stack and, when the spacing to the third or grounded stack was less than 8 feet, this second gap would flash to ground. If a switch were built with the factor of safety required on a 60 cycle basis and with arcing rings of 4 foot diameter on all insulator posts, the necessary center-to-center distance of a single pole unit between outside posts would be approximately 28 feet.

Similar tests were made with the center post dry and clean and without leakage, a condition which should have less disturbing effect on the voltage distribution between the 2 gaps. The spacing between the 2 arcing rings to withstand the 110 per cent voltage was found to be 7 feet 6 inches, thus the necessary separation between centers of the outside posts of a horizontal double break switch under ideal conditions would have to be 7 feet 6 inches, plus 7 feet 6 inches for the 2 gaps, plus the 8 feet for the arcing rings, or a total of 23 feet.

The transmission line was insulated for a clearance to ground of 7 feet under wind conditions, against normal switching surge voltages. Since it was required that the disconnecting switches, when open, should insulate against 110 per cent normal switching surge voltages, the separation between the adjacent insulator stacks for one pole of a double break switch would still have to be practically the same as already determined, or 23 feet. Each gap would need to be slightly more than 7 feet.

The spacing between phases tentatively had been established at 22 feet because of the limited space at the Boulder switch yard. The minimum arm length for a double break switch would be the 2 gap lengths, plus the diameter of one ring, or 19 feet. Arms of these dimensions could not be rotated without practically touching at the 90 degree position. A 45 degree to 60 degree position would have to establish the opening position of the switches. Tests were made on 2 poles of the switch with the blades set at approximately the 60 degree position and with 18 feet between arcing rings of adjacent phases (22 feet center-to-center of phases), and with each gap in each pole unit of the switch set at 7 feet 6 inches. The 60-cycle 3-phase voltage was applied to the line terminals and for this condition flashover would always occur to the center posts rather than between adjacent phases, thus establishing the adequacy of the 22 foot phase-to-phase operation.

The result of the 60 cycle tests clearly indicated that considerably more floor space was required for the horizontal double break than for the single vertical break disconnecting switch. It also indicated that the vertical break switch from an electrical standpoint afforded a higher factor of safety under all climatic conditions.

Impulse Requirements

The same conditions were set up at the Ohio Brass Company's laboratory to determine the impulse characteristics of the 2 types of disconnecting switches. All flashover tests were made with the impulse generator adjusted for $1^1/_2$x40 microsecond waves, breakdowns being made to occur within 3 microseconds. In order to check the minimum horizontal dimensions of 10 feet for the vertical break switch found to be necessary by the 60 cycle tests, 2 stacks equipped with 4 foot rings were tested under impulse conditions at varying separations until

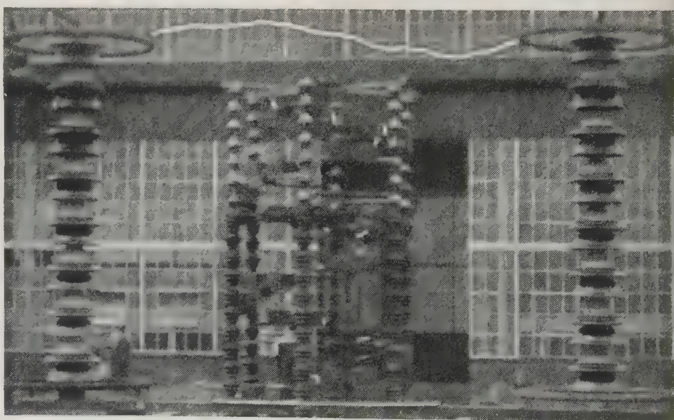


Fig. 4. Flashover with 2 insulator stacks

Conditions same as in figure 3 except insulator stacks moved together until flashovers occurred equally well between rings and across the energized stacks; 1 x 40 microsecond impulse wave used, with flashover in 3 microseconds

flashovers occurred half horizontally between stacks and half vertically down the energized stack. The top ring of the other stack was grounded in all cases. It was found that a horizontal spacing of 109 inches between the nearest edges of the rings caused this

equal division of flashover paths. Figures 3 and 4 illustrate these conditions. As this spacing is appreciably less than the minimum spacing of 10 feet established by the 60 cycle flashover condition, in this case the 60 cycle condition is the controlling factor.

To determine the possible effect upon flashover voltage of 2 stacks being closely mounted, as is necessary at the hinge-end of the conventional vertical break disconnecting switch, a special double stack test was made. In these tests oblong rings were used with parallel sides and radii of 2 feet at their ends in order to agree with the conditions associated with the use of rings of 4 foot diameter on the single stack test. The upper oblong rings were mounted in the same position as the rings on the single stack. Spacings ranging from 2 to 3 feet between the center lines of the parallel stacks were tried, and in all cases flashovers occurred between rings without cascading, and no measurable reduction in flashover voltage was noted from that obtained with the single stack.

To check the minimum dimensions for impulse flashover of the horizontal double break switch, 3 pillar stacks were mounted on the test rack, all equipped with 4 foot rings. The horizontal separation between rings was first set at 6 feet. With approximately 20 impulse waves applied to the line stack, flashover was found to occur in all cases through a vertical path to ground over the energized stack. The stacks then were moved together so as to give a separation of 4 feet 10 inches between rings. Again impulse waves were applied and all discharges occurred to ground over the line stack. From these tests it was obvious that the 7 foot minimum spacing as dictated by the 60 cycle flashover tests was ample to take care of the conditions to be guarded against from impulse waves; namely, a discharge over an open switch.

Realizing the leakage over the center stack caused by contaminated conditions of the insulation would have little effect on its flashover value for short-time-lag impulse waves the same tests were repeated, using 1x100 microsecond waves, with voltage just sufficient to produce flashover. A 6 foot spacing between rings of the line and center stack would balance the flashovers between line and center stack and from line to ground. To obtain the 10 per cent safety factor required would have necessitated approximately 6 feet 6 inches, approaching the requirement of the 60 cycle test.

Specifications Established

The impulse tests served as a check, but in every condition the 60 cycle flashover conditions governed the horizontal spacing between terminals and between phases. As a result of these tests, the horizontal double break type was eliminated and the proper spacings were established for the single vertical break type of disconnecting switches ultimately specified for the transmission line switching facilities and the Boulder power plant switch yard serving the line.

To reduce maintenance and improve reliability, ball and roller bearings were specified throughout the switch. Because of the high temperatures that occur and the wear caused by the blowing sand of the desert, it was required that the bearings be designed to retain the grease and prevent the entrance of dust or sand.

High pressure contacts were specified, but it was required that the pressure be released before any movement of the blade should occur. Full contact and current carrying capacity was required for a motor mechanism travel tolerance of 15 per cent, $7^1/_2$ per cent overtravel and $7^1/_2$ per cent undertravel. It was specified further that parts subject to line potential be designed to obviate visible corona discharge in complete darkness when energized at 200 kv to ground with the switch in either the open or closed position.

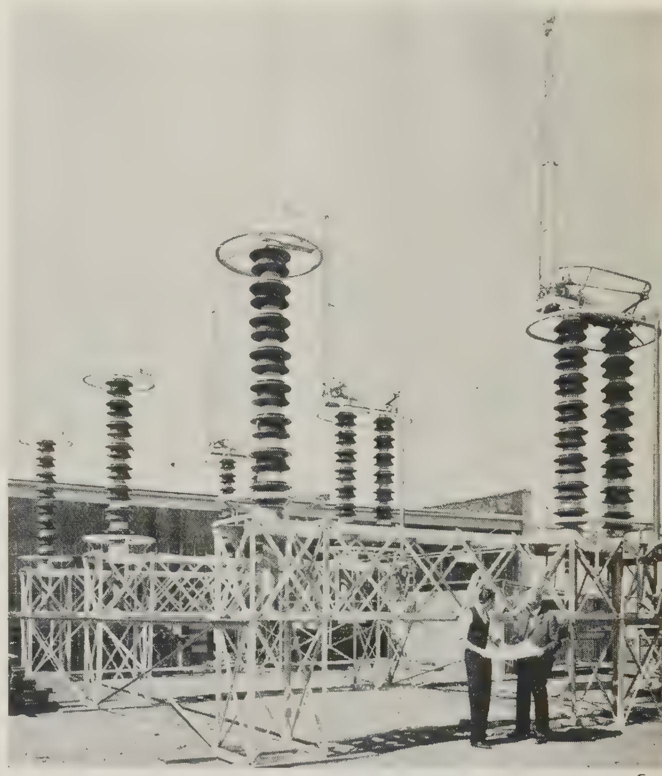


Fig. 5. A 3-pole 287-kv disconnecting switch set up for factory tests

All the switching equipment in the switching stations on the transmission line, and the terminal facilities, will be operated by carrier current supervisory control. Because the time of operating each piece of equipment is cumulative, it was necessary that the disconnecting switches operate from full open position to full closed position in not to exceed 10 seconds.

All the disconnecting switches now have been built, tested, and installed. The disconnecting switches purchased for the switching stations were designed and built by the Delta Star Electric Company while those for the switchyard at Boulder Canyon power plant were designed and built by the Bowie Switch Company. Each type of switch has been tested, in accordance with the specifications, and the test

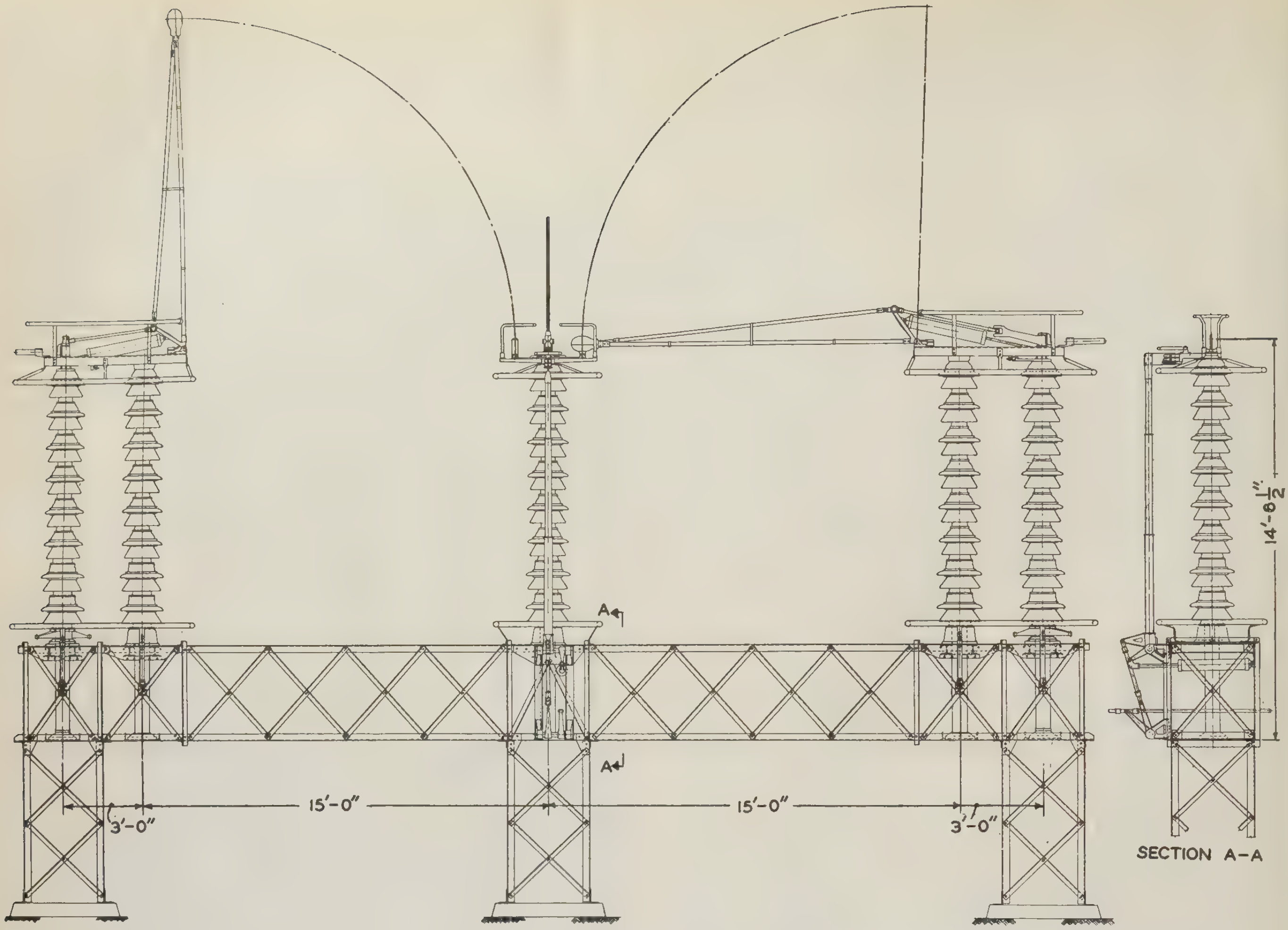


Fig. 6. A 287-kv selector switch

results as well as the finished product are very gratifying.

Boulder Dam Switches

The only 287 kv disconnecting switch installation at Boulder Dam comprises 12 3-pole gang-operated switches. (Figure 5.)

The original requirements were for all switches to be of practically similar construction, the base for each individual pole to be mounted on 2 supports. Some of the switches have gang operated grounding switches. All the main and grounding switches were to be operated by individual motor mechanisms. It was possible to effect a material saving in space, as well as in cost, by making some of the switches of the selector type, each single pole base consisting of 5 insulator pedestals, with the central pedestal supporting the clips with which the blades on each side contact. (Figure 6.) Consequently, it was decided to use 2 selector switches, each comprising 2 3 pole independently operated gang switches.

As the blades of the switch were about 13 feet in length, it was important to minimize both their weight and wind resistance. To insure the proper contact operation, the blade position with reference to the clip when the switch is closed should not vary more than $^1/_2$ inch from the desired position. With the multiplicity of moving parts, it was essential to provide a structure with exceptional rigidity and lightness of blade, and to eliminate practically all lost motion. For this reason, all bearings were of the precision ball or roller type, provided with weather seals and "alemite" lubrication. This construction resulted in smooth operation entirely without shock, although the speed of operation was 30 per cent faster than specified.

There was a time in the past, when the voltages to be switched were lower, when practically any type of contact and operating mechanism was considered to be sufficient. With higher voltages and larger power installations, there came a demand for high grade construction involving accurate engineering design. High voltage switches now require a carefully considered structure, with a precision in the manufacture of many parts quite comparable with that obtaining in high grade automotive work. The failure of any one part of the switch might involve an untold amount of trouble and damage, and it was particularly essential in the Boulder Dam project to employ a thoroughly deptndable construction immune from failure.

It is true that the switch operation may be quite infrequent, but satisfactory operation under this condition often is more difficult of attainment than where frequent operation is required. The requirements

for assured operation and perfect performance of outdoor equipment exposed continuously to the elements, but receiving little or no attention, seldom are given sufficient consideration.

Corona

To prevent radio interference in the Boulder Dam area, where radio reception means so much, no visible corona was allowed when as much as 200 kv was applied between the switch parts and the ground. A great deal of special design was necessitated to meet these requirements. The high voltage tests of the assembled switch were made at Stanford University and did not show visible corona until the voltage from line to ground was raised to 240 kv, or 20 per cent above the specified voltage.

Each insulator stack was equipped with 2 inch pipe rings, one at the top and the other at the bottom. On the hinge insulator, which was adjacent to the rotating insulator, single oblong rings were furnished surrounding the 2 insulators at the top and bottom, whereas, with the clip insulator, rings for both ends of the insulator were circular. On the hinge-end of the switch an additional upper ring was provided, oblong in shape and having a single gap, with balls on the ends, to provide space for the passage of the blade. This ring protected all the upper parts of the hinge and rotating insulator construction. On the clip-end, additional small horizontal rings were used merging into 2 vertical legs to protect the tongue-type clip when the blade was open, the vertical part serving as a guide for centering the blade.

The blade-end, consisting of 2 parallel busses which contact the clip tongue, is provided with a spheroidal corona shield of high strength aluminum alloy made in 2 sections, the bottom section being provided with swinging doors normally closed when the blade and clip are disengaged, but which allow the entrance of the clip tongue. This very effectively prevents corona and also acts as a sleet and dirt shield. A great multiplicity of tests were required to meet the corona specifications, in view of the different grounding clips, single and double switches, varying directions of the incoming conductors, etc.

Insulator Pedestals

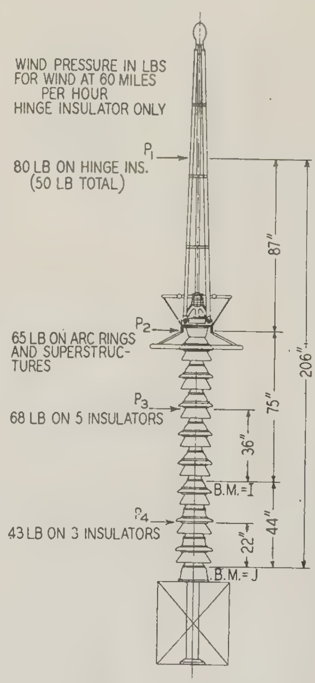
Both mechanical and electrical considerations required the development of new insulators. One novel feature of the insulator units was a porcelain ring projection, integral with the top shell and concentric with the cap, that permitted a stack voltage considerably higher than allowable with conventional designs.

Especially for the rotating insulator where the top and bottom shafts had to align, the mechanical accuracy of alignment in the cementing of the insulators was of particular importance to secure a stack which would be true to form. The results were most gratifying, as the first pedestal set up aligned without adjustment of any kind within $^{1}/_{32}$ inch.

In practically every outdoor installation, consideration should be given to the climatic conditions under which the equipment must operate. As the temperatures at Boulder Dam in the summer are very high, it was essential to provide equipment conservatively rated, and that would not be damaged by the ambient heat, plus the necessary rise of temperature of the current carrying parts.

Earthquake Stresses

Until recently, no consideration had been given to the possible stresses occasioned in electrical switches by earthquakes. It is manifestly advisable in many sections of the country to make adequate provision therefore in the strength calculations. Where equipment rests on ground that will yield and flow it is often impossible to have an adequate basis of figuring stresses. However, in the case of the Boulder Dam site, where the ground is solid, the stresses in any structure are due to direct earthquake vibration, which results in a horizontal component directly proportional to the product of the weight times the acceleration due to earthquake. The proportion of the weight acting horizontally is measured by the ratio of the maximum acceleration divided by the acceleration of gravity. (See Appendix I.) The vibration rate of earthquakes generally is low, perhaps of the order of one second, whereas the corresponding vibration periods of structures employed in switches is much higher. Such frequencies are so far apart that resonance usually is not a factor and the horizontal weight component generally will suffice.



	WIND ONLY	QUAKE ONLY	INSULATOR STRENGTH B.M.	FACTOR OF SAFETY WIND	FACTOR OF SAFETY QUAKE
P_1	30	12.6			
P_2	65	60			
P_3	68	120			
P_4	43	84			
I	12,183	10,845	86,000	7.1	7.9
J	20,301	21,161	139,000	6.9	6.6
B.M. AT BOTTOM OF INSULATOR FROM WIND ON BLADE ONLY = 6180				22.5	
BASIS OF STRESS					
FLAT SURFACES				17 LBS SQ FT	
ROUND SURFACES – PROJ. AREA				8½ LBS SQ FT	
FROM QUAKE				20% OF WT.	

Fig. 7. Diagram of switch showing stresses

Stresses on hinge insulator from wind at 60 miles per hour; earthquake stresses at 20 per cent horizontal component

Whereas a 10 per cent component is frequently used, the buildings for the San Francisco high pressure fire system allowed for a 20 per cent component, and the latter figure was assumed for switches for Boulder Dam.

Wind stresses were specified for a velocity of 60 miles per hour, with a pressure of 17 pounds per

square foot of projected area for flat surfaces and half of this for cylindrical surfaces. Under this assumption, a 60 mile wind allows a factor of safety of about 7 at the bottom of the insulator stack, while with a 20 per cent horizontal weight component the factor of safety is about the same. (See figure 7 for stress diagram.)

Switch Contacts

In the design of the contacts it was desirable to provide a construction that as far as possible would be immune from the necessity of servicing after installation. For this reason the contacts were constructed of heavy wrought silver, made integral with the copper base contact bars. A large area of silver contact was provided to eliminate the mechanical and electrical difficulties liable to occur where line or point contacts of small area are employed. The contacts are engaged under only a light and adjustable pressure, but after they have come into full engagement and relative motion has ceased, a pressure of approximately 500 pounds is applied, clamping them rigidly in place. In opening the switch, the high pressure is relieved entirely before the contacts start to move.

In the past, most switch contacts have been made of copper, particularly for outdoor switches. In a few instances silver contacts have been used. Both these metals, of course, will oxidize from exposure to the weather, resulting in an increase in contact resistance far greater for copper than for silver, for copper oxide is an insulator, whereas silver oxide is a conductor. Pure silver being somewhat softer than copper, it is not advisable to move the silver contacts under any material pressure if damage to the surfaces is to be prevented. A light wiping pressure is desirable for contacts to clean the surfaces, but heavy pressure should not be applied to either silver or copper while the contacts are moving.

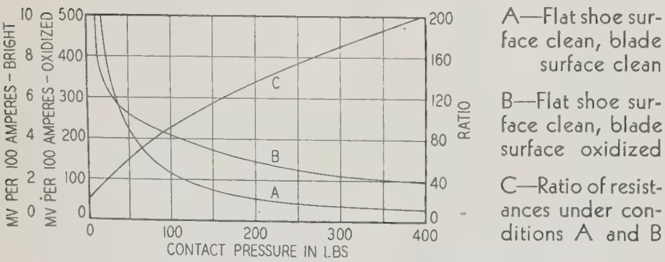


Fig. 8. Relation between resistance and contact pressure for copper

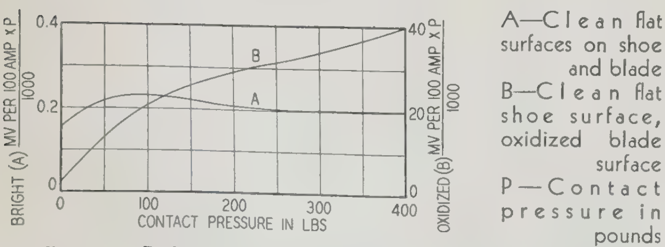


Fig. 9. Relation between pressure and the product of pressure times resistance

Numerous tests have shown that the contact resistance for similar surfaces of clean conducting metals varies approximately inversely with the total applied pressure. Figure 8 shows the effect of pressure in diminishing the contact resistance between clean surfaces, and corresponding tests where the blade of wrought copper was oxidized by exposure to the weather for about a year. Figure 9 shows corresponding results to another scale, giving the relation of pressure to the product of resistance times pressure. Further tests have shown that with a given pressure there is materially less resistance when the contact is of appreciable area than will obtain with a line contact having the same total pressure applied. Since the resistance of any contact will increase in time from oxidation, it is desirable to have high pressure applied to the contacts after engagement to avoid future servicing. The efficiency of these contacts is shown by the fact that they had a rise under full load test of 16 degrees centigrade although 30 degrees was permissible. The high pressure with silver contacts of large area has practically eliminated contact resistance, the rise of temperature under load being caused mainly by the heating of the blade conductors.

Blade Construction

The relatively large size of the blades and of the general construction necessitated special care in mechanical design to eliminate distortion and to

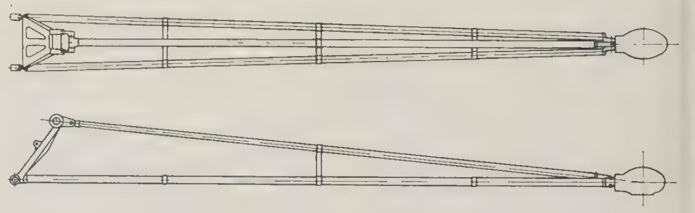


Fig. 10. Typical triangular switch blades

keep down the wind and earthquake stresses as well. The approximate pedestal height is 10 feet and the blade length 13 feet, making a total height above the insulator base of about 23 feet when the switch is open. The switch blades, which are exceptionally light, are made in tripod form of 3 pieces of tubing rigidly joined, forming a very stiff, light structure particularly free from corona. The 3 cylindrical tubes act like a cage in distributing the electrostatic stress, avoiding local concentration. (See figure 10.) Each blade is counterbalanced by a housed compression spring, so accurately designed that the blade will remain in any position of the stroke with the connecting rod disengaged.

As it is highly important to eliminate lost motion, it is essential to keep the blade constantly under positive control. In the closed position of the switch, the top crank of the operating insulator, which operates the blade through a connecting rod having universal joints at each end, is practically on dead center, thus bringing the blade to rest like an engine piston at the end of its stroke. When the top crank is

near dead center there is no appreciable motion of the blade, and advantage is taken of this fact to have the high pressure applied to the contact through a roller carried by the top cap of the moving insulator. This roller sets the high pressure through a link motion which operates through a rod running between the ends of the blade. It is particularly essential, where high speed of operation is required, to maintain the blade at all times *positively in definite control*, since, if the blade be freed from the operating mechanism, there will be probable slamming from the sudden stopping of the moving 13 foot blade.

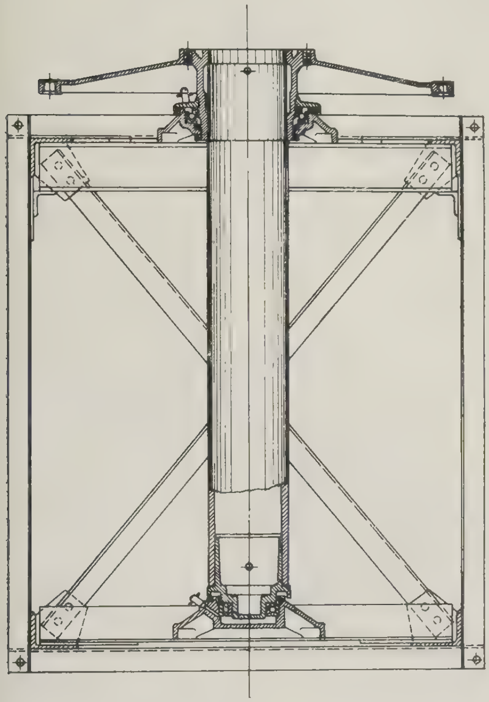


Fig. 11. Base crank and base bearing for rotating insulator

The framework, which is 42 inches deep, is very stiff although of light steel construction, and has the bending stresses at the bases of each insulator column carried down in cantilever through large steel tubes between the top and bottom frames. Figure 11 shows the base crank and base bearing for the rotating insulator.

The ground switch blades consist of tapered tubing, each blade being provided with its own counterbalance. An independent rockshaft, the crank of which is on dead center when the switch is closed, is used for operating the ground blade, insuring a very definite blade position when the blade and clip are closed and preventing undue operating stresses in disengaging the blade from the clip.

Motor Operators

The operating mechanism, which is mounted on one of the switch towers, will operate the switch in $7^1/_2$ seconds, although a limit of 10 seconds was permitted. The operator is driven by a $^1/_2$-horsepower 220-volt motor connecting to the operating shaft through a transmission consisting of triple reduction spur gearing. The operator is provided with 3 hinged steel doors and is exceptionally accessible. (See figure 12). Mounted in the box are relays that insure continued operation once the switch starts to move. Different types of interlocks as specified are provided for interlocking the mechanism in various ways. The brake is set mechanically by cams operating a few degrees from the dead center of the top crank, at which time the mechanism is doing no appreciable work in operating the blade.

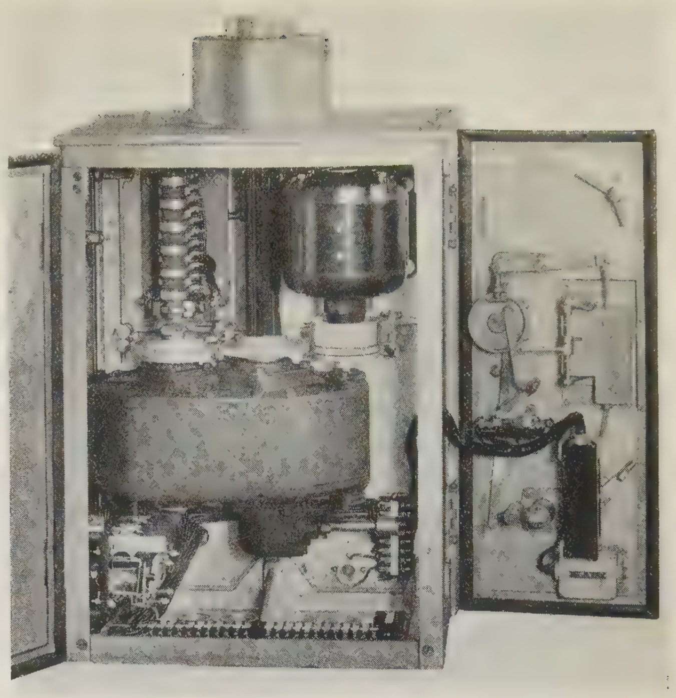


Fig. 12. Typical motor operator

An important aim in the design of this equipment has been not only to provide easy and simple means of servicing where necessary, but also to eliminate as far as possible any necessity for servicing where inevitable physical conditions render it inconvenient.

Appendix I

The matter of protection against earthquake stresses has been called into prominent consideration by damage in structures incurred recently in some parts of the country, and by the definite knowledge that in these districts ordinary precaution demands provisions to meet such contingencies. In most cases where the damage from earthquake does not occur directly over a fault, the resultant damage is occasioned by the secondary motion of the earth. Where the ground is solid such stresses are a minimum, but where there is an alluvial soil the stresses are greatly intensified by the further movement and settling of the loose ground. The worst conditions occur with a soft fill on a sloping foundation where the ground will both flow and settle. The earthquake in San Francisco in 1906 was of such exceptional severity that the results thereof have a bearing on the probable stresses in structures; they furnished an important basis for the design of an independent high pressure fire system subsequently built in San Francisco.

The data obtained in San Francisco may be applied equally well to other structures, such as those used in high voltage switch work. In many places it is common to assume a maximum earthquake stress of 10 per cent of the weight; namely, a 10 per cent horizontal component. This would correspond to the maximum acceleration of 3.2 feet per second per second. The general stresses set up in a structure on solid ground consist of such a component which may be increased by vibration due to resonance.

Tests on Oil Impregnated Paper

About 2 years ago a series of researches was started to study the causes of dielectric loss, chemical deterioration, and electrical failure of oil impregnated paper insulation. Miniature technique has been developed so that at moderate cost specimens can be assembled in glass under accurately controlled conditions, subjected to high voltage life tests, tested periodically for electrical properties and finally examined minutely for changes in electrical, physical, and chemical properties. The results obtained in the first part of this program are presented herewith. Hypotheses now held regarding the effects of variations in several different factors on the life of oil impregnated paper insulation are discussed, and a description of the techniques developed for preparing and testing specimens is given. Illustrative examples are given of the results obtained to date from the correlation of life tests with microscopic physical and chemical examination.

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THE COMBINATION of specially treated mineral oil and wood pulp paper from which water, oxygen, and other contaminants very carefully have been removed makes the best flexible high voltage insulation yet discovered. The major application of such insulation is in high voltage underground power cables, although the same components are used in other apparatus, such as high voltage capacitors. An understanding of the causes of the excellence as well as the failure of such a combination of materials is a major factor in making improvements and requires many types of investigation. One type of investigation involves building full sized apparatus and testing it to destruction. Such tests are very expensive and, therefore, are applied usually to proving the quality of designs for which experience indicates that the probability of successful operation will be very high.

At the other end of the scale of types of investigation are researches in which miniature specimens made in the laboratory under the most carefully controlled conditions are subjected to much more severe tests than ever will be expected of apparatus in service. Such experiments are much less expensive than the large scale tests; thus, the number of combinations of materials and conditions that can be investigated economically is correspondingly much greater.

In this paper are described methods that have been developed for making small laboratory oil-impregnated paper test specimens, subjecting them to high voltage life tests, observing by electrical measurements any progressive changes during life, and finally examining the specimens minutely for physical and chemical changes which might be indicative of the causes of failure.

Short-time dielectric-strength tests are the quickest and easiest quality tests that can be applied to electrical insulation. However, such tests are of doubtful value in predicting service life on systems having *differing physical conditions.* For example, the so-called "solid" and "oil filled" types of cables of the same insulation thickness will have about the same short time dielectric strength when new. However, in actual service the "oil filled" cable will operate at double the electric stress that can be tolerated in the "solid" type.

Another test that often is used to evaluate insulation quality is to measure a-c dielectric loss (usually interpreted by the engineer in terms of power factor or tangent of the loss angle). This again is no measure of probable life unless all other conditions are identical.

Without citing further examples, it generally is admitted that no quick measure of insulation quality has yet been developed that is a sure indication of service life. For most insulation systems, service experience on similar systems is the major guide to the probable life of a new design. However, accelerated life tests on short lengths of commercial cables have been found to correlate fairly well with service. This paper suggests that accelerated life tests on miniature oil impregnated paper specimens can be used as an additional tool to compare the behavior of different systems and to study causes of failure.

Theories and Hypotheses

Some of the statements listed hereinafter are founded upon considerable experimental evidence and therefore may deserve to be called theories. Others have very meager experimental proof and have been advanced as explanations of a few obser-

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The information reported in this paper is obviously the results of the co-operative efforts of many individuals. In particular, the writer wishes to express appreciation to those who have been responsible for important phases of the work, namely: S. I. Reynolds—preparation of specimens, preliminary and post mortem electrical measurements; H. M. Bousman and C. Zubal—life tests; J. L. Oncley and A. F. Winslow—dissection of specimens, and acid and hydrophil measurements; R. W. Dornte and C. V. Ferguson—molecular weight and unsaturation measurements; Mrs. A. P. O'Hara—spectrographic determinations; and F. A. Benford—photometric evaluation of spectrograms.

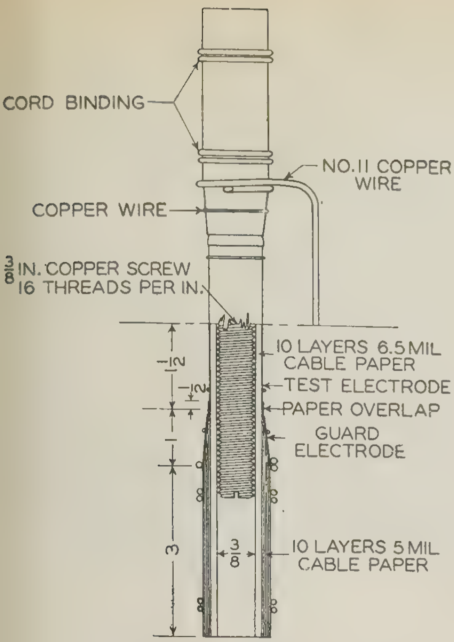


Fig. 1. Impregnated paper test specimen for electrical life test

All dimensions are in inches; diagram is not to scale

vations. Many of these conditions are conflicting or coexistent so that the conditions under which each factor has a dominant influence on life must be determined. An attempt has been made to list the factors that may be of major importance in determining the quality and behavior of impregnated paper insulation. It is hoped that experiments may be devised to isolate single experimental variables and determine their relative importance, and thus either to confirm or disprove present conceptions, thereby laying the groundwork for new designs and better service. In reviewing these theories and hypotheses the reader should consider that each statement is preceded by the thought "other conditions being the same. . . ."

1. *Electrical failure (breakdown) of impregnated paper insulation is preceded by gaseous ionization* so that:

(*a*). In a gas saturated system the dielectric strength should increase with the gas pressure.

(*b*). In a nominally gas-free liquid-filled system the dielectric strength should increase with the hydraulic pressure and with the degree of degasification.

(*c*). In a nominally gas free system the presence of hydrophilic materials in the impregnating liquid should aid the wetting of the solid by the liquid components, thereby increasing the dielectric strength of the composite insulation because of the decreased tendency toward ionization in gas spaces at the liquid-solid interfaces.

(*d*). In a nominally gas free system the dielectric strength of the liquid component is usually lower than that of the solid component; therefore, the dielectric strength of the composite insulation is higher the nearer the ratio of the dielectric constants of the liquid to the solid component approaches the inverse ratio of their dielectric strengths.

2. *Gas evolution and electrical breakdown are caused by the generation of heat in localized regions* so that:

(*a*). The dielectric strength should increase if the temperature be decreased, or if the average dielectric loss be decreased.

(*b*). Dielectric loss can be decreased by eliminating ionic contaminants if the source be ionic conduction or by eliminating polar constituents or changing the frequency-viscosity ratio away from resonance conditions if the source be molecular orientation.

3. *The life of composite insulation depends upon its chemical and physical stability* so that:

(*a*). The chemical constitution of the component materials and the methods used in processing them should be important factors in

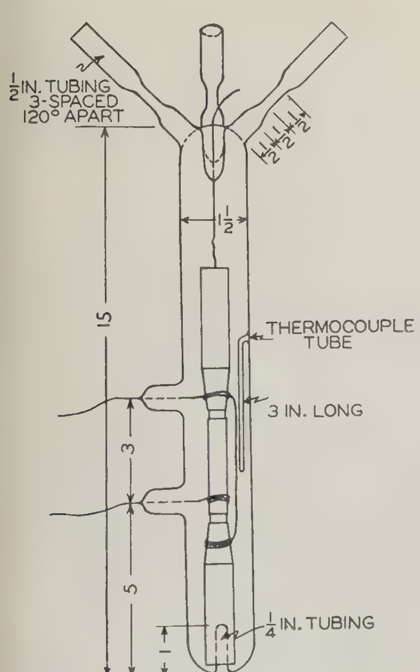


Fig. 2. Gas saturated cell (Type A) for electrical life test on impregnated paper specimens

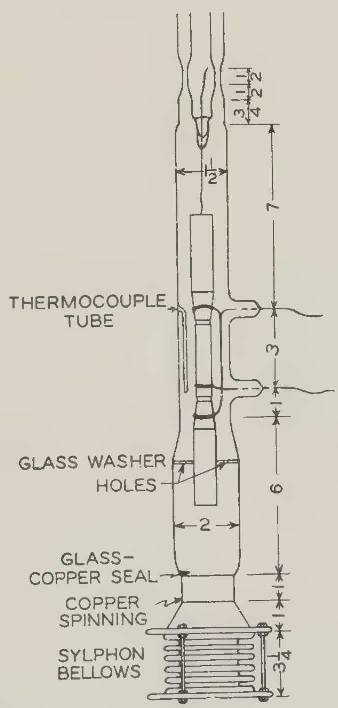


Fig. 3. Gas free cell (Type B) for electrical life test on impregnated paper specimens

All dimensions are in inches

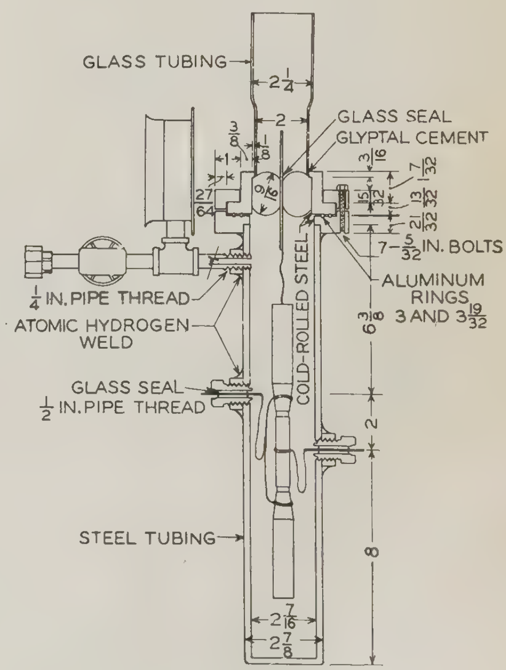


Fig. 4. High pressure cell (Type C) for electrical life test on impregnated paper specimens

their ultimate stability, because different organic materials when subjected to heat and electric stress decompose or polymerize at different rates.

(*b*). If the presence of limited amounts of oxygen is inevitable in practice, the use of oxidation inhibitors should be beneficial if they do not affect adversely any other property, such as conductivity or the ultimate chemical stability.

Test Specimens

Figure 1 shows the detailed construction of a test specimen. These specimens are made to accentuate conditions in a high voltage cable. Because of possible catalytic or chemical action, the inner electrode is made of copper and the outer of lead. The copper rod is threaded to accentuate the effect of regions of high electrical stress at the outer edges of the threads and to provide oil channels at their base. The edges of the threads are carefully smoothed and freed from copper slivers by using emery cloth and high speed wire brushes. The lead foil outer electrode is rolled from standard cable sheath lead.

The usual thickness of insulation in the test section is about 0.065 inch obtained, for example, by wrapping as tightly as possible 10 layers of 0.0065 inch paper. If paper of a different thickness is to be tested, the number of layers is changed to give the same total thickness. Clean rubber gloves are worn during assembly of a specimen. The ends under the guard rings are built up with additional layers of

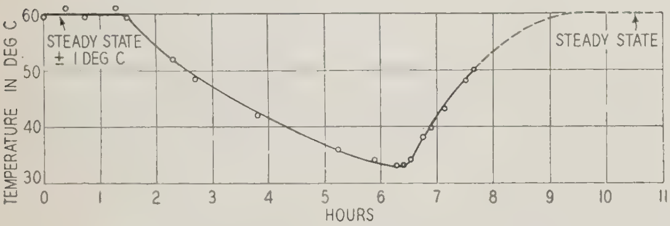


Fig. 5. A typical daily temperature cycle for life test specimens

paper so that the maximum stress is in the measuring section and so as to prevent creepage over the ends.

The design of this specimen has been found to be very satisfactory. About 50 specimens have been examined to date and all failures occurred within the test section.

Cell Construction

In order to prepare and keep the specimens under carefully controlled conditions they are assembled in glass cells of one of 3 types depending upon the gas pressure to be maintained.

Type A: Gas Saturated Cell (15 Pounds per Square Inch Pressure). The first type of glass cell is shown in figure 2 and is used to determine the effects of different gases at about 1 atmosphere pressure on the electrical properties of oil impregnated paper. A mercury manometer is sealed to one of the side arms at the top. Completely degasified oil is admitted under vacuum to cover the specimen. Then the de-

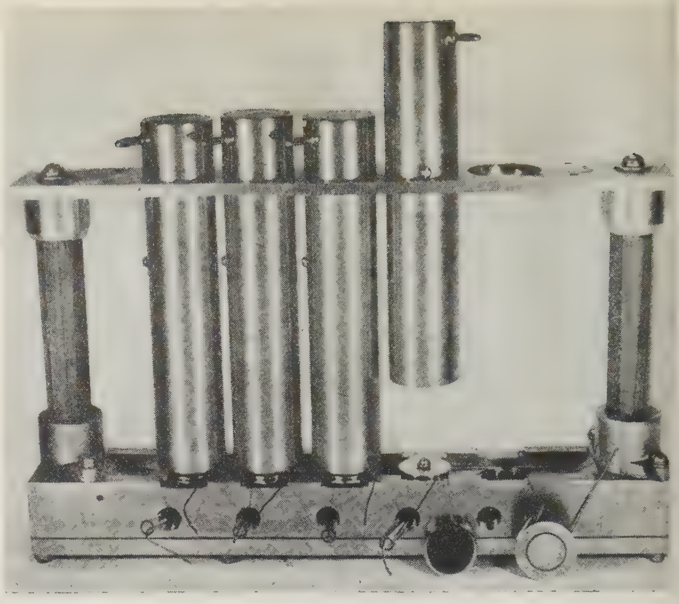


Fig. 6. Cells for measuring dielectric constant and power factor of one inch paper strips

sired gas is admitted to the cell, which then is sealed off at atmospheric pressure. The manometer is used to indicate subsequent changes in gas pressure.

Type B: Gas Free Cell (Less Than 0.1 Micron Pressure). The type of cell shown in figure 3 simulates the ideal oil-filled gas-free cable. During evacuation and filling, the bellows is held open with small bolts. After evacuation and filling the glass tubes are sealed off, and then the bolts are removed so that thereafter the oil in the cell is maintained at atmospheric pressure.

Type C: Gas Pressure Cell (200 Pounds per Square Inch). The third type of cell, shown in figure 4, was designed to test all kinds of insulating materials under gas pressures up to 200 pounds per square inch. Special high-voltage high-pressure glass bushings were made so that the specimens could be tested at the same voltage as in the other cells.

Drying and Impregnation

The following procedure has been adopted as standard: The cell containing the unimpregnated specimen is evacuated for 24 hours to less than 0.1 micron pressure. During this time the specimen is heated radiantly at about 100 degrees centigrade. After the drying period, degasified oil is admitted slowly under vacuum and kept at about 100 degrees centigrade. The rate of admission is such that the level rises about 2 inches per hour. After sufficient oil has been admitted, the cell is sealed off at 0.1 micron pressure or any desired gas is readmitted.

Life Tests

The life tests are designed to accelerate the effects of high voltage with normal operating temperature and temperature cycles. Alternating potential at a frequency of 60 cycles is applied continuously to the test specimens; 22 and 30 kv potentials are avail-

able, corresponding to average gradients of 340 and 460 volts per mil, respectively, in the standard specimen. Maximum gradients at the edges of the threads on the copper rod must be considerably greater than these values. The specimens are mounted in large ovens having air circulating fans and automatic temperature cycle control. A typical temperature cycle taken by a thermocouple in the oil of one of the specimens is shown in figure 5.

Selector switches and connections to the test electrodes of all specimens are so arranged that periodic 60 cycle capacitance and dielectric loss measurements can be made by means of a Schering bridge on all specimens without removing them from the ovens.

Examination After Life Test

Measurements of the following quantities (where possible) are made on each specimen immediately after opening the glass cell:

1. Electrical properties of paper and oil.
2. Electrical properties of oil extracted from paper.
3. Acid number of oil.
4. Hydrophil content of oil.
5. Molecular weight of oil.
6. Iodine unsaturation number of oil.
7. Copper and lead compounds dissolved in oil.

These measurements were attempted because of the *hypothesis that under certain conditions prolonged exposure to heating and cooling cycles under continued severe electrical stress may cause chemical and physical changes resulting in ever-increasing dielectric losses and eventual thermoelectric failure.*

If the electric stress be a factor in such changes, the effects should be most pronounced nearest the inner electrode where the stress is greatest. Therefore, all the foregoing properties were measured as functions of the radial distance from the inner (copper) to the outer (lead) electrode. This procedure is an extension of the work of K. S. Wyatt and others[1,2] who have studed the variations in hydrophil content of oil and power factor of oil impregnated paper as functions of radial position in cables.

As soon as each cell is opened, the oil soaked paper is unrolled, untouched by hand, and is clamped in a frame; a 1 inch strip then is cut at the edge of the test section farthest from the puncture. This strip is used for test 1, one end having been next to the copper and the other end next to the lead. The remainder of the test section, exclusive of visibly burned portions, is divided into 3 parts: (1) the 2 inner layers, (2) the middle layers, and (3) the 2 outer layers. With a high speed centrifuge the free oil is thrown out of each of the 3 sections of paper and used for tests 2, 5, 6, and 7. The oil remaining in the paper after centrifuging is extracted with highly purified benzene and used for tests 3 and 4. All measurements are made as soon as possible after opening a cell. Normally, tests 1, 2, and 7 are completed and samples for the other tests are prepared on the same day the cell is opened.

1. Electrical Properties of Paper Plus Oil. Using a Schering bridge, the dielectric constant and dielectric loss of a 1 inch strip of oil saturated paper are measured at 60 degrees centigrade as functions of the radial distance from the inner to the outer electrode of the test specimen. The cell shown in figure 6 was developed for this work. The lower guard ring electrodes were made by sealing a one millimeter glass insulating ring between the inner 1.9-centimeter "fernico"-disk measuring electrode and the outer guard ring. ("Fernico" is an iron-nickel-cobalt alloy having the same coefficient of expansion as glass.) The whole surface then was ground flat and polished, giving a guard ring electrode of fixed dimensions that can be cleaned very easily. The upper electrode is a disk of nickel 2.6 centimeters in diameter on the flat with edges rounded on a 1 millimeter radius. A constant pressure of 450 grams per square centimeter is applied to all specimens by weights held by guides in a central vertical position. The thickness of paper at each test point is measured with micrometers after the electrical measurements are complete.

2. Electrical Properties of Oil Alone. The quantities of oil removed from each section of paper by centrifuging are of the order of 0.1 to 0.3 gram. No equipment was available for measuring the electrical properties of such small quantities of oil, so the cell shown in figure 7 was developed. As in the paper strip cell, an easily cleanable guard ring electrode was made conveniently by sealing the guard ring to the measuring disk with a ring of glass and then grinding

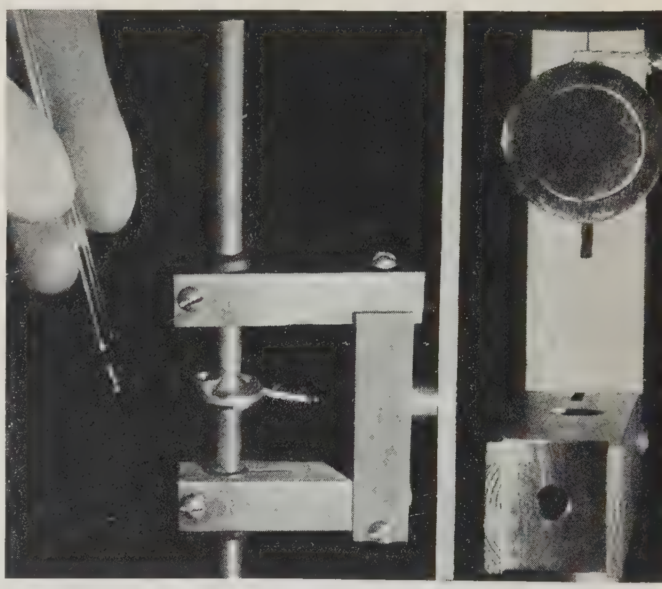


Fig. 7. Cell for measuring electrical properties of 0.1 gram of oil. View on right shows details of electrode; note glass insulating ring between measuring and guard electrodes

and polishing the surface. The measuring electrode is about one centimeter in diameter and the glass insulation about 0.5 millimeter wide.

The technique employed for making measurements is as follows: The upper electrode is adjusted to give a spacing between electrodes of about 0.1 millimeter.

1. For all numbered references see list at end of paper.

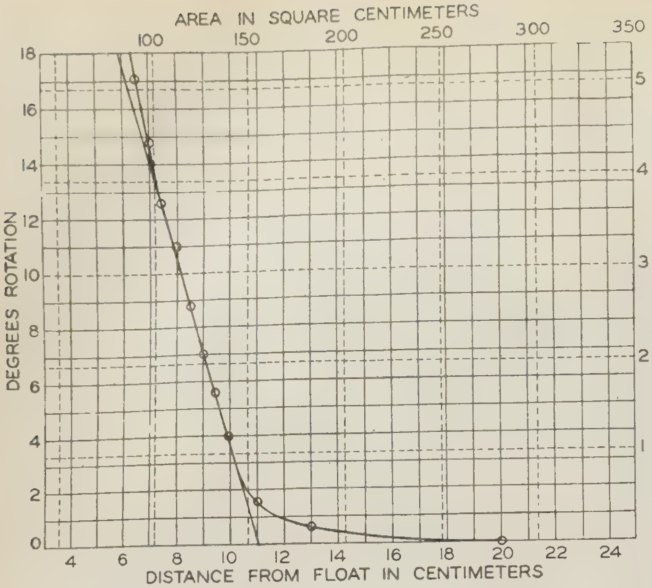


Fig. 8. A typical hydrophil curve

Uncorrected distance extrapolated = 11.1 centimeters
Corrected distance (0.1 cubic centimeter benzene =1 centimeter) = 10.1 centimeters
Corrected area = 141 square centimeters
Weight of oil = 3.00 milligrams
Corrected area per milligram of oil = 47.0 square centimeters

The cell is heated in an oven at 60 degrees centigrade, and after coming to thermal equilibrium the air capacitance is measured. The oil to be measured is introduced from a small pipette to the funnel at the edge of the upper electrode from which it spreads between the electrodes by capillarity, forcing all air out ahead of it. Electrical measurements then are made with a Schering bridge at 60 cycles and 250 volts. Using this technique it has been found possible to measure to about 1 per cent accuracy the dielectric constant and power factor of 0.1 gram of oil.

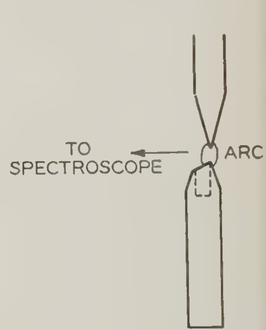
3. Acid Number. An attempt has been made to modify the usual method for determining the acidity of oil using phenolphthalein solution as an indicator so as to make a determination on one drop of oil. The procedure adopted is as follows: The oil sample of 0.03–0.05 gram is weighed into a small flask made from a test tube of approximately 9 cubic centimeter capacity; 6 cubic centimeters of alcohol-benzol mixture is added and the solution is boiled gently on a steam bath for 5 minutes. The flask then is placed in a wire clip holder and is stirred with air free from carbon dioxide which is introduced from a fine capillary tube. A burette then is introduced and placed so that the end of the burette tip extends about half way to the bottom of the flask. The stirring capillary is moved constantly for thorough stirring, and the titrating solution, a $^1/_{10}$ normal solution (one liter of a normal solution contains one gram atom of replaceable hydrogen or its equivalent) of potassium hydroxide ($N/10$ KOH), is run in to the phenolphthalein end point. A white background aids in seeing this end point.

The operator's judgment of the end point is somewhat subjective and quite dependent upon variation in light conditions. Using the "micro" technique, an acid number less than 0.1 cannot be distinguished from neutral, and even values of 0.2 and 0.3 are subject to 50 per cent error. For low acid numbers the method is qualitative rather than quantitative. Effort is being made to increase the sensitivity of this micro method for determining the acid content of 0.1 gram of oil.

4. Hydrophil Content of Oil. The hydrophil content or proportion of polar constituents of insulating oils as measured by Langmuir's spreading methods on the surface of water have been correlated with electrical properties by several authors.[1-4] Unoxidized highly-refined mineral oil contains no polar constituents and therefore is not hydrophilic. However, hydrophilic compounds are formed by oxidizing mineral oil.[3] Therefore, *hydrophil measurements detect either oxidation products or polar contaminants that have been dissolved by the oil from the materials with which it has come in contact.*

A simplified torsion head hydrophil balance[5] is used for all measurements. Variations in time, temperature, rate of application of pressure, and acidity or alkalinity of the water surface are known to cause considerable changes in pressure-area diagrams for films containing mixtures of unknown complicated polar molecules that exist in contaminated mineral oil. Therefore, in order that the results for all samples should be comparable, the following technique is followed very closely: The tray and float are heavily paraffined. A pipette having an accuracy of about 2 per cent is used to drop onto the water surface 0.1 cubic centimeter of a solution having a concentration of 0.03 gram of oil per cubic centimeter of carefully redistilled benzene. Most consistent results can be obtained using acid water; therefore, a $^1/_{100}$ normal solution of hydrochloric acid ($N/100$ HCl) was used as a spreading liquid. The sensitivity of the balance is such that one degree of the torsion head corresponds to a force of 3.89 dynes on the float, or a surface pressure of 0.2994 dyne centimeter. After 0.1 cubic centimeter of solution has been dropped on a clean water surface and the benzene allowed to evaporate, a barrier is moved up and pressure-area readings made from 0 to 20 degrees angular deflection of the torsion head. The pressure-area curve then is plotted and the best straight line drawn through the points between 1 and 4 dyne centimeters. The intercept of this line with the zero force axis is taken as the spreading area, a correction for the benzene blank is subtracted, and the hydrophil number is calculated as square centimeters of spreading area per milligram of oil. Because of the

Fig. 9. Arrangement of electrodes for arc spectrograph



complicated nature of the polar compounds present in different samples, various types of curves have been found, some of which were straight to forces higher than 4 dyne centimeters, and some of which departed considerably from linearity below 2 dyne centimeters. In all cases the best line possible is drawn through the linear range. A typical curve is shown in figure 8.

5. *Molecular Weight of Oil.* Another hypothesis of this series of experiments is that *polymerization of the oil may occur under the influence of high electric stress.* If an appreciable amount of such polymerization occurs, it should be detectable as an increase in the molecular weight and in the viscosity of the oil, and in advanced stages by the formation of visible wax. Accurate viscosity measurements on 0.1 gram samples of oil appeared impracticable, so it was decided to attempt direct molecular weight measurements by the Rast method, which depends upon the freezing point depression of camphor. The Rast method has been adopted for liquids by A. Soltys[6] and the procedure followed is described in ample detail by Pregel.[7]

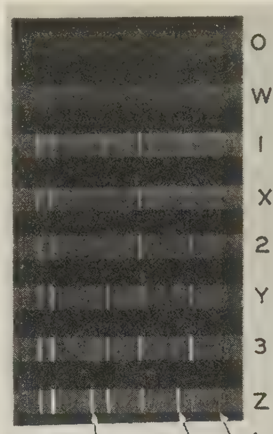
The recrystallized camphor sample has a melting point of 176.7 degrees centigrade, and the molecular freezing point depression k is determined by observations using para-nitrochlorobenzene and pure tetracosane ($C_{24}H_{50}$). The equation

$$M = \frac{1{,}000\ ks}{L\Delta}$$

is used, where M is the molecular weight of the solute; s is the weight of solute used; L is the weight of camphor used; and Δ is the freezing point depression of the camphor. The constant k increases with decreasing concentration of the solute, but at a concentration corresponding to a solute to camphor weight ratio of 1 to 10, k had a value of 36.3. All determinations are made with this ratio of weights. The results on compounds containing no solids should be reproducible within 5 per cent.

The samples are weighed on an assay balance which is sensitive to ± 0.01 milligram. The oil samples vary from 0.20 to 1.00 milligram, and for each oil sample 10 times its weight of camphor is taken. Two sealed ampoules for duplicate determinations are attached to the bulb of a tenth degree thermometer by thread and immersed in a motor-stirred glycerine bath. The samples are melted and mixed by rotating the thermometer between the palms of the hands. The melting point is taken as the temperature at which the last crystal of camphor disappears. A low power (x 20) microscope is used for observing the crystals. Care must be used to distinguish the paper fibers, which are usually present, from the needle-like camphor crystals. The presence of these paper fibers or other solid impurities in the centrifuged oil probably causes the large differences occasionally observed between duplicate samples.

6. *Iodine Unsaturation Number of Oil.* Another hypothesis of this series of experiments is that *the life of a gas-free oil-impregnated paper specimen will be better the smaller the tendency of the oil to give off gas under high electrical stress.* Of the gas given off from an oil during bombardment, about 80 per cent is hydrogen. A further hypothesis is then that *if the oil be partly unsaturated, hydrogen given off during bombardment may recombine with unsaturated oil molecules thus preventing its liberation as a free gas and reducing the tendency toward ionization in gas pockets.*



Cu—Copper, 2,824.4 angstrom units
Pb—Lead, 2,873.3 angstrom units
Bi—Bismuth, 2,898.0 angstrom units

Fig. 10 (above). Typical spectrogram for copper and lead determination

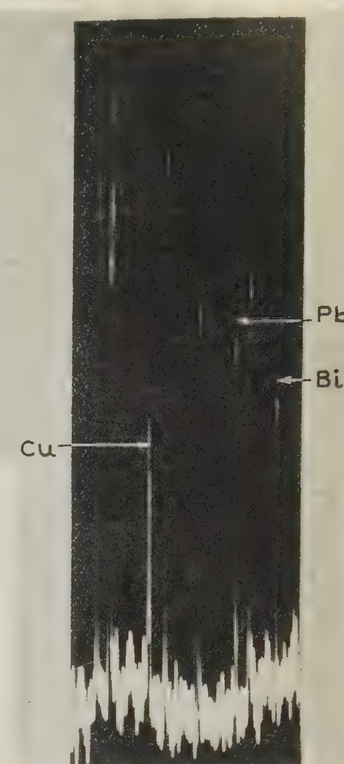


Fig. 11 (right). Photometric evaluation of standard solution Y of the spectrogram shown in figure 10

For these reasons it was desired to measure the degree of unsaturation of the oils after removal from life test, and the method of Francis, as described by Mulliken and Wakeman,[8] was followed. The only changes are the use of a 10–20 milligram sample of oil, which is weighed out in a small capillary, and the use of microburettes for the solutions. The results obtained to date are not entirely satisfactory, but the method appears worthy of further development.

7. *Copper and Lead Compounds Dissolved in Oil.* Another hypothesis under investigation is that *if the oil be acid to any degree whatever, so-called metal soaps gradually will be formed at the copper and lead electrodes during long life at elevated temperature.* The presence of such metal compounds is known to increase the conductivity and power factor of oil alone, although their effect on the electrical properties of oil impregnated paper is not so well established. These metal compounds appear in such low concentrations and the available quantities of oil centrifuged from small sections of paper are so small that chemical methods for quantitative analysis were abandoned, although considerable effort was expended to use them. The following arc spectroscopic method then was developed.

For each oil sample a lower carbon is drilled,

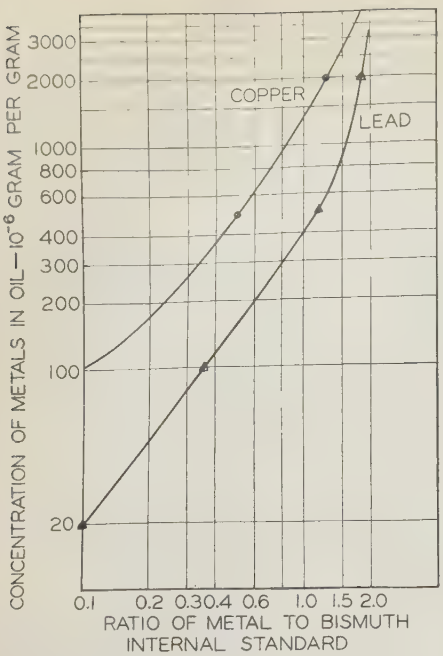


Fig. 12. Calibration of spectrographic method

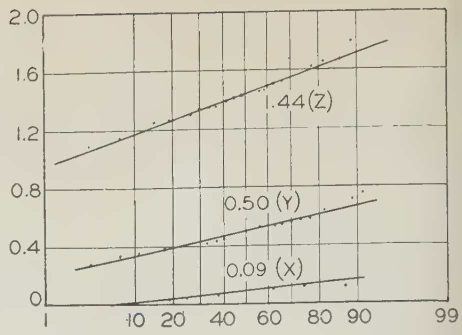


Fig. 13. Probability plots for copper calibration of figure 12

Designations on curves indicate ratios of copper intensity to bismuth intensity as indicated on photographic records of densitometer measurements, for the various standard solutions

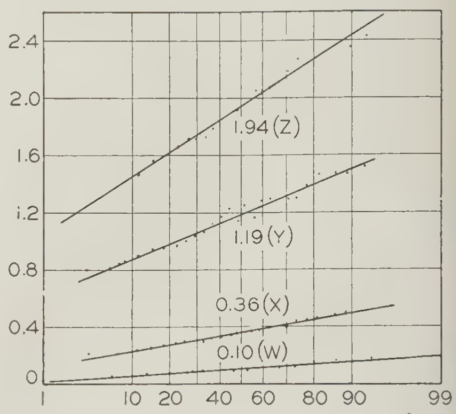


Fig. 14. Probability plots for lead calibration of figure 12

Designations on curves indicate ratios of lead intensity to bismuth intensity as indicated on photographic records of densitometer measurements, for the various standard solutions

tapered, and the top edge cut at an angle of about 30 degrees. The upper electrode is tapered to a sharp point and adjusted to make contact with the highest point of the lower electrode, the lowest point of the lower electrode edge being toward the spectroscope slit (see figure 9). The lower carbons are charged first with 0.1 cubic centimeter of acid solution of bismuth of known concentration (10^{-4} gram of bismuth per cubic centimeter). The carbons then are placed in an aluminum holder on a hot plate and the temperature gradually is increased to about 180 degrees centigrade, thus evaporating the water from the bismuth solution. Then 0.05 cubic centimeter of either an unknown oil or of an oil containing known quantities of copper and lead in solution is dropped from a pipette into each carbon crater and is allowed to stand at this temperature until the electrode appears dry and stops smoking.

From each cable specimen there are 3 unknown oil samples: (1) from the 2 inner layers next to the copper, (2) from the middle layers, and (3) from the 2 outer layers next to the lead. On each spectrographic plate in addition to the spectrograms of these 3 unknowns, the spectrograms of 4 known standard solutions, each containing equal amounts of copper and lead, are taken (see table I).

The use of an internal bismuth standard in each carbon and the taking of spectrograms of the 4 standards with each set of unknowns was adopted in an attempt to eliminate the effects of variations in arc stability, arcing time, development procedure, and photographic plate sensitivity. A typical spectrogram is shown in figure 10.

Densitometer measurements of these spectrograms then were made photometrically and a photographic record was obtained. The record for standard solution Y for the spectrogram of figure 10 is shown in figure 11. It was found that the intensities of the strongest lines for each of the 3 metals was too great to give good comparative results. Therefore, the next strongest lines have been used throughout to obtain a quantitative estimate of the concentration of these metals. The wave lengths of the lines used, in angstrom units, are: copper —2,824.4, lead —2,873.3, and bismuth —2,898.0. From the photometric curves the amplitude of these lines above the background is measured, after which the ratios of the copper and lead intensities to the bismuth intensity are calculated. The whole sequence of measurements then is calibrated by plotting these ratios for the known standards against the known concentrations as shown in figure 12. Data from 25 plates are recorded in this graph. To obtain the mathematically most probable calibration values for this method, the ratios of each of the copper and lead standards to the internal bismuth standard from 25 plates are shown in figures 13 and 14. The calibration curves shown in figure 12 are drawn through these most probable points.

Two general statements may be made regarding this attempt to use a spectrographic method for quantitatively determining small amounts of copper and lead in 0.05 gram of oil:

1. The minimum detectable concentrations are 10^{-4} gram of copper and 10^{-6} gram of lead per gram of oil.

2. Quantitatively, the method gives only the order of magnitude of the concentration of these 2 metals in oil from 10^{-5} to 10^{-2} gram per gram of oil.

Life Tests on Gas Saturated Specimens

To illustrate the information that may be obtained from tests such as are outlined hereinbefore, the data obtained from a series of experiments using type A cells is presented next. These specimens were dried and impregnated under vacuum and heat, and then either dry oxygen, nitrogen, or carbon di-

oxide gas at atmospheric pressure was readmitted.

Table II gives a summary of the results of tests on gas-saturated specimens arranged in order of their length of life. The first column of power factor data gives results of the first measurements at 60 degrees centigrade on each specimen. The second and third columns give the minimum and maximum readings during life, and the fourth column gives the last measurement before failure. When specimen 196 failed, it blew up, spilling mercury and oil in the oven and causing a fire; therefore, specimens 162, 175, and 194, which indicated a gas pressure greater than 2 atmospheres, were removed before failure and opened.

Table III gives the results of closed manometer measurements of the gas pressure above the oil for the 10 best gas-saturated specimens. The graduations were difficult to read so that it is believed that the values for specimen 173 marked (?) were errors in reading and not actual pressure changes. These measurements were made so that any consumption of gas (particularly oxygen) by chemical combination with paper and oil would be observed. Any increase in pressure during life caused by gas generation from high stress decomposition of the oil also would be observed. Specimen 194, for example, increased in pressure slowly throughout its entire life; then on or before August 23 it generated gas very rapidly and was removed from test before actual failure so as to prevent it from bursting and possibly destroying other specimens. All other specimens in this list failed electrically before being removed from test.

Table I—Concentrations of Copper and Lead in Standard Solutions

Designation of Standard Solution	Copper and Lead, Grams per Cubic Centimeter of Solution	Solution, Grams per Carbon	Copper and Lead, Grams per carbon
W	20×10^{-6}	0.05	1×10^{-6}
X	100×10^{-6}	0.05	5×10^{-6}
Y	500×10^{-6}	0.05	25×10^{-6}
Z	$2{,}500 \times 10^{-6}$	0.05	125×10^{-6}

Table IV gives typical power factor measurements as a function of applied voltage. All routine power factor measurements are made at 6 kv, which corresponds to an average gradient of about 90 volts per mil. Specimens 156 and 157 contained carbon dioxide and oxygen with new oil *A*, and 6.5 mil kraft paper *D*. Specimens 172 and 173 were like 156 and 157 except that they were made with 0.5 mil kraft capacitor paper *E*. These were chosen because if any ionization were observed in the spaces between tapes of 6.5 mil cable paper, this ionization should have been much less in specimens made from thin capacitor paper.

Figure 15 shows the life histories of these specimens in graphical form. The top line shows the periods for which the temperature has been either above or below the desired constant value of 60 degrees centigrade. The temperature accidentally rose above this value twice, once in April to about 90 degrees, and again in September to about 140 degrees. These excess temperatures may have contributed to the failure of specimens 157, 164, 168, and 169, although, of course, this conclusion cannot be verified. In the second line, the black portions indicate the time that 22 kv was applied to the specimens, and the intervening spaces indicate the time that voltage was off for any appreciable period for repairs to the testing equipment.

Table II—Summary of Data for Gas Saturated Specimens

Specimen	Contents: Gas	Contents: Paper	Contents: Oil	†Special Treatment	Power Factor: Start	Power Factor: Minimum	Power Factor: Maximum	Power Factor: Finish	Life, Days
161	Oxygen	*D*	*B*	+ 0.1% F	0.0027	0.0027	0.0359	0.0048	150
165	Carbon dioxide	*D*	*A*	+ 0.01% G	0.0071	0.0051	0.0133	0.0104	150
170	Oxygen	*D*	*A*	+ 0.1% H	0.0214	0.0214	0.0564	0.0301	143
156	Oxygen	*D*	*A*	New oil	0.0128	0.0122	0.0286	0.0204	110
172	Carbon dioxide	*E*	*A*	New oil	0.0105	0.0049	0.0171	0.0171	105
173	Oxygen	*E*	*A*	New oil	0.0077	0.0077	0.0362	0.0236	102
196	Oxygen	*D*	*A*	+ 0.1% F	0.0068	0.0068	0.206	0.206	75
*194	Oxygen	*D*	*C*	+ 0.1% F	0.0053	0.0053	0.056	0.056	72
197	Carbon dioxide	*D*	*A*	+ 0.1% F	0.0118	0.0118	0.0336	0.0295	68
190	Oxygen	*D*	*C*	+ 0.1% I	0.0082	0.0082	0.0300	0.0300	61
157	Carbon dioxide	*D*	*A*	New oil	0.0089	0.0089	0.0170	0.0162	52
168	Nitrogen	*D*	*A*	New oil	0.0064	0.0064	0.0175	0.0132	50
169	Carbon dioxide	*D*	*A*	+ 0.1% H	0.0103	0.0103	0.0348	0.0348	50
159	Oxygen	*D*	*A*	+ 0.5% H	0.371	0.361	0.371	0.361	49
164	Carbon dioxide	*D*	*A*	+ 0.1% G	0.0125	0.0028	0.0111	0.0111	38
160*A*	Carbon dioxide	*D*	*B*	+ 0.1% F	0.005	0.0044	0.082	0.082	29
*175	Carbon dioxide	*D*	*A*	Paper washed 24 hr.	0.0024	0.0024	0.0252	0.0252	29
157*A*	Carbon dioxide	*D*	*A*	New oil	0.0046	0.0026	0.0488	0.0151	26
*162	Nitrogen	*D*	*A*	+ 0.1% F	0.0031	0.0031	0.062	0.062	15
192	Oxygen	*D*	*C*	+ 0.1% I + 0.1% F	0.0088	0.0088	0.0347	0.0347	13
158	Carbon dioxide	*D*	*A*	+ 1% H	0.454				8
193	Carbon dioxide	*D*	*C*	+ 0.1% I + 0.1% F	0.0148				2
163	Oxygen	*D*	*A*	+ 1% G					0

*These specimens indicated a gas pressure exceeding 2 atmospheres and were removed before failure and opened.

†Special characters F, G, H, and I indicate polar substances of undisclosed nature used in the laboratory experiments.

Table III—Pressure Changes During Life of the 10 Best Gas Saturated Specimens Listed in Table II

Date	Temperature, Degrees Centigrade	Manometer Pressure Readings From Fixed Point, in Millimeters									
		Specimen									
		161	165	170	156	172	173	196	194	197	190
1/25/34				−54	−63						
2/28				−47	−68						
3/23	62			−67	−53						
4/5	62			−85	−62						
4/12	62					−22	−16				−68
4/19	60			−60	−48	−20	−57				−52
5/8	62				−50	−18	−55	− 8			−25
5/14	59			−60	−45	−20	−35?	−25	−35	−12	−14
6/7	58			−60	−48	−20	−35?	−19	−27	−22	− 7
6/20	58				−48	−19	−35?	−20	−22	−20	− 5
7/6	60	−37	−62	−55		−24	−55	−24	−18	−22	Failed
7/9	64	−38	−50	−55		−20	−53	−23	−16	−21	
7/13	61	−47	−50	−58		−20	−52	−26	−16	−20	
7/19	61	−40	−52	−50		−19	−52	−32	−13	−32	
8/1	59	−41	−52	−52		−20	−54	−34	−12	−22	
8/23									+35		
8/24				Failed					Opened		
8/25								Failed		Broken	
8/19					Failed	Failed	Failed				
3/6/35			Failed								
3/7/35		Failed									

In the body of the graph, the power factor for each specimen has been plotted as a function of time, the total height of the space allotted to each specimen representing 10 per cent power factor.

These specimens were tested at an electric stress many times greater than is used in cables, but the total thickness and, therefore, the total voltage was much less than is used in cable practice. The potential gradient is comparable to that used in capacitors, but the total thickness, and therefore the total voltage, is much greater. Because of the greater tendency toward ionization, these results are not directly comparable with standard oil-filled gas-free cable or capacitor practice, but should be indicative of changes that may occur in much longer time under service conditions where any appreciable amounts of gas may be present.

Examination After Life Tests

Three of the specimens listed in table II were examined by the micro methods already outlined. The results are given in table V and particular comments in succeeding paragraphs.

Cell 157A. Specimens 157 and 157*A* were made by different operators with supposedly the same ingredients. Differences in their life history indicated the need for extreme personal care in assembly, evacuation, and impregnation, and led to the establishment of the schedule outlined previously in this paper. Ionization throughout the insulation was the most probable cause of the short life of this specimen, possibly resulting from incomplete evacuation. Test 1 shows maximum deterioration in the inner layers where the stress was highest, particularly in the second, third, and fourth layers. The fact that the first layer showed less deterioration than those just mentioned indicates that the major changes in the oil impregnated paper were caused by ionization rather than by chemical changes catalyzed by copper. However, tests 2, 3, and 4 all indicate that the oil alone had been affected more in both the inner and outer layers than in the middle layers. Figure 15 indicates that the average deterioration was gradual, and examination showed that it was not localized but was general throughout the specimen.

Cell 160. By accident, this cell was sealed off with a gas pressure of only about $^1/_2$ atmosphere. This accounts for its almost immediate failure on life test. Even this short period on voltage caused large charred areas in the third and fourth layers and heavy stress markings in the copper. Test 1 shows high losses in the paper, especially in the middle layers. Test 2, however, shows little deterioration of the oil in the inner and outer layers.

Cell 161. This specimen was impregnated with a highly refined naphthenic base oil plus 0.1 per cent polar material *F* and then saturated with oxygen at atmospheric pressure. It had the longest life of any of the gas saturated specimens. Its power factor continued to decrease after the first few days of life

Table IV—Power Factor Measurements as a Function of Applied Voltage of Typical Gas Saturated Specimens

Date	Specimen	Power Factor at Various Voltages (in Kv)				
		0.5	1.2	3	6	10
2/28	156	0.0365	0.0365	0.0344	0.0286	0.0226
	157	0.0177	0.0187	0.0158	0.0112	0.0091
3/8	156	0.0276	0.0288	0.0263	0.0218	0.0191
	157	0.0267	0.0284	0.0242	0.0170	0.0133
3/23	156	0.0250	0.0251	0.0224	0.0186	0.0169
	157	0.0230	0.0227	0.0200	0.0156	0.0134
4/5	156	0.0169	0.0169	0.0157	0.0142	0.0134
	157	0.0169	0.0172	0.0170	0.0162	0.0160
4/12	172	0.0105	0.0106	0.0105	0.0105	0.0107
	173	0.0073	0.0073	0.0073	0.0077	0.0084
4/19	156	0.0200	0.0198	0.0185	0.0170	0.0168
	172	0.0102	0.0102	0.0100	0.0100	0.0102
	173	0.0163	0.0162	0.0164	0.0180	0.0188
5/8	156	0.0220	0.0216	0.0202	0.0183	0.0182
	172	0.0103	0.0100	0.0090	0.0090	0.0095
	173	0.0154	0.0194	0.0205	0.0217	0.0230
5/14	156	0.0180	0.0182	0.0170	0.0149	0.0146
	172	0.0075	0.0076	0.0076	0.0078	0.0080
	173	0.0199	0.0201	0.0200	0.0199	0.0207
6/7	156		0.0203	0.0183	0.0163	0.0160
	172		0.0100	0.0094	0.0093	0.0095
	173		0.0315	0.0307	0.0295	0.0292

(see figure 15). Tests 1 and 2 show that even after this long life the oil had considerably lower power factor than the impregnated paper. Oil from the layers nearest the copper had both highest power factor, highest acid number, and highest hydrophil number. Also, test 7 showed a small amount of copper but no lead in solution in the oil. The paper was brittle and very heavily marked next to the copper. No guess can be made as to the possible life of this specimen if it had been left at 22 kv. Apparently, 30 kv, which gives an average gradient of nearly 500 volts per mil, is too high for any specimen gas-saturated at atmospheric pressure to last any appreciable length of time.

Summary

1. By the use of small-scale carefully-made laboratory specimens subjected to long-time accelerated life tests and subsequently examined by micro electrical and chemical methods, it should be possible to determine the important factors leading to long life of oil impregnated paper insulation.

2. Gas saturated specimens having longest life have contained definite small quantities of purposely introduced hydrophilic materials or limited amounts of oxygen which by combining with oil produce polar compounds (cell 161, 170, 156, etc.). For these specimens, hypothesis 1(*c*) was apparently a predominant factor.

3. Specimens containing too much conducting material had short life; for example, compare specimens 165, 164, and 163 having 0.01, 0.1, and 1.0 per cent by weight of polar material *G*. Also compare specimens 169 and 158 having 0.1 and 1.0 per cent by weight of polar material *H* added to the oil. For these specimens, hypothesis 2(*b*) was apparently a predominant factor.

4. From the last 2 statements it may be concluded also that an optimum amount of hydrophilic material may be used to give tolerable dielectric loss and maximum life.

5. From table III it may be seen that only specimens 190 and 194

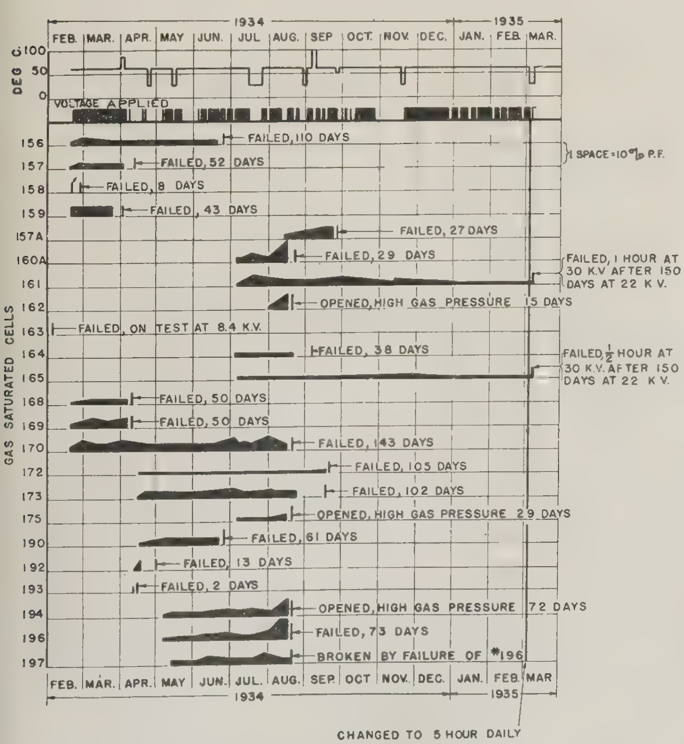


Fig. 15. Results of electrical life tests on gas saturated cells

Table V—Data From Examination of Gas Saturated Specimens

	†Portion	157A		160		161	
Cell		157A		160		161	
Days Life 22 Kv		26.5		0.5		149	
Days Life 30 Kv		—		—		0.04	
Oil		A		B+F		B+F	
Paper		D		D		D	
Gas		Carbon Dioxide		Carbon Dioxide		Oxygen	
Gas Pressure		1 Atmosphere		About 1/2 Atmosphere		1 Atmosphere	
Test 1 Paper plus oil Dielectric constant (ε) and power factor (P. F.)	†Portion	ε	P. F.	ε	P. F.	ε	P. F.
	1	3.21	0.077	2.99	0.037	2.82	0.018
	2		0.104	burnt		2.79	0.019
	3	3.22	0.087	2.35	0.054	2.94	0.018
	4	2.83?	0.075	3.09	0.079	2.84	0.015
	7	3.00	0.047	2.52	0.060	2.79	0.006
	8	—	0.046	2.79	0.044	—	—
	9	3.23	0.032	3.02	0.047	2.88	0.008
	10	3.31	0.037	3.38	0.025	3.15	0.019
Test 2 Oil Dielectric constant (ε) and power factor (P. F.)	†Portion	ε	P. F.	ε	P. F.	ε	P. F.
	1	2.15	—	2.11	0.0023	2.10	0.0075
	2	2.18	0.028	2.11	—	2.10	0.0042
	3	2.18	0.041	2.12	0.0023	2.10	0.0025
Test 3 Acid (Milligram per gram)	1	0.3		0.2		0.3	
	2	0.1		0.1		0.1	
	3	0.2		0.1		0.2	
Test 4 Hydrophil (Square centimeters per milligram)	1	57		54		72	
	2	47		13		44	
	3	53		25		24	
Test 5 Molecular weight (Gram/mol)	1	404	390	321	333	—	323
	2	402	387	316	346	355	335
	3	—	396	385	333	379	375
Test 6 Unsaturation (Grams per 100 grams)	1	18		4		9	
	2	21		5		1	
	3	21		5		9	
Test 7 Copper and lead (10^{-6} grams per gram)	1	*	*	*	*	—	*
	2	*	*	*	*	20	*
	3	*	*	*	*	20	*

Note: A dash (—) denotes quantities not measured; (*) indicates less than minimum detectable concentration.

†For test 1, "portion" indicates layer of paper, numbering from copper electrode out toward lead electrode; for all other tests, portions 1, 2, and 3 designate oil samples extracted from inner 2 layers, center layers, and outer 2 layers, respectively.

showed a steady increase in gas pressure during life. These cells contained a highly refined paraffin base oil manufactured for special lubricating rather than insulating use. Specimens 161, 162, and 162*A*, containing a highly refined naphthenic base oil, did not show an increase in gas pressure during life. For specimens 190 and 194, hypothesis 3(*a*) was apparently a predominant factor.

6. In general, it seems that from experiments such as are outlined in this paper, materials may be chosen intelligently to give simultaneously the optimum balance of the desired properties, namely, low dielectric loss, stability, and long life in oil impregnated paper insulation.

References

1. A New Method of Investigating Cable Deterioration and Its Application to Service Aged Cable, K. S. Wyatt, E. W. Spring, and C. H. Fellows. A.I.E.E. Trans., v. 52, Sept.-Dec. 1933, p. 1035–43.

2. A Criterion of Quality of Cable Insulation, K. S. Wyatt and E. W. Spring. Elec. Engg. (A.I.E.E. Trans.), v. 54, April 1935, p. 417–21.

3. Progress in High Tension Underground Cable, G. B. Shanklin and G. M. J. Mackay. A.I.E.E. Trans., v. 48, April 1929, p. 338–67.

4. Changes in Physical and Electrical Properties of a Mineral Insulating Oil, Heated in Contact in Air, Hubert Race. *Jl. Phys. Chem.*, v. 36, 1932, p. 1928–41.

5. The Physics and Chemistry of Surfaces (a book), N. K. Adam. Oxford Press, 1930.

6. *Jl. Prakt. Chem.*, v. 105, 1923, p. 27.

7. Quantitative Organic Microanalysis (a book), F. Pregl. Blakiston, Philadelphia, Pa., second English edition, 1930.

8. Estimation of Unsaturation in Aliphatic Hydrocarbons by Bromide-Bromate Titration, S. P. Mulliken and R. L. Wakeman. *Industrial Engg. Chem.*, anal. ed., v. 7, 1935, p. 59.

Cable and Damper Vibration Studies

A theoretical analysis of dampers and conductor vibration is presented in this paper and certain new methods of eliminating or reducing conductor vibration are discussed.

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THE interesting and important subject of mechanical vibration of transmission line conductors lends itself readily to a mathematical discussion if certain assumptions be made. Although these assumptions are not exactly true, they are quite close to the physical case, and produce results which should not be far in error. It is the purpose of this paper to present a fairly comprehensive analysis of several aspects of conductor vibration and of some of the methods which have been employed or suggested to lessen it.

Imperfect Flexibility

In the discussion of transmission line vibrations it is customary to assume the cable tension to be of such great magnitude that it may be considered constant, and that the effect of rigidity may be neglected. In such an analysis the motion of the cable is described by the familiar wave equation in one dimension for small deviations from the static catenary curve produced by oscillations. The same equation is employed in the study of stringed instruments. If rigidity be taken into account, however, the equation of motion is:

$$m\frac{\partial^2 u}{\partial t^2} + EI\frac{\partial^4 u}{\partial s^4} = T\frac{\partial^2 u}{\partial s^2} + F(s, t) \qquad (1)$$

where

u = lateral displacement of the cable from the catenary position of equilibrium, measured in a vertical plane and having a direction at right angles to the cable
E = Young's modulus of elasticity for the material of the cable
I = moment of inertia of the plane area cut by a plane normal to the cable about a diameter
m = mass per unit length of cable
T = tension, assumed constant throughout the span. This may be done with very little error for the usual span sag
s = distance measured along the catenary curve as position of static equilibrium
F = any arbitrary force per unit length acting along the conductor
t = time

Traveling Waves

A paper by W. B. Buchanan[1] has stimulated a great deal of interest in the study of traveling waves along conductor spans. It may be profitable, therefore, to investigate the effect of conductor rigidity on such waves.

In the absence of any impressed or frictional forces equation 1 becomes

$$m\frac{\partial^2 u}{\partial t^2} + EI\frac{\partial^4 u}{\partial s^4} = T\frac{\partial^2 u}{\partial s^2} \qquad (1a)$$

It is this equation which a traveling wave must satisfy. If f is the frequency of the traveling wave, and the quantity

$$x = \frac{16\pi^2 f^2 mEI}{T^2}$$

is so small that x^2 may be neglected in comparison with x, then the velocity of propagation v of the wave is given by $v = \sqrt{T/m}$ and is thus independent of the frequency of the traveling wave. The criterion that x be small is satisfied at frequencies of from 10 to 30 vibrations per second, the usual aeolian frequencies for the size of conductor ordinarily employed, if the size of cable is not very much greater than one inch in diameter. If the conductor is assumed to be a solid cylinder having a density ρ, an expression is obtained for x in terms of r, the radius of the conductor: $x = \dfrac{4\rho\pi^4 E r^6 F^2}{T^2}$, showing that the effect of rigidity is dependent upon the sixth power of the radius. At high frequencies or with large radii x^2 is not negligible compared to x. In such cases the effect of rigidity is marked and the law for the velocity of propagation is not a simple one. The velocity of propagation then depends upon the frequency and the other constants. The expression may be written

$$v = \omega\sqrt{\frac{2EI}{T}}\left[\sqrt{\frac{1}{\sqrt{1 + \frac{4EIm\omega^2}{T^2}} - 1}}\right] \qquad (2)$$

where $\omega = 2\pi f$

The conclusions to be drawn from this are:

1. For the usual size of transmission conductors and for frequencies and tensions that are met in practice, the criterion that x be small is met. In such cases the velocity of propagation is the same as that given by equation 1 if the term EI involving rigidity is neglected. Hence, for the usual transmission line span, the effect of rigidity may be neglected for traveling waves.

2. For an impressed wave of a complex character, which may be resolved into a Fourier series, the high frequency components will not be propagated with a velocity independent of frequency and there will be marked distortion.

3. Since the quantity x varies as the sixth power of the radius of a solid conductor, it may be seen that the effect of rigidity becomes

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1. For all numbered references, see list at end of paper.

marked as soon as the conductor size passes a certain dimension. (x is quite small for conductors having a diameter of one inch or less.)

Standing Waves on Rigid Conductors

It is not surprising that there is an appreciable difference between the behavior of a perfectly flexible conductor and a rigid conductor. If standing waves exist, it is natural to find them distorted because of the effect of rigidity near the supports.

If equation 1 is solved under the assumption that no external forces exist, that $F = 0$, and that absolute fixity exists at the 2 extremities of the span, the quantity x is encountered again, and if it is assumed to be small, a solution for standing waves is possible. The effect of rigidity under this assumption is merely to disturb the nodal points, making their position a function of the distance from the points of support as well as the frequency of vibration. The nodal points are defined by a somewhat complex transcendental equation, but it is possible to calculate their position near the middle of the span. The general effect of rigidity appears to be to raise the frequency of all harmonics as given by the formula

$$f = \frac{r}{2l}\sqrt{\frac{T}{m}}\left[1 + \frac{2}{l}\sqrt{\frac{EI}{T}}\right] \text{ where } r = 1, 2, \ldots, \text{etc.} \quad (3)$$

and to shorten the wave length.

The effect of the small amount of rigidity present in a transmission line cable does not influence materially the propagation of traveling waves or the formation of standing waves; however, at higher harmonic frequencies the result becomes complicated. Fortunately the criterion that the quantity x be small is met in a practical transmission line, and there is no need for a discussion of this complicated phenomenon.

Damped Vibrations

Since the effect of rigidity is not appreciable in a practical transmission line it is possible to study the effect of damping on the behavior of the vibrations by omitting the term due to rigidity in equation 1. The physical law of damping actually followed by the cable is very complicated, and it cannot be formulated exactly; nevertheless, a law of damping that is approximately true and that can be analyzed mathematically can be assumed. Although the results are not exactly correct, they should give at least a qualitative view of the effects of damping.

If it is supposed that the force F in equation 1 is a force of viscous damping, or damping proportional to the transverse velocity of the cable,

$$m\frac{\partial^2 u}{\partial t^2} = T\frac{\partial^2 u}{\partial s^2} - R\frac{\partial u}{\partial t} \quad (4)$$

where the damping coefficient R is assumed to be small.

Traveling Waves With Damping

In case a traveling wave on a conductor obeys the foregoing law of damping produced, say, by having one end of the span execute forced vibrations of a character $u = A \cos \omega t$ at $s = 0$ it is found, on the supposition that R is small, that

$$u = Ae^{-\frac{Rs}{2}\sqrt{\frac{1}{mT}}} \cos \omega(t - s/a) \quad (5)$$

Unless otherwise defined $a = \sqrt{T/m}$ in this and subsequent equations.

Equation 5 represents a traveling wave progressing in the positive s direction with a velocity a independent of the frequency. The wave undergoes a diminution in amplitude because of the attenuation factor, and the greater the tension and mass per unit length of the conductor, the smaller will be the attenuation. If the damping coefficient is very small, as it is usually, the amplitude diminishes in a manner proportional to the distance traveled; however, if R is appreciable the damping is great, particularly at low values of tension.

It is interesting to observe that Buchanan[1] found experimentally a greater attenuation of the traveling wave at low values of tension, and that the loss in amplitude is proportional approximately to the distance traveled. This effect can be seen from equation 5, for if R is quite small,

$$\epsilon^{-\frac{Rs}{2}\sqrt{\frac{1}{mT}}} = 1 - \frac{RS}{2}\sqrt{\frac{1}{mT}}$$

approximately.

This shows that the loss in amplitude is proportional to the distance traversed. Hence, a rough experimental check is available for the assumed law of damping. The rate of decay is seen to be independent of the frequency, so that no distortion of a complex wave occurs.

Standing Waves With Damping

With the justifiable assumption that the damping coefficient R is very small, it is possible to obtain a general solution of the damped wave (equation 4). This solution is:

$$u = \epsilon^{-\frac{Rt}{2m}}\left[\phi_1(s - at) + \phi_2(s + at)\right] \quad (6)$$

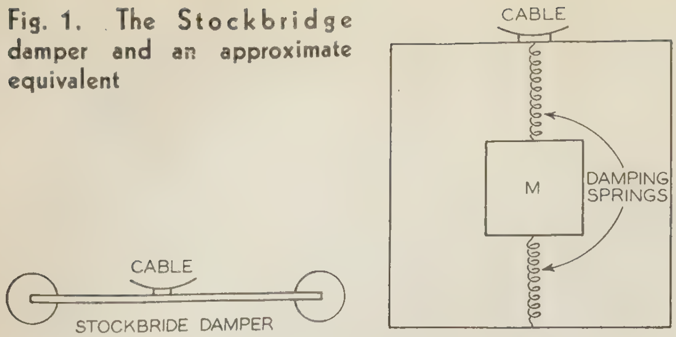
where ϕ_1 and ϕ_2 represent arbitrary functions of the argument $(s \pm at)$. Physically ϕ_1 is a wave of displacement traveling in the positive s direction and ϕ_2 is a wave traveling in the negative s direction. The whole disturbance is subjected to a decay because of the term $\epsilon^{-\frac{Rt}{2m}}$. If a solution is sought to satisfy the restriction of fixity at the extremities of the span, on the supposition that R is small,

$$u = \epsilon^{-\frac{Rt}{2m}}\sum_{n=1}^{n=\infty}(A_n \sin \beta_n t + C_n \cos \beta_n t) \sin \frac{n\pi s}{l} \quad (7)$$

where A_n and C_n are arbitrary constants depending upon the initial distortion of the cable from the position of equilibrium and

$$\beta_n = \frac{1}{2}\sqrt{\frac{4n^2\pi^2a^2}{l^2} - \frac{R^2}{m^2}}$$

Fig. 1. The Stockbridge damper and an approximate equivalent



which is the solution for standing waves when damping is present. The position of the nodes is not changed by damping, but the whole disturbance undergoes a rapid decay in amplitude with time, and the periods of the various frequency components are increased by damping. The damping of standing waves, unlike the damping of traveling waves, is independent of the tension, and is greater for lower values of mass per unit length of the conductor. The powerful damping effect of the exponential term suggests the desirability of having a large damping coefficient R. If R could be made large enough there would be no tendency for transient oscillations of the cable to arise.

Forced Oscillations

When a circular cylinder is placed in a moving fluid there is a tendency, in a real fluid such as air, to form a region of immobility behind the cylinder. Because of the shearing effect of the fluid stream on this dead wake, however, the fluid is set in rotation in the form of 2 eddies as shown in figure 3.

This dynamic system is most unstable, and when the stream attains a sufficient velocity, the vortexes formed in the wake no longer remain attached to the cylinder, but on reaching a sufficient size are carried downstream as if they were solid bodies.

Kármán and Rubach[3] investigated the stability of the vortex system and Rayleigh[4] gave an empirical formula for the frequency of detachment of these vortexes. Rayleigh's formula is:

$$\text{frequency} = \frac{0.195}{D} V\left(1 - \frac{20.1}{R}\right)$$

where

V = velocity of wind in feet per second
D = diameter of cylinder in feet
R = Reynold's number or the critical value of VD/ν at which eddies form expressed in suitable constants, and $\nu = \mu/\rho$ the kinematic coefficient of viscosity
μ = coefficient of viscosity of the fluid
ρ = density of fluid

For the range of values of practical interest in transmission line vibrations use may be made of Relf and Ower's[5] value for the frequency, $f = 0.185\ V/D$, in which V = velocity of wind relative to the cylinder in feet per second, and D = diameter of cylinder in feet.

The problem is the finding of an expression for the transverse oscillatory force produced by the detachment of the vortexes and the drag force due to the wind. If use is made of the dynamically similar case discussed by Thom,[6] F_D per square foot (drag force per square foot) $= K_D\ \rho\ V^2$, where K_D is a constant varying but slightly with the velocity V, and ρ is the density of the fluid.

For the maximum amplitude of the transverse force or pressure

$$F_{T\text{per square foot}} = K_T \rho V^2 \tag{8}$$

The values of K_D and K_T vary with the velocity, but for the usual value of Reynold's number for flow of air around a cylinder having a diameter of about one inch, they may be taken to be approximately $K_T = 0.45$ and $K_D = 0.64$.

Using the average density of air, and converting units to the English system,

P_T = maximum transverse pressure $= (1.04 \times 10^{-3})V^2$ pounds per square foot
P_D = drag pressure $= (1.47 \times 10^{-3})V^2$ pounds per square foot
V = velocity of wind in feet per second
If D = diameter of wire in feet,
$F_T = M_T D V^2$ = maximum transverse force per foot of length
$F_D = M_D D V^2$ = drag force per foot of length

where the quantities M_T and M_D vary somewhat with the velocity of the wind but may be taken as reasonably constant throughout the usual range of wind velocities of from 0 to 10 miles per hour. For such a case the values $M_T = 1 \times 10^{-3}$ and $M_D = 1.5 \times 10^{-3}$ produce the correct order of magnitude.

Having obtained this empirical expression for the maximum transverse force and drag force, the problem can be solved as follows: Assume that the transmission line span is subjected to a uniform horizontal wind having a velocity component V_W at right angles to the length of the line. Assume further that the transmission line conductor is a cylinder suspended in space, acted on by the forces shown in figure 4, and has a velocity v transverse to the wind.

Now if the transverse force F_T may be assumed as $F_T = M_T D V_T^2 \cos \omega t$ where $\omega = \dfrac{(2\pi)(0.185)V_T}{D}$ and $F_D = M_D D V_T^2$ components may be taken in the u direction to obtain

$$\sum F_u = F_T \cos\theta - F_D \sin\theta \text{ where } \theta = \tan^{-1}\frac{v}{V_W}$$

It may be assumed that in a practical case $V_T = \sqrt{V_W^2 + v^2} = V_W$ approximately; therefore,

$$\sum F_u = M_T D V_W^2 \cos\omega t - M_D D V_W v \tag{9}$$

The action of the wind is seen to be that of a driving force $M_T D V_W^2 \cos\omega t$ and a viscous damping force $M_D D V_W v$.

Assuming the velocity of the wind to be reasonably constant throughout the span, it is permissible to substitute into the general equation of motion and obtain

$$m\frac{\partial^2 u}{\partial t^2} + R\frac{\partial u}{\partial t} = T\frac{\partial^2 u}{\partial s^2} + M_T D V_W^2 \cos\omega t - M_D D V_W \frac{\partial u}{\partial t} \tag{10}$$

It will be noticed that the coefficient of the damping

term contains 2 factors; R due to the bending and twisting of the material, and $M_D D V_W$ due to the drag of the wind.

On the assumption that there is negligible damping,

$$\frac{2\pi(0.185)V_W m}{D} > R + M_D D V_W$$

and with the limitations imposed by the boundary conditions of fixity at the extremities, the solution of equation 10 is:

$$u = \frac{23.8 M_T D^3}{w \sin Kl} [\sin Ks + \sin K(l - s) - \sin Kl] \cos \omega t \qquad (11)$$

where

w = weight of line, pounds per foot
M_T, a constant $= 1 \times 10^{-3}$
$K = \omega \sqrt{\frac{w}{gT}}$
$\omega = \frac{1.16 V_W}{D}$
l = length of span, feet
D = cable diameter, feet
g = 32.2 feet per second per second
T = tension, pounds
V_W = wind velocity, feet per second

This solution satisfies the conditions of fixity of the supports and represents a superposition of 2 waves, each having a wave length corresponding to the forced frequency, each with a node at one end, and with the proper compensating constant.

It must be realized, however, that the tentative assumption of negligible damping has been made. If

$$\sin Kl = 0 \text{ or } V_W = \frac{2.7 D_n}{l} \sqrt{T/m} \text{ and } n = 1, 2, \ldots$$

a condition of resonance exists, and the amplitude apparently is infinite except at the nodal points $s = rl/n$.

However, if the small terms due to damping had not been rejected, $\sin Kl$ is not zero for any value of K, because K then is complex and approximately equal to

$$K = \frac{\omega}{a}\left[1 - \frac{1}{2} j \frac{p}{\omega}\right] \qquad (12)$$

where

$p = \frac{g}{w} [R + M_D D V_W]$

$\omega = 1.16 V_W / D$

$|\sin Kl|_{\text{resonance}} = \sinh pl/2a$

and the vibration is slightly out of phase with the impressed force.

Analysis of Vibration Damper

A mathematical analysis of the action of a damper of the Stockbridge[2] type is possible if certain simplifying assumptions are made.

The usual Stockbridge damper consists of 2 masses supported at the ends of a rod and clamped to the conductor by means of parallel groove planks. The masses have weights of about 7 pounds each, and the distance between them is about 5 feet. In this analysis it is proposed to replace such a complex dynamic system by the simpler system of figure 1.

The approximation is permissible because the action of the rod is essentially that of a damped spring at whose ends are connected masses.

Power Consumed by Damper

Essentially, a damper is a device for extracting energy. Its action in suppressing vibration is due to its dissipation of the energy contained in the transient oscillations which the cable persists in following even after the action of the wind has stopped.

The average power consumed by the damper of figure 1 is:

$$P_{\text{avg.}} = \frac{RM^2D^2\omega^4}{2[R^2 + (\omega M - \mu/\omega)^2]} \qquad (14)$$

where

D = displacement of cable at the damper
R = damping coefficient of the damper spring
μ = spring constant
M = moving mass of the damper
$\omega = 2\pi f$
f = frequency

Hence it is apparent that in the steady state the average amount of energy dissipated in the damper is much greater at the high frequencies; however, the higher frequencies have smaller amplitudes D and the effect balances out.

The maximum power dissipated by the damper occurs at the resonant frequency $F_R = 1/2\pi \sqrt{\mu/M}$. It is $P_{\text{avg.}} = \frac{M^2D^2\omega_R^4}{2R}$ where $\omega_R = 2\pi F_R$. It appears that if R is quite small the damper will extract considerable energy at the resonant frequency. A damper should be designed, therefore, so that its

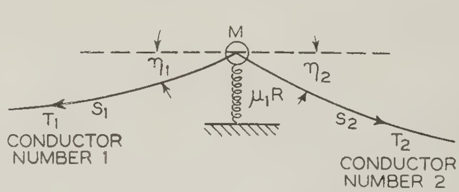


Fig. 2. System of forces acting on the end of a span with movable supports

natural or resonant frequency coincides with the frequency of the most troublesome vibration. If the damper contains several springs and masses, a band of frequencies may be damped. The dynamic displacement factor D of the cable at the damper indicates that if the damper be placed at a node, the energy absorbed will be zero. Because of the irregularity of the wind and slight yielding of the supports, however, the nodes are not fixed points, and energy

absorption always is taking place. The more fundamental action is that of a traveling wave on the damper which will be discussed later.

By maximizing the expression for the average power absorbed with respect to the various quantities the following results are obtained:

1. *Optimum R* (other constants fixed)

$$R^2 = (\omega M - \mu/\omega)^2 \tag{15}$$

2. *Optimum M* (other constants fixed)

$$M = R^2/\mu + \mu/\omega^2 \tag{16}$$

3. *Optimum* μ (other constants fixed)

$$\mu = \omega^2 M \tag{17}$$

The last condition indicates that the spring should be adjusted for resonance for best results, and suggests the greater flexibility of a damper whose spring constant or mass may be adjusted.

There is, of course, a danger that too great a flexibility of the spring may give rise to dangerous amplitudes and their corresponding destructive effect on the damper.

Reaction of Damper on Conductor

An expression for the dynamic reaction of a damper on an oscillating conductor shows rather clearly the mechanism by which energy is abstracted from the conductor. The expression is:

$$F = \frac{\omega MD\sqrt{\mu^2 + R^2\omega^2}}{\sqrt{R^2 + (\omega M - \mu/\omega)^2}} \cos(\omega t - \theta) \tag{18}$$

where

$$\theta = \phi + \beta \qquad \phi = \tan^{-1}\frac{\omega M - \mu/\omega}{R}$$

$$\beta = \tan^{-1}\mu/\omega R$$

in which the quantities have been defined previously. The point of support of the damper is experiencing the motion $u = D\cos\omega t$ and the static weight of the damper is neglected. With the condition that $\mu = \infty$ a correct expression is obtained for the reactive force due to a mass M fixed to the conductor. The energy-abstracting action of the conductor is due to the phase displacement θ. The reactive force is proportional to the amplitude of the motion and, at the resonant frequency, to the mass of the damper.

Extremities Subject to Yielding

An analysis of the behavior of standing waves may be carried out when the extremities are subject to yielding in a vertical direction if it is assumed that the ends of the cable are constrained to move in a vertical direction by the action of springs and masses attached to the ends. If the spring constants are denoted by μ and the masses by M, and if it is assumed that there is negligible damping in the conductor ($R = 0$), the solution is dependent upon the equation

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial s^2} \text{ where } a = \sqrt{T/m}$$

subject to the conditions:

1. At $s = 0$ $\qquad M\dfrac{\partial^2 u}{\partial t^2} + \mu u = T\dfrac{\partial u}{\partial s}$

2. At $s = l$ $\qquad M\dfrac{\partial^2 u}{\partial t^2} + \mu u = -T\dfrac{\partial u}{\partial s}$

3. At $t = 0$ $\qquad u = 0$

The solution is:

$$u = (A\sin rs + D\cos rs)\sin rat \tag{19}$$

in which

$$\frac{A}{D} = \frac{\mu - Mr^2a^2}{rT} \qquad A = \text{an arbitrary constant}$$

$$\tan rl = \tan 2\theta \qquad \theta = \tan^{-1}\frac{\mu - Ma^2r^2}{rT}$$

The analysis of the general case thus leads to a somewhat complicated transcendental equation for the possible values of r. The accuracy can be

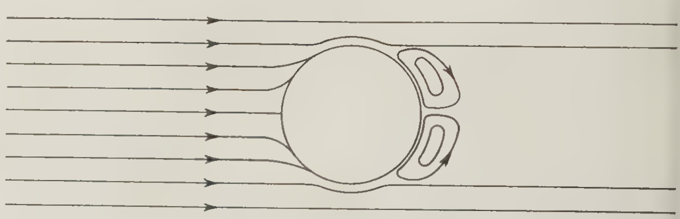


Fig. 3. Formation of vortexes behind a cylinder in a moving fluid

checked, however, by considering first whether it reduces to the correct solution for the rigidly fixed case. This can be done by placing $\mu = \infty$ and $M = 0$. The result is:

$$\tan\theta = \infty, \quad \theta = \pi/2 + 2s\pi \text{ and } s = 0, 1, \ldots$$

$$\tan 2\theta = \tan(\pi + 4s\pi) = 0 = \tan rl$$

therefore

$$r = \frac{n\pi}{l} \text{ and } \frac{A}{D} = \infty$$

and

$$u = \sum_0^\infty A_n \sin\frac{n\pi s}{l}\sin\frac{n\pi a}{l}t \tag{19a}$$

This agrees with the solution obtained for rigid ends which may be obtained by elementary methods, and is the solution for standing waves.

Slight Mobility of Supports

By placing $M = 0$ and letting μ be great, the practical case of slight yielding at the supports is obtained, and the following expressions are applicable:

$$f = \text{frequency of vibration} = \frac{na}{l}\left(1 - \frac{2T}{l\mu}\right)$$

$$\frac{A}{D} = \frac{\mu}{rT} \text{ and } r = \frac{n\pi}{l}\left(1 - \frac{2T}{l\mu}\right)$$

by which it is indicated that the frequencies of the harmonics are slightly lowered. The ratio A/D, however, yields the important information that there are no nodes. This is evident from the observation that even though A/D is large and A is arbitrary, nevertheless D is not zero, and from the equation

$$u = (A \sin rs + D \cos rs) \sin rat \tag{19b}$$

it appears that there are no true nodes although there are points at which the displacement u is always small as a consequence of the smallness of D. Perhaps this conclusion explains the action of the damper at almost any point. The solution of the general case requires a graphical analysis of equation 19*b*, for which given values of the constants must be used.

Reflection Coefficients

The fundamental position of traveling waves in the theory of vibration suggests the value of an analysis of their reflection at points of discontinuity. The complete derivation of these equations is found in the appendix.

Reflection of Traveling Wave at End of Span Having Movable Supports. In this analysis the system of figure 2 is assumed. The span terminates on a mass M, which is given a degree of vertical freedom by a spring that is supposed to have a spring constant μ and a viscous resistance coefficient R. For generality it may be assumed that the 2 spans have the masses per unit length m_1 and m_2, respectively, and the tensions T_1 and T_2. It is found that for the size of cable ordinarily used (about one inch in diameter) and for frequencies of the order of magnitude of the objectionable frequencies present in transmission lines, the effect of rigidity in the reflection and transmission coefficients may be neglected, provided that the tensions are not such as to allow a sag exceeding $^1/_{10}$ of the span length.

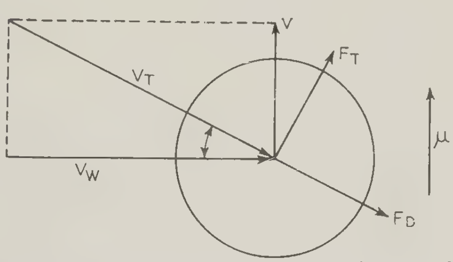


Fig. 4. Diagram of wind forces acting on a transmission line conductor

v = velocity of cylinder in u direction as a result of its oscillation $\partial u/\partial t$. V_w = component of wind velocity acting in horizontal direction at right angles to the span. V_t = velocity of wind relative to the moving cylinder

General Reflection Coefficient. If the incident wave traveling along conductor number 1 of figure 2 be represented by the equation

$$u_I = A \cos \omega(t + s_1/v_1) \text{ where } v_1 = \sqrt{T_1/m_1}$$

The reflected wave along conductor number 1 is given by

$$u_R = \frac{AQ}{P} \cos\left[\omega\left(t - \frac{s_1}{v_1}\right) + \theta\right] \tag{20}$$

where

$$Q = \sqrt{\alpha^2 + (\beta - \gamma)^2}$$
$$P = \sqrt{\alpha^2 + (\beta + \gamma)^2}$$
$$\alpha = \omega^2 M - \mu$$
$$\beta = \omega\sqrt{m_1 T_1}$$
$$\gamma = \omega(\sqrt{m_2 T_2} + R)$$
$$\theta = \theta_1 - \theta_2$$
$$\theta_1 = \tan^{-1}\frac{\beta - \gamma}{\alpha}$$
$$\theta_2 = \tan^{-1} - \frac{(\beta + \gamma)}{\alpha}$$

This complex expression may be simplified by the assumption that the adjacent spans are identical; that is, $m_1 = m_2$ and $T_1 = T_2 = T$, then

$$\frac{Q}{P} = \sqrt{\frac{\alpha^2 + \omega^2 R^2}{\alpha^2 + \omega^2(R + 2\sqrt{mT})^2}}$$
$$= \sqrt{\frac{\omega^2(M - \mu/\omega^2)^2 + R^2}{\omega^2(M - \mu/\omega^2)^2 + (R + 2\sqrt{mT})^2}} \tag{20a}$$

showing greater reflection at the higher frequencies. At the frequencies for which the spring and mass at the ends are in resonance,

$$\omega = \sqrt{\mu/M} \text{ and } \frac{Q}{P} = \frac{R}{R + 2\sqrt{mT}}$$

Since R is small, it is apparent that there is a small amount of reflection at the resonant frequency.

Transmission Coefficients

If the supports are movable, a traveling wave in span 1 (figure 2) proceeds to span 2. The transmitted wave is defined by

$$u_{\text{trans.}} = \frac{\delta_1}{\delta_2}\frac{AQ_1}{P} \cos\left[\omega(t - s_2/v_2) + \epsilon_1\right] \tag{20b}$$

in which

$$P = \sqrt{\alpha^2 + (\beta + \gamma)^2} \qquad Q_1 = 2\omega\sqrt{m_1 T_1}$$
$$\alpha = \omega^2 M - \mu \qquad \gamma = \omega(\sqrt{m_2 T_2} + R)$$
$$\delta_1 = \cos \eta_1$$
$$\delta_2 = \cos \eta_2 \qquad \epsilon_1 = \pi/2 - \theta_3 \quad \theta_3 = \tan^{-1}\frac{\alpha}{(\beta + \gamma)}$$

For ordinary spans, the angles η_1 and η_2 are small, and the ratio $\frac{\delta_1}{\delta_2}$ may be taken as unity, if as before, the spans are identical. The expression Q_1/P reduces to

$$\frac{Q_1}{P} = \frac{2\omega\sqrt{mT}}{\sqrt{(\omega^2 M - \mu)^2 + \omega^2(R + 2\sqrt{mT})^2}}$$

where

$m_1 = m_2 = m$ and $T_1 = T_2 = T$

This expression reduces to zero if $\mu = \infty$ or the end is fixed. At resonance the expression is a maximum, showing that in this case there is a minimum of reflection and a maximum of transmission. If M is appreciable the reflection coefficient becomes quite small at the higher frequencies.

Since both the reflection and transmission coefficients involve the frequency in a nonlinear manner, it is apparent that when a complex wave, which may be resolved into simple waves of several frequencies, impinges on a movable support it is partially reflected and partially transmitted in a very complex manner. Since the transmission coefficient is highest for the wave that is in resonance with the movable supports it might be possible in this manner to build up objectionable vibrations in several adjacent spans as a result of the discriminating action of the supports in favor of waves of that particular frequency.

Reflection From Damper

In an analysis of the action of a vibration damper it is of interest to observe the effect of a damper on a traveling wave. On the assumption that the frequency of vibration and the size of wire are such that the effects of rigidity may be neglected as discussed previously, and that the vibrations are of the small amplitudes

$$u_{\text{incident}} = A\epsilon^{j\omega(t-s/v)}$$

$$u_{\text{reflected}} = B\epsilon^{j\omega(t+s/v)}$$

$$u_{\text{transmitted}} = C\epsilon^{j\omega(t-s/v)}$$

because of the linearity of the equations, only the real parts of the solutions are retained. If equation 18 is used for the dynamic reacting force of the damper the boundary conditions can be defined as usual, and the reflection and transmission coefficients may be obtained. Because of the complex character of the reactive force the expressions for these coefficients are quite involved.

Reflection Coefficient

If the incident wave is represented by the equation

$$u_I = A \cos \omega(t - s/v) \text{ where } v = \sqrt{\frac{T}{m}}$$

the reflected wave is defined by

$$u_R = -\frac{Ak}{V} \cos [\omega(t + s/v) + (\epsilon - \beta_1)] \tag{21}$$

in which

$$\frac{K}{V} = \frac{1}{\sqrt{1 - \dfrac{4\omega^2 R\sqrt{mT}}{z_1^2} + \dfrac{4z^2 mT}{z_1^2 M^2}}}$$

$$z = \sqrt{R^2 + (\omega M - \mu/\omega)^2} \qquad z_1 = \sqrt{\omega^2 R^2 + \mu^2}$$

$$\epsilon = -(\phi + \beta) \qquad \phi = \tan^{-1}\frac{\omega M - \mu/\omega}{R}$$

$$\beta = \tan^{-1}\frac{\mu}{\omega R} \qquad \beta_1 = \tan^{-1}\frac{K \sin \epsilon + 2T\omega/v}{K \cos \epsilon}$$

$$K = \frac{\omega M\sqrt{R^2\omega^2 + \mu^2}}{\sqrt{R^2 + (\omega M - \mu/\omega)^2}}$$

The quantities M, R, and μ are those defined previously for the damper. When the denominator of the expression for K/V becomes imaginary, it is interpreted as a 90 degree further change of phase in the reflected wave. The complexity of this expression makes it difficult to analyze. Since the damping coefficient R is small, however, it appears that the denominator of the expression for K/V is approximately equal to unity if the frequency of the traveling wave is the same as the resonance frequency of the damper; thus, it appears that traveling waves having frequencies close to the resonance frequency of the damper are quite strongly reflected. Of course, perfect reflection does not occur in this case because of the viscous damping coefficient R of the damper. Much information cannot be obtained from the above equations, however, without the use of numerical values for the constants. This reduction may be done in a practical case.

Energy Considerations of Traveling Waves

Any vibration suppressing device must in some manner extract energy from the vibrating transmission line. On the hypothesis that the tension of the cable is great in comparison with its mass per unit length, an expression may be written for the average energy per unit length of the wire. The expression for a traveling wave of amplitude A and of frequency $f = \omega/2\pi$ is:

$$W = \frac{\text{energy}}{\text{length}} = \frac{1}{2} m\omega^2 A^2 \tag{13}$$

This relation shows that the high frequency components of a complex traveling wave carry an appreciable amount of energy even if their amplitudes be small. Since the energy varies as the square of the amplitude, it is apparent that any factor that tends to increase the damping coefficient R has a great effect in suppressing transient oscillations. This may be seen from the equation of damped traveling waves

$$u = A\epsilon^{-\frac{Rs}{2}\sqrt{\frac{1}{mT}}} \cos \omega(t - s/a)$$

In case R is appreciable the amplitude is reduced rapidly with distance. Any factor such as interstrand friction, which tends to increase R, has great influence in absorbing energy and in mitigating vibration.

Effect of Wind Velocity

The amplitude of the forced oscillation is dependent on V_W in the following ways:

1. The angular frequency $\omega = 1.16V_W/D$ varies directly with the wind velocity.

2. The imaginary part of K, or the part that prevents $\sin Kl$ from going to zero and limits the amplitude at quasi-resonant values of ω, increases with an increase of ω or of wind velocity. This is due to the wind velocity viscous damping term. See equation 11.

3. The factor M_T increases somewhat with wind velocities.

Effect of Cable Constants

The weight of the cable appears to lessen the damping effect of the wind and the damping factor R of the material. An increase in the weight per unit length lessens the damping effect; however, the factor w in the denominator of the expression for u lessens the amplitude of the nonresonant vibrations.

The equation for K shows the effective lessening of the damping caused by an increase in tension.

It may be seen that as long as damping is considered, or K is taken as complex, there are no true nodes except at the ends. There are, of course, points of minimum and maximum amplitude, but they are not true nodes or antinodes. If the wind is of variable velocity, these false nodal points must continually shift their positions because of changing values of ω. The amplitude appears to be a function of the cube of the diameter, suggesting that cables of large diameter are most prone to large amplitudes.

The fact that at the resonant frequency

$$|\sin Kl|_{\text{resonance}} = \sinh\left[\frac{l}{2}\sqrt{\frac{m}{T}}\,(R + M_D D V_W)\right]$$

shows most clearly the influence of several line constants on the magnitude of the amplitude at resonance.

Hollow Cables

The Swiss engineer, Max Preiswerk, has proposed a theory of a vibrationless cable. He proposes to have a hollow conductor inside of which is a steel core. The steel core is loose and at a different tension than the envelope. Preiswerk argues that in such a system 2 natural frequencies exist and one or the other is always opposing the establishment of all types of standing waves resulting from an accumulation of traveling waves. The situation, he argues, is similar to that of 2 coupled vibrating systems which, when coupled together, have a natural frequency differing from that of either system taken separately.

A mathematical analysis of this theory produces many interesting conclusions. Since a mathematical analysis of a loose core is impossible because of the discontinuous character of the motion, consideration will be given to a system that is more responsive to analysis and less destructive in its action, for it does not allow the 2 conductors to bump together.

Assume that some material of an elastic character is inserted between the inner and outer cables. For the purpose of analysis, assume that this material produces an interaction between the cable and the core proportional to the difference of the absolute displacements at right angles to the lengths of the conductors. Assume also that the packing material absorbs energy in such a way as to produce viscous damping proportional to the relative velocities between the inner and outer cables.

Let all quantities with a subscript 1 refer to the inner cable, and all quantities with a subscript 2 refer to the outer hollow cable. Then let

u_1 = transverse displacement of the inner cable from the normal catenary form as position of static equilibrium
m_1 = mass per unit length of the inner cable
T_1 = tension of the inner cable
k = elastic coefficient of the packing material
R = damping coefficient of the packing material
t = time
s = distance measured along the conductor
A_2 = amplitude of impressed force per unit length acting on outer cable
$\omega_1 + \beta$ = frequency and phase constant

Under the assumption that the tension is practically constant throughout the span length, and that the amplitudes of the transverse vibrations are small, the equations of motion are:

$$m_1\frac{\partial^2 u}{\partial t^2} = T_1\frac{\partial^2 u}{\partial s^2} + k(u_2 - u_1) + R\frac{\partial}{\partial t}(u_2 - u_1) \tag{22a}$$

$$m_2\frac{\partial^2 u}{\partial t^2} = T_2\frac{\partial^2 u}{\partial s^2} + k(u_1 - u_2) + R\frac{\partial}{\partial t}(u_1 - u_2) + A_2\cos(\omega t + \beta s) \tag{23a}$$

Since the inner and outer conductors must hang in identical catenary curves in the static state, each static curve must be equal to $y = H/w \cosh (xw/H)$ in which $m = w/g$, and g is the constant of acceleration due to gravity. To a great degree of accuracy for a taut cable $H = T$. The static curves must be identical so that there will not be a static stress, and the condition that

$$\frac{T_1}{m_1 g}\cosh\frac{x m_1 g}{T_1} = \frac{T_2}{m_2 g}\cosh\frac{x m_2 g}{T_2}$$

must be valid for all values of x. If, in particular, $x = 0$ then $T_1/m_1 g = T_2/m_2 g$ or $T_1/m_1 = T_2/m_2 = a^2$. This equality satisfies all values of x.

Standing Waves

Eliminating u_1 from equations $22a$ and $23a$, placing $A_2 = 0$ for the freely vibrating case, and assuming a solution of the type $u = C\epsilon^{j(\omega t + \beta s)}$ produces the following solution for u_2:

$$u_2 = \sum_0^\infty\left(A_n \sin\frac{\pi a n t}{l} + B_n \cos\frac{\pi a n t}{l}\right)\sin\frac{\pi n s}{l} + \epsilon^{-Rt/2m}\sum_0^\infty (C_n \sin \alpha_n t + D_n \cos \alpha_n t)\sin\frac{n\pi s}{l}.$$

in which

$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} \qquad \alpha_n = \sqrt{\frac{k}{m} + \frac{a^2 n^2 \pi^2}{l^2} - \frac{R^2}{4m^2}}$$

l = span length $\qquad A_n, B_n, C_n, D_n$ = arbitrary constants

Use has been made of the fact that the supports are rigid and are nodal points. The first summation

in the solution for u_2 represents the usual standing wave solution for a span with undamped vibration. This represents the trivial case in which the inner and outer cables are vibrating in such exact synchronism that they do not react with one another. Since any force tending to cause the system to oscillate necessarily acts on the exterior conductor, asymmetry is introduced from the beginning of the motion, and synchronism cannot exist.

The second summation represents the actual physical case when the 2 cables react one against the other, but it will be noticed that the whole disturbance dies out because of the factor $\epsilon^{-Rt/2m}$. If the damping coefficient R of the packing material is large, the quantity α_n is imaginary for small values of n, showing that in such a case vibrations having great loop lengths do not occur. In any case the exponential damping term soon nullifies the entire motion.

Distinction Between Distributed and Lumped Systems

It is evident that there is a difference between coupled systems having lumped constants and systems having distributed constants. In lumped systems the coupling causes the natural frequency of the resulting system to be removed from that of the individual systems, but in distributed systems both uncoupled systems have an infinite number of natural frequencies, and so does the coupled system. It does not seem possible, therefore, by coupling 2 cables to remove the natural frequency of the system beyond the range of the most prevalent objectionable frequency of the system produced by wind eddies.

The chief advantage resulting from the type of system here considered is the fact that a material having a large damping coefficient R may be employed; hence, vibrations of a transient character are reduced promptly.

Traveling Waves

Before analyzing the forced vibration it is of interest to study the behavior of a traveling wave on the system under consideration. If a traveling wave of angular frequency ω and velocity v is considered, the following solutions are applicable:

1. *Inner and Outer Cables in Exact Synchronism.* This represents the trivial case in which synchronism between inner and outer conductors is so perfect that there is no interaction.

$$u_2 = A \cos \omega(t - s/v) + B \cos \omega(t + s/v) \qquad (24)$$

2. *Loose Coupling or High Frequencies.* If the spring effect of the packing is not excessive, it may be assumed that $k/\omega^2 m$ is much less than unity. The solution is:

$$u_2 = A\epsilon^{\frac{R\omega s}{2ma\sqrt{\omega^2-k/m}}} \cos \omega\left[t + \frac{s\sqrt{\omega^2 - k/m}}{\omega a}\right] + B\epsilon^{-\frac{R\omega s}{2ma\sqrt{\omega^2-k/m}}} \cos \omega\left[t - \frac{s\sqrt{\omega^2 - k/m}}{\omega a}\right] \qquad (25)$$

This equation indicates damped traveling waves whose damping and velocity of propagation depend upon the frequency; hence, a complex wave would suffer distortion.

3. *Frequency ω Approximately Equal to $\sqrt{k/m}$.* In this case the waves are of the form

$$u_2 = A\epsilon^{-\alpha_1 s} \cos \omega[t - \beta_1 S] + B\epsilon^{\alpha_1 s} \cos \omega[t + \beta_1 s] \qquad (26)$$

where

$$\alpha_1 = \frac{1}{a\sqrt{2}}\sqrt{\frac{R\omega}{m}} \text{ and } \beta_1 = \frac{1}{a\sqrt{2}}\sqrt{\frac{R}{\omega m}}$$

and distortion is produced as before.

4. *Low Frequencies or Close Coupling.* This case, which may be approximated closely in practice, represents waves traveling with a velocity $v = 2a\sqrt{km/R}$ which is great, since in general R is small. The attenuation in this case is extremely great, and there is no distortion.

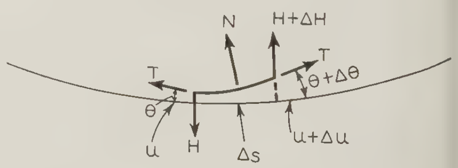
$$u_2 = A\epsilon^{-\alpha s} \cos \omega[t - \beta s] + B\epsilon^{+\alpha s} \cos \omega[t + \beta s] \qquad (27)$$

in which $\omega^2 < k/m$, $\alpha = (1/a)\sqrt{k/m}$ and $\beta = R/2ma\sqrt{k/m}$.

Forced Vibration

If a force of the form $A_2 \cos \omega t$ is assumed to be of constant amplitude throughout the length of the conductor and of an oscillatory character acting at

Fig. 5. Forces acting on a cable in a condition of equilibrium



every point of the outer cable, the following equation is obtained for the vibration of the outer cable in the steady state:

$$u_2 = \frac{A_2}{\omega^2}\sqrt{\frac{(k - \omega^2 m_1) + R^2\omega^2}{\left[\omega^2 m_1 - \left(1 + \frac{m_1}{m_2}\right)k\right]^2 + R^2\omega^2\left(1 + \frac{m_1}{m_2}\right)^2}} \cos(\omega t + \theta_3) \qquad (28)$$

where

$$\theta_1 - \theta_2 = \theta_3 \quad \theta_1 = \tan^{-1}\frac{R/m_1}{k/m_1 - \omega^2} \quad \theta_2 = \frac{\omega\delta R}{\omega^2 - \delta k} \quad \delta = \frac{1}{m}$$

The transient solution must be added to equation 28 in order to satisfy the boundary values. This complex expression gives an approximate indication of the behavior of the system under the action of a hypothetical wind.

It will be noticed that in every case it is the factor R that is active in reducing the vibration. This is only a special application of the principle that energy must be absorbed in order to damp vibra-

tion. The energy absorption in the case of Preiswerk's loose core cables must come from the shock of bumping, and, of course, this action must be destructive. In the above case no such destructive action exists.

Appendix

Derivation of Equations

The equation of the catenary curve, which is the static equilibrium position of the cable, is:

$$y = \frac{H}{w} \cosh \frac{xw}{H}$$

As a typical case for a cable having a diameter of one inch and a span length of 1,200 feet, let

H = the horizontal component of T
T = tension
w = weight per unit length
l = span length

but

H = 8,500 pounds = T approximately
w = 0.858 pounds per foot

then

$$\frac{w}{H} = \frac{0.858}{8{,}500} = 10^{-4} \text{ approximately}$$

and the equation of the span can be taken to be approximately that of a parabola

$$y = wx^2/2H$$

then

$$\frac{dy}{dx} = \frac{wx}{H} \text{ and } \frac{d^2y}{dx^2} = \frac{w}{H}$$

The curvature of this span is given by

$$K = \frac{1}{R} = \frac{d^2y/dx^2}{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}} = \frac{w}{T}$$

approximately, since for all points (wx/H^2) may be neglected in comparison to unity. Thus it is seen that the curvature of the static curve may be regarded as having the small constant value w/H, or the radius of curvature R as the large constant value T/w for a typical span.

The fact that the equilibrium curve can be considered with very little error as a circle of large radius of curvature T/w simplifies the analysis greatly.

Let s be the position of static equilibrium of the cable and measure u in the normal direction to the static position of equilibrium s. Now consider the shearing force H at the point s (figure 5) and the shearing force $H + \Delta H$ at the point $S + \Delta S$.

Since small deviations are being considered from the static position of equilibrium, which takes into account the weight of the cable, the weight of the cable may be neglected and components of shearing force and tension may be taken in the direction u normal to the cable at the point s. Consider, in addition, an outside force N per unit length acting normal to the cable.

Equating forces in the u direction,

$$T \sin \theta + H \cos \theta = T \sin (\theta + \Delta\theta) + (H + \Delta H) \cos (\theta + \Delta\theta) + NdS$$

$$\lim_{\Delta\theta \to 0} \sin (\theta + \Delta\theta) = \sin \theta + \frac{\partial^2 u}{\partial s^2} dS$$

$$\lim_{\Delta\theta \to 0} \cos (\theta + \Delta\theta) = \cos \theta$$

then

$$T \frac{\partial^2 u}{\partial s^2} dS + \Delta H + NdS = 0$$

$$\Delta H = \frac{\partial H}{\partial s} dS$$

and

$$T \frac{\partial^2 u}{\partial s^2} + \frac{\partial H}{\partial s} = -N$$

Since only transverse vibrations are of importance here, the components of force along s need not be written.

There is another condition expressing the fact that there is no

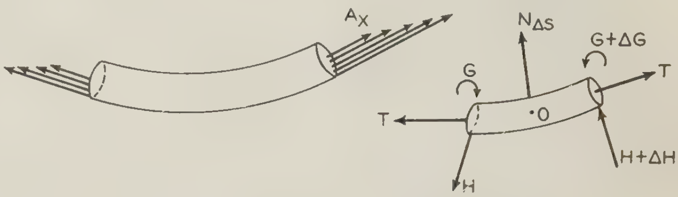


Fig. 6. Free body diagram of a differential length of a conductor vibrating transversely

tendency to turn the element about an axis parallel to the z-axis.

Suppose that the moment of all the normal forces A_x on one end of a differential element is $-G$ and at the other end is $G + \frac{\partial G}{\partial s} ds$ as indicated in figure 6.

Taking moments about o

$$G = G + \Delta G + H\frac{\Delta S}{2} + (H + \Delta H)\frac{\Delta S}{2}$$

then

$$-\Delta G = H\Delta S + \frac{(\Delta H)(\Delta S)}{2}$$

$$\lim_{\Delta s \to 0} H = -\frac{\partial G}{\partial s} \text{ and } \frac{\partial H}{\partial s} = -\frac{\partial^2 G}{\partial s^2}$$

The moment G may be considered as a result of the stresses of the longitudinal fibers of the cable in bending. Consider a bent section of wire so that it has a radius of curvature R and let q be the distance of a given fiber from the neutral surface. Since the neutral surface is the surface at which the fibers are in a state of equilibrium,

$$\frac{\text{stretched length}}{\text{unstretched length}} = \frac{R - q}{R} = 1 - \frac{q}{R}$$

Let E be Young's modulus of elasticity for the material of the wire, then

$$\text{normal stress} = A_x = -\frac{Eq}{R}$$

The moment about the elastic surface o-o (figure 7) is:

$$G = -\int_s\int A_x q dS = \frac{E}{R}\int_s\int q^2 dS$$

but $\int_s\int g^2 ds$ is the moment of inertia I of the plane area of the normal cross-sectional area of the conductor with respect to the diameter o-o.

It has been found that for a typical static span the curvature is very small, being of the order w/T; hence, any appreciable curvature must be due to the dynamic wave. For the radius of curvature of the dynamic waves occurring on a tightly stretched transmission line $1/R = \partial^2 u/\partial S^2$ and the unbalanced force N is defined by

d'Alembert's principle $-N = m\partial^2u/\partial t^2 - F$ where m is the mass per unit length, $\partial^2u/\partial t^2$ is the acceleration, and F is any other force per unit length.

The value of N is

$$-N = T\frac{\partial^2 u}{\partial s^2} + \frac{\partial H}{\partial s} \quad \text{where} \quad \frac{\partial H}{\partial s} = -\frac{\partial^2 G}{\partial s^2} \text{ and } G = EI\frac{\partial^2 u}{\partial s^2}$$

and the equation of motion becomes

$$m\frac{\partial^2 u}{\partial t^2} + EI\frac{\partial^4 u}{\partial s^4} = T\frac{\partial^2 u}{\partial s^2} + F(s, t) \tag{1}$$

The EI term involves the effect of rigidity. Monroe and Templin[7] have made some measurements of EI for a cable subjected to tension.

Traveling Waves

Placing $F = 0$, assuming $u = \cos\omega(t - s/v)$ and substituting in

$$m\frac{\partial^2 u}{\partial t^2} + EI\frac{\partial^4 u}{\partial s^4} = T\frac{\partial^2 u}{\partial s^2} \tag{1a}$$

where

$k = 1/v \quad T/m = a^2$ and $EI/m = \beta^2$

produces the following result:

$$2k^2 = -\frac{a^2}{\beta^2\omega^2} \pm \sqrt{\frac{a^4}{\beta^4\omega^4} + \frac{4}{\beta^2\omega^2}}$$

Expanding, $k = \pm 1/a$ or $v = \pm a\sqrt{T/m}$ provided that x^2 is negligible in comparison with x when f = frequency, $\omega = 2\pi f$ and

$$x = \frac{4EIm\omega^2}{T^2}$$

If $m = \pi\rho r^2$ and $I = \pi r^4/4$ in the above expression,

$$x = \frac{4\rho\pi E r^6 f^2}{T^2}$$

The units used are the foot and the pound. E must be expressed in pounds per square foot and I in (feet)^4.

In the general case in which x is not small, the above equation may be solved for k, and since $k = 1/v$,

$$v = \omega\sqrt{\frac{2EI}{T}}\sqrt{\frac{1}{\sqrt{1 + \frac{4EIm\omega^2}{T^2}} - 1}} \tag{2}$$

Standing Waves on Rigid Conductors

Equation 1a may be solved by the usual method of separation of variables with the assumption that $u = \phi(S)\psi(t)$. Making use of the boundary conditions $u = o$ at $s = o$ and $u = o$ at $s = l$ leads to an extremely complicated transcendental equation for n, as follows:

$$\frac{\sinh r_1 l \sin r_2 l}{1 - \cosh r_1 l \cos r_2 l} + \frac{2n\beta}{a^2} = 0$$

where

$$r_1^2 = \frac{a^2}{2\beta^2} + \frac{1}{2}\sqrt{\frac{a^4}{\beta^4} + \frac{4n^2}{\beta^2}} \qquad r_2^2 = -\left(\frac{a^2}{2\beta^2} - \frac{1}{2}\sqrt{\frac{a^4}{\beta^4} + \frac{4n^2}{\beta^2}}\right)$$

$$\beta^2 = \frac{EI}{m} \qquad f = \frac{n}{2\pi}$$

It is found for the usual constants of a transmission line that r_1 is large and r_2 is small; therefore, the approximations $\cosh r_1 l = \sinh r_1 l$ and $\cos r_2 l \to 1$ are justifiable.

The above expression then reduces to

$$\tan r_2 l = \frac{2n\beta}{a^2} \text{ where } r_2^2 = \frac{n^2}{a^2}$$

and is subject to the very accurate approximate solution

$$n = \frac{r\pi}{l}\sqrt{\frac{T}{m}}\left(1 + \frac{2}{l}\sqrt{\frac{EI}{T}}\right) \tag{3}$$

or

$$f = \frac{n}{2\pi} = \frac{r}{2l}\sqrt{\frac{T}{m}}\left(1 + \frac{2}{l}\sqrt{\frac{EI}{T}}\right)$$

in which $r = 0, 1, 2, \ldots$.

Damped Vibrations

In this case the important results are the values of v in the traveling wave solution $u = A\cos\omega(t - s/v)$. By substituting these values in the equation $m\partial^2u/\partial t^2 = T\partial^2u/\partial s^2 - R\partial u/\partial t$, making use of the exponential form for the trigonometric relations, and assuming that the damping factor R is small, it is found that

$$u = A\epsilon^{-\frac{Rs}{2}\sqrt{\frac{1}{mT}}}\cos\omega(t - s/a) \tag{5}$$

Standing Waves With Damping

An approximate general solution of the equation $m\partial^2u/\partial t^2 = T\partial^2u/\partial s^2 - R\partial u/\partial t$ may be obtained by making the substitution $u = y(s, t)\epsilon^{-\frac{Rt}{2m}}$ which produces the expression

$$\frac{\partial^2 y}{\partial t^2} - \frac{R^2}{4m^2}y = \frac{T}{m}\frac{\partial^2 y}{\partial s^2}$$

Since R/m is small, however, this term may be neglected on the supposition that y is small. Then

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\frac{\partial^2 y}{\partial s^2}$$

whose solution is:

$$y = \phi_1(s - at) + \phi_2(s + at)$$

Hence

$$u = \epsilon^{-\frac{Rt}{2m}}[\phi_1(s - at) + \phi_2(s + at)] \tag{6}$$

Standing Waves

The solution for standing waves with damping may be obtained easily by the method of separation of variables. The boundary conditions of fixity at the extremities must be imposed, and with the condition that R is small

$$u = \epsilon^{-\frac{Rt}{2m}}\sum_1^\infty (A_n \sin\beta_n t + C_n \cos\beta_n t)\sin\frac{n\pi s}{l} \tag{7}$$

$$\beta_n = \frac{1}{2}\sqrt{\frac{4n^2\pi^2a^2}{l^2} - \frac{R^2}{m^2}}$$

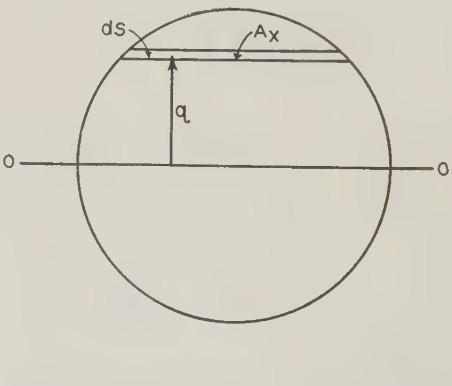


Fig. 7. Normal cross section of a vibrating conductor, showing the quantities involved in the derivation of the dynamic equations

Forced Oscillations

Consider the equation

$$m\frac{\partial^2 u}{\partial t^2} + R\frac{\partial u}{\partial t} = T\frac{\partial^2 u}{\partial s^2} + M_T D V_W^2 \cos \omega t - M_D D V_W \frac{\partial u}{\partial t} \quad (10)$$

Let

$$p = \frac{1}{m}\left[R + M_D D V_W\right] a^2 = T/m \text{ and } A = \frac{M_T}{m} D V_W^2$$

then

$$\frac{\partial^2 u}{\partial t^2} + p\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial s^2} + A\cos\omega t$$

Assuming $u = v(s)\epsilon^{j\omega t}$ and retaining only the real part of the solution,

$$\frac{\partial^2 u}{\partial t^2} + p\frac{\partial u}{\partial t} = a^2\frac{\partial^2 u}{\partial s^2} + A\epsilon^{j\omega t}$$

if

$$K^2 = \frac{1}{a^2}[\omega^2 - j\omega p]$$

then

$$\frac{d^2 v}{dS^2} + K^2 v = -\frac{A}{a^2}$$

or

$$v = C_1 \cos Ks + C_2 \sin Ks - \frac{A}{a^2 K^2}$$

The boundary conditions are: 1. At $s = o, v = o$. 2. At $s = l$, $v = o$

Then

$$v = \frac{A}{a^2 K^2 \sin Kl}[\sin Ks + \sin K(l - s) - \sin Kl]$$

where

$$K^2 = \frac{\omega}{a^2}[\omega - jp]$$

It is required that

$$[\omega - jp]^{1/2} = \omega^{1/2}[1 - {}^1/_2 jp/\omega + {}^1/_8 p^2/\omega^2 \ldots]$$

Since in general

$$p < \omega$$

$$[\omega - jp]^{1/2} = \omega^{1/2}[1 - {}^1/_2 jp/\omega]$$

and

$$K = \frac{\omega}{a}[1 - {}^1/_2 jp/\omega]$$

for practical purposes. However

$$\sin(x - jy) = \sin x \cosh y - j\cos x \sinh y$$

therefore

$$\sin Kl = \sin\left[\frac{l}{a}\left(\omega - j\frac{p}{2}\right)\right] = \sin\frac{\omega l}{a}\cosh\frac{pl}{2a} - j\cos\frac{\omega l}{a}\sinh\frac{pl}{2a}$$

At resonance frequency $\sin(\omega l/a) = 0 \quad (\omega l/a) = n\pi \quad n = 0, 1, \ldots$

and $f = (n/2l)\sqrt{T/m}$

In addition, $\cos(\omega l/a) = \pm 1$ depending upon whether n is odd or even. Then

$$|\sin Kl|_{\text{resonance}} = \sinh\frac{pl}{2a} = \sinh\left\{\frac{l}{2}\sqrt{\frac{m}{T}}(R + M_D D V_W)\right\}$$

The imaginary part of the other terms is of no interest, and may be neglected in the solution

$$u = \frac{A}{a^2 K^2 \sin Kl}[\sin Ks + \sin K(l - s) - \sin Kl]\cos\omega t$$

It will be noticed that $\sin Kl$ never can equal zero and that its imaginary part may be neglected except in the discussion of resonance; hence,

$$K = \frac{\omega}{a} \text{ and } |\sin Kl|_{\text{resonance}} = \sinh\frac{pl}{2a}$$

The phase angle which should be introduced in the $\cos \omega t$ term also may be discarded as having no bearing on the discussion, since $\omega > p$ in the practical case.

Hence

$$u = \frac{23.8\ M_T D^3}{w \sin Kl}[\sin K_s + \sin K(l - s) - \sin Kl]\cos\omega t$$

by obvious transformations and changes in units.

Energy Considerations

General expressions for the energy of transverse vibrations in an oscillating span may be obtained easily for the usual case in which the tension is great in comparison to the mass per unit length. The kinetic energy per unit length is seen to be ${}^1/_2 m(\partial u/\partial t)^2$ from general principles.

The potential energy equation requires a little more care. Let s be the length of the cable in the normal catenary form as the position of equilibrium and s_1 be the dynamic length of the cable.

Since the weight per unit length is small compared to the tension, the potential energy is the work necessary to stretch against the tension because of the loops formed by the oscillation. Potential energy stored in bending is neglected.

The potential energy V per unit length is:

$$V = T\left(\frac{\partial s_1}{\partial s} - 1\right) \text{ but } ds_1 = \sqrt{ds^2 + du^2}\ \frac{\partial s_1}{\partial s} = \sqrt{1 + \left(\frac{\partial u}{\partial s}\right)^2}$$

and

$$\frac{\partial s_1}{\partial s} = \left[1 + \left(\frac{\partial u}{\partial s}\right)^2\right]^{1/2} = 1 + \frac{1}{2}\left(\frac{\partial u}{\partial s}\right)^2 + \ldots +$$

then

$$V = \frac{1}{2}\left(\frac{\partial u}{\partial s}\right)^2 T$$

$\partial u/\partial s$ is small but not zero and lower order quantities have been neglected.

Assume now a traveling wave of the form $u = A\cos(\omega t + \beta s)$ where β and ω must be such as to satisfy $m\partial^2 u/\partial t^2 = T\partial^2 u/\partial s^2$. By carrying out the above differentiations and averaging throughout a cycle it is found that average energy per unit length

$$W = {}^1/_2 m\omega^2 A^2. \quad (13)$$

Analysis of Vibration Damper

Let the damper be represented schematically as in figure 8, and let

- x_F = absolute displacement of the damper frame relative to a point fixed in space
- x_M = displacement of the mass M from the point of equilibrium relative to the frame
- x_A = absolute displacement of the mass M relative to a fixed point in space
- μ = combined spring constant of both springs
- R = damping coefficient of the springs

The equations of motion are:

$$M\frac{d^2 x_A}{dt^2} + R\frac{dx_M}{dt} + \mu x_M = 0$$

and

$x_A = x_M + x_F$

Assume that the cable is vibrating in such a manner that at the point at which the damper is attached it is performing an oscillation of the type $x_F = D \cos \omega t$

Then the equation of motion is

$$\frac{d^2x_M}{dt^2} + \frac{R}{M}\frac{dx_M}{dt} + \frac{\mu}{M} x_M = \omega^2 D \cos \omega t$$

which has the steady state solution

$$x_M = \frac{M\omega D \sin(\omega t - \phi)}{\sqrt{R^2 + (\omega M - \mu/\omega)^2}} \text{ and } \tan\phi = \frac{\omega M - \mu/\omega}{R}$$

Power Consumed

Let W be the amount of energy dissipated in the damper. From fundamentals $dW = FdS$. Now at time t

$$dS = dx_M \text{ and } F = R\frac{dx_M}{dt}$$

Let

$$MD\omega = E_0$$

$$Z = \sqrt{R^2 + (\omega M - \mu/\omega)^2}$$

$$R\frac{dx_M}{dt} = \frac{RE_0\omega \cos(\omega t - \phi)}{Z}$$

then

$$dW = \frac{RE_0{}^2\omega^2 \cos^2(\omega t - \phi)dt}{Z^2}$$

Since the average power dissipated is of importance, this expression must be integrated through a complete cycle and divided by the elapsed time of the cycle. The result of these operations is:

$$P_{\text{average}} = \frac{RM^2D^2\omega^4}{2\left[R^2 + \left(\omega M - \frac{M}{\omega}\right)^2\right]} \quad (14)$$

The various expressions given by equations 15, 16, and 17 for the optimum values of the constants may be obtained by maximizing equation 14.

Reaction of Damper on Conductor

The reaction of the moving mass of the damper on its frame, which is equivalent to its reaction on the conductor, if the mass of the frame be neglected, is defined as

$$F = \mu x_M + R\frac{dx_M}{dt} = \frac{\omega MD\sqrt{\mu^2 + R^2\omega^2}}{\sqrt{R^2 + (\omega M - \mu/\omega)^2}} \cos(\omega t - \phi - \beta) \quad (18)$$

where

$$\phi = \tan^{-1}\frac{\omega M - \mu/\omega}{R} \text{ and } \beta = \tan^{-1}\mu/\omega R$$

by obvious transformations.

Extremities Subject to Yielding

The vertical component of tension at the support (figure 9) is

$$-T\sin(\theta + \alpha) = -T(\sin\theta\cos\alpha + \cos\theta\sin\alpha)$$

but α is small; therefore $\cos\alpha \rightarrow 1$, $\sin\alpha \rightarrow 0$ and the vertical component of tension is

$$-T\left[\sin\theta + \cos\theta\left(\frac{\partial u}{\partial s}\right)_{s=0}\right]$$

The first term of this expression represents the downward pull due to the weight of the cable and the second term represents fluctuations in force due to the dynamic oscillation of the cable. For the usual cable span the angle θ is quite small, as may be seen from the equations of the catenaries in which these spans lie; therefore, the approximation $\cos\theta = 1$ is valid.

The boundary equations at the extremities are:

$$M\frac{\partial^2 u}{\partial t^2} + \mu u = T\frac{\partial u}{\partial s} \quad \text{at } s = o$$

$$M\frac{\partial^2 u}{\partial t^2} + \mu u = -T\frac{\partial u}{\partial s} \quad \text{at } s = l$$

where the supports are constrained to move in a vertical line and M and μ are the mass and the spring constant of the support, respectively. The wave equation $m\partial^2 u/\partial t^2 = T\partial^2 u/\partial s^2$ must be solved subject to these boundary conditions. By the usual method of separation of variables, the solution is

$$u = (A\sin rs + D\cos rs)\sin rat \quad (19)$$

where A is arbitrary, but is connected to D by the relation

$$\frac{A}{D} = \frac{\mu - Mr^2a^2}{rT}$$

and the possible values of r are given by the complicated transcendental equation

$$\tan rl = \tan 2\phi$$

where

$$\phi = \tan^{-1}\frac{\mu - Ma^2r^2}{rT}$$

Having determined r, use may be made of the principle of superposition, which always applies to linear equations, and a particular solution capable of expressing any arbitrary initial configuration will result. The case of slight immobility has been discussed previously.

Reflection of Traveling Waves From Movable Supports

Consider 2 adjacent spans connected by a movable support that has some degree of freedom in a vertical line. Let this support have a mass M which is constrained to move in a vertical line by a spring having a spring constant μ and a damping coefficient R.

Let

T_1 = the tension in span 1
T_2 = the tension in span 2
m_1 = the mass per unit length of span 1
m_2 = the mass per unit length of span 2
H_1 = the shearing force exerted on the support as a consequence of rigidity of cable 1
H_2 = a similar quantity for cable 2
θ_1 = angle with the horizontal made by the catenary of cable 1 at the point of support
θ_2 = similar quantity for cable 2

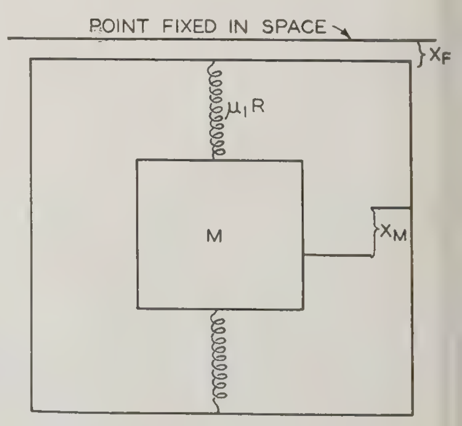


Fig. 8. Diagram of one type of vibration damper

Note that the angles θ_1 and θ_2 are given respectively by

$$\theta_1 = \tan^{-1}\left[\sinh\frac{l_1 w_1}{T_1}\right]$$

$$\theta_2 = \tan^{-1}\left[\sinh\frac{l_2 w_2}{T_2}\right]$$

where l_1 and l_2 and w_1 and w_2 are the respective lengths and weights per unit length of the 2 cables. In general, it may be seen that these angles are quite small.

The vertical component of force exerted by the cables (figure 10) on the support is

$$F_y = T_2 \sin\left(\frac{\partial u_2}{\partial s_2} - \theta_2\right) + H_2 \sin\left(\frac{\pi}{2} - \theta_2\right) +$$
$$T_1 \sin\left(\frac{\partial u_1}{\partial s_1} - \phi_1\right) + H_1 \sin\left(\frac{\pi}{2} - \phi_1\right)$$

where u_1 and u_2 are measured at right angles to the static position of equilibrium.

The boundary conditions are:

$$u_1 \sin\left(\frac{\pi}{2} - \theta_1\right) = u_2 \sin\left(\frac{\pi}{2} - \theta_2\right)$$

and

$$M\frac{\partial^2 u_v}{\partial t^2} + R\frac{\partial u_v}{\partial t} + \mu u_v = F_y$$

Let

$$u_v = u_2 \cos\theta_2 = u_I \cos\theta_1 \text{ and } u_1 = u_I + u_R$$

where

u_I = incident wave traveling on span number 1
u_R = reflected wave traveling on span number 1
u_2 = the refracted wave on span number 2

Then

$u_I = A\cos\omega(t + s_1/v_1)$ = real part of $A\epsilon^{j\omega(t+s_1/v_1)}$
$u_R = B\cos\omega(t - s_1/v_1)$ = real part of $B\epsilon^{j\omega(t-s_1/v_1)}$
$u_2 = C\cos\omega(t - s_2/v_2)$ = real part of $C\epsilon^{j\omega(t-s_2/v_2)}$

Substituting in the boundary conditions,

$$F_y = C\epsilon^{j\omega(t-s_2/v_2)}[-\omega^2 M + Rj\omega + \mu]$$

Making use of the expression for the shearing force H,

$$H = -\frac{\partial G}{\partial S} = -EI\frac{\partial^3 u}{\partial s^3}$$

Let

$$\sin\frac{\partial u_2}{\partial s_2} \longrightarrow \frac{\partial u_2}{\partial s_2} \text{ and } \cos\frac{\partial u_2}{\partial s_2} \longrightarrow 1$$

since oscillations of small amplitude are being considered. The expression for the vertical components of forces becomes

$$F_y = \cos\theta_2\left(T_2\frac{\partial u_2}{\partial S_2} - E_2 I_2\frac{\partial^3 u_2}{\partial s_2^3}\right) +$$
$$\cos\theta_1\left(T_1\frac{\partial u_1}{\partial s_1} - E_1 I_1\frac{\partial^3 u_1}{\partial s_1^3}\right) - (W_1 + W_2)$$

where W_1 and W_2 represent $1/2$ the weights of the cables of span number 1 and span number 2, respectively. These quantities represent a static load on the spring assumed at the supports, and do not influence the dynamic condition.

Substituting the assumed wave solutions of the differential equations of motion, and equating the above 2 expressions for F_y, it is found that

$$\delta_2 C\{\mu + Rj\omega - \omega^2 M\} = -\frac{\delta_2 T_2 j\omega C}{v_2} + \frac{\delta_1 T_1 j\omega(A - B)}{v_1}$$

$$\delta_1(A + B) = \delta_2 C,\ \delta_2 = \cos\theta_2 \text{ and } \delta_1 = \cos\theta_1$$

in which the effect of the shearing force H has been neglected, because

$$H_2 = -EI\frac{\partial^3 u_2}{\partial s_2^3} = -EI\frac{j\omega^3}{v_2^3}C\epsilon^{j\omega(t-s_2/v_2)}$$

It has been indicated previously that $v_2 = \sqrt{T_2/m_2}$ and for the usual size of cable and frequency of vibration this term may be neglected with respect to the term, $T_2\partial u_2/\partial s_2$ provided that $\omega < v_2$ and $EI < T$.

Solving the 2 equations above for C and B in terms of A and retaining only the real part of the solutions results in equations 20, 20*a*, and 20*b*.

Reflection From Damper

The calculation of the reflection from the damper proceeds in the same manner as above. As a boundary condition the force of re-

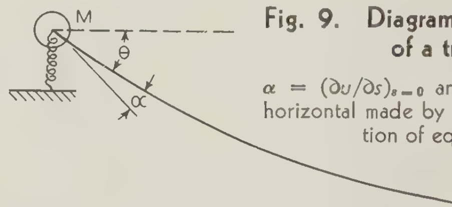


Fig. 9. Diagram of the end support of a transmission line span

$\alpha = (\partial u/\partial s)_{s=0}$ and θ = angle with the horizontal made by the catenary in the position of equilibrium at the supports

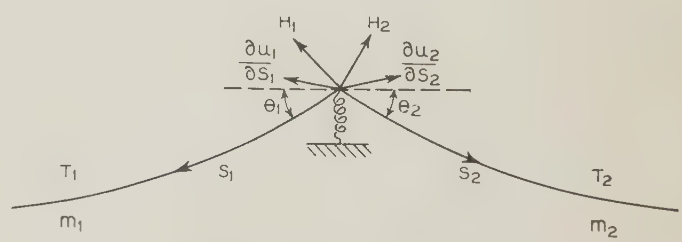


Fig. 10. Force diagram of adjacent spans connected by a movable support

action of the damper as calculated in equation 18 (converted to the exponential form) must be equated as follows:

$$\frac{\omega MC\sqrt{\mu^2 + R^2\omega^2}}{\sqrt{R^2 + (\omega M - \mu/\omega)^2}}\epsilon^{j\omega t}\epsilon^{j\theta} = T\left(\frac{\partial u_T}{\partial s}\right) - T\left(\frac{\partial u_I}{\partial s} + \frac{\partial u_R}{\partial s}\right)$$

where

$$\theta = -(\phi + \beta) \text{ and } u_T = u_I + u_R$$

Equation 21 is produced by substituting the assumed expressions for the traveling waves, satisfying the boundary equations, retaining only the real terms, and making certain trigonometric and algebraic reductions.

Hollow Cables

The analysis of coupled hollow cables is merely a solution of the simultaneous equations

$$p^2 u_1 = a^2\sigma^2 u_1 + K_1(u_2 - u_1) + E_1 p(u_2 - u_1) \quad (22)$$

$$p^2 u_2 = a^2\sigma^2 u_2 + K_2(u_1 - u_2) + E_2 p(u_1 - u_2) + A_2\cos(\omega t + \beta s) \quad (23)$$

where

$$p = \frac{\partial}{\partial t} \quad \sigma = \frac{\partial}{\partial s} \text{ and } a^2 = \frac{T_1}{m_1} = \frac{T_2}{m_2}$$

If u_1 is eliminated from these 2 equations, and only the real parts

are retained, the solution is:

$$[\alpha^2 + (s+1)\alpha\delta]u_2 = A_2(-\omega^2 + a^2\beta^2 + K_1 + Ej\omega)\epsilon^{j(\omega t + \beta s)}$$

where the operators are defined as

$$\alpha = p^2 - a^2\sigma^2 \quad \delta = K_1 + E_1 p \text{ and } s = m_1/m_2$$

FREE VIBRATIONS, STANDING WAVES

In this case A_2 is zero and the expression

$$[\alpha^2 + (s+1)\alpha\delta]u_2 = 0$$

must be solved by the assumption that

$$u_2 = C\epsilon^{j(\omega t + \beta s)}$$

The boundary condition of fixity at the extremities produces the result

$$\beta = n\pi/l \text{ where } n = 0, 1, 2, \ldots \tag{28}$$

Equation 28 connects ω with β. Using the principle of superposition because of the linearity of the equation

$$u_2 = \sum_0^\infty \left(A_n \sin \frac{\pi ant}{l} + B_n \cos \frac{\pi ant}{l}\right) \sin \frac{\pi ns}{l} +$$

$$\epsilon^{-\frac{R}{2m}t} \sum_0^\infty (C_n \sin \alpha_n t + D_n \cos \alpha_n t) \sin \frac{n\pi s}{l}$$

where

$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2} \text{ and } \alpha_n = \sqrt{\frac{k}{m} + \frac{a^2n^2\pi^2}{l^2} - \frac{R^2}{4m^2}}$$

TRAVELING WAVES

Substitute the expression $u_2 = A\epsilon^{j\omega(t - s/v)}$ representing a traveling wave, in the equation $[\alpha^2 + (s+1)\alpha\delta]u_2 = 0$. This substitution produces the expressions

$$v = \pm a \text{ and } \frac{a^2}{v^2} = 1 - \frac{1}{\omega^2 m}(k + Rj\omega)$$

Solving for v in the second expression and substituting the result in the above assumed solution, introduces the various expressions of equations 24, 25, 26, and 27 with the approximations that are mentioned in connection with them.

FORCED VIBRATIONS

In the case of forced vibrations A_2 is not zero and the equation

$$[\alpha^2 + (s+1)\alpha\delta]u_2 = A_2(-\omega^2 + a^2\beta^2 + K_1 + E_1 r\omega)\epsilon^{j(\omega t + \beta s)}$$

must be solved. This is easily done by assuming

$$u_2 = C\epsilon^{j(\omega t + \beta s)}$$

and substituting and solving for C. If the real value of the expression is retained and simplified, the result is equation 28.

References

1. VIBRATION ANALYSIS, W. B. Buchanan. ELEC. ENGG. (A.I.E.E. TRANS.), v. 53, Nov. 1934, p. 1478–85.

2. OVERCOMING VIBRATION IN TRANSMISSION CABLES, G. H. Stockbridge. *Electrical World*, v. 86, Dec. 26, 1925, p. 1304–05.

3. RESISTANCE OF A BODY MOVING IN A FLUID, T. von Kármán and H. Rubach. *Gesell. Wiss. Gottingen, Nach.*, Math. Phys. Klasse 5, 1911, p. 509–17.

4. AEOLIAN TONES, J. W. S. Rayleigh. SCIENTIFIC PAPERS (a book), Cambridge University Press, London, 1920, v. 6, p. 315–25.

5. NOTES ON THE VIBRATION OF TRANSMISSION LINE CONDUCTORS, Theodore Varney. A.I.E.E. TRANS., v. 45, 1926, p. 791–5.

6. EXPERIMENTS ON CYLINDERS OSCILLATING IN A STREAM OF WATER, A. Thom. *Phil. Mag.*, v. 12, series 7, July-Dec. 1931, p. 490–503.

7. VIBRATION OF OVERHEAD TRANSMISSION LINES, R. A. Monroe and R. L. Templin. A.I.E.E. TRANS., v. 51, December 1932, p. 1059–73.

8. HYDRODYNAMICS (a book), Horace Lamb. Cambridge University Press, London, 1932.

Better Visibility Needed on Highways at Night

More than half of the highway fatalities occur during the hours of darkness with a quarter of the daytime traffic volume. This is significant of conditions beyond the control of driver or pedestrian. Campaigns for education of driver and pedestrian, to be effective both day and night, must be accompanied by measures to improve night visibility, such as systems of fixed lights along the highways. The use of automobile headlights is ineffective and unscientific. The financing of highway lighting service is a state and not a county or township function. It is a vital necessity for preservation of life and property on the highway, and may readily be financed by surplus funds from gasoline taxation.

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FEW PEOPLE in fields closely related to the problem of public safety, even in fields dealing directly with the problems of traffic safety, have realized the vital relationship of darkness with death. Few people have realized that existing traffic hazards remain unchanged throughout the 24 hours of the day, except that darkness comes down like a veil over all hazards once every day—comes down to confuse and blind the driver and pedestrian—comes down to reduce the time allowance for life saving reflexes—comes down to subject the eye to conditions with which it cannot cope—comes down, in fact, to increase fourfold the lethal potentialities of all other traffic hazards.

Proof, if proof be needed, that those responsible for public safety do not appreciate the relationship between highway darkness and death is found in press and radio campaigns to educate the driver and pedestrian and to enforce traffic regulations, with never a suggestion for the elimination of the greater hazard of darkness.

A paper recommended for publication by the A.I.E.E. committee on production and application of light, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted April 7, 1936; released for publication May 6, 1936. An address embracing the substance of this paper was presented at the A.I.E.E. North Eastern District meeting, New Haven, Conn., May 6–8, 1936.

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Unlighted Highways Danger Zones

At night each foot of highway, even though it stretches far and straight into the night, becomes a danger zone that no sign can mark. No signal can safeguard the almost invisible pedestrian on his quiet progress or minimize the fatal blinding glare of headlights. No warning can dispel the illusion of safety or sharpen deadened or confused judgment. There is no way of localizing the problem, for it exists wherever and whenever there is darkness. It presents itself as a separate field of research and effort because it cannot be treated and studied as a factor applying only to a specific situation. It applies to every foot of road and for every hour of the night.

The facts are clear, simple, and adequate, beyond all question. The conclusion they establish is not so astonishing as the apparent failure to meet it seriously, or at least seriously enough to take suitable action to improve highway lighting conditions.

During 1935, according to the Travelers Insurance Company, automobiles in the United States killed 21,480 persons during the hours of darkness, as compared with 14,600 killed in daylight hours. This experience, indicating that highway darkness is more deadly than daylight, is recorded despite the facts that the number of night accidents is less by 33,000 than the daylight total and that the volume of night traffic only $^1/_4$ that of the daytime traffic.

In the early days of the depression many public authorities, in an endeavor to reduce expenses, curtailed urban street lighting and in some cases such limited highway lighting as was provided at that time. The savings in lighting appropriations thus effected were found to be more than offset by the losses incurred through the enormous increase in night traffic fatalities, with the result that normal service later was restored. This experience conclusively demonstrated that highway illumination will reduce night traffic fatalities.

The rapidly mounting toll of automobile traffic fatalities has shocked the American public. Prcss and radio carry almost daily warnings against careless and drunken driving. Highway improvements and severe punishment of traffic violators are suggested and thought even has been given to attaching governors to cars to limit maximum speeds.

Safety campaigns, directed as they are to public education and enforcement of traffic regulations, together with better roads and improved automobile design, have proved effective in reducing daytime accidents despite increased traffic volume and speed. They have had little or no effect, however, on the night traffic accident rate, which has risen from a point where in 1917 it represented 30 per cent of all accidents to a point where today it represents nearly 60 per cent of the total.

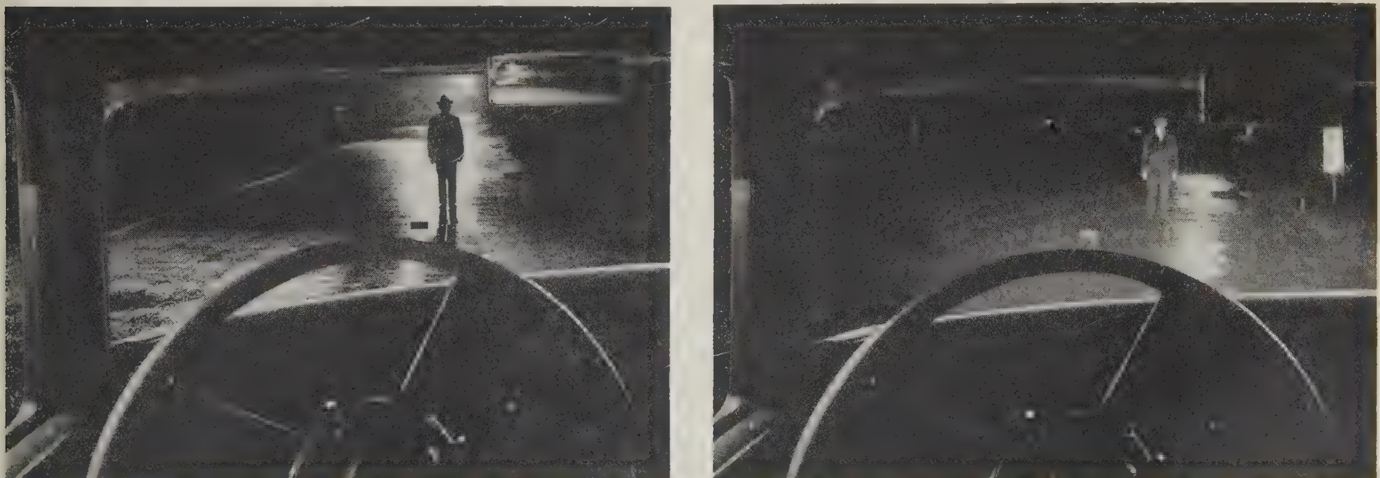
Light a Separate Field in Traffic Safety

These accident trends are significant of nighttime conditions which are beyond the control of the automobile driver. Safety propaganda, to be effective, must be accompanied by measures that will improve night visibility, such as adequate systems of fixed lights along the highway.

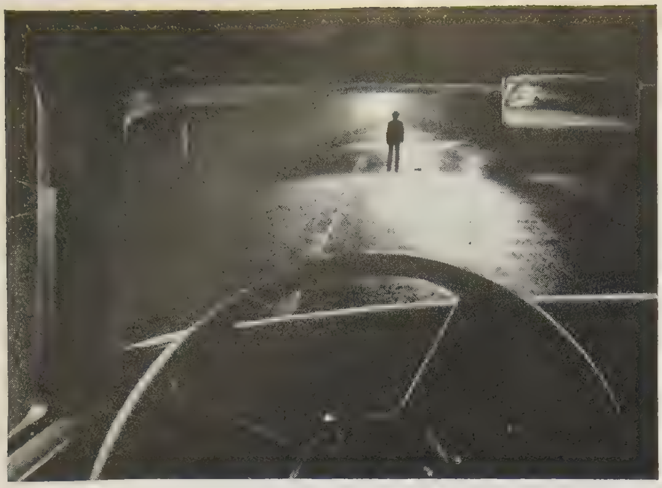
Light as a factor in traffic safety differs from all others in that it possesses the inevitable and constant element of change. As a problem, it is not limited to a study of conditions existing at one type of intersection, or one particular corner; to the number of traffic lanes, or the placement of signs or signals. It is, in effect, an over-all factor, the significance of which can be appreciated only through a general study and broad view of traffic accident statistics, and a comparison of night and day traffic experience.

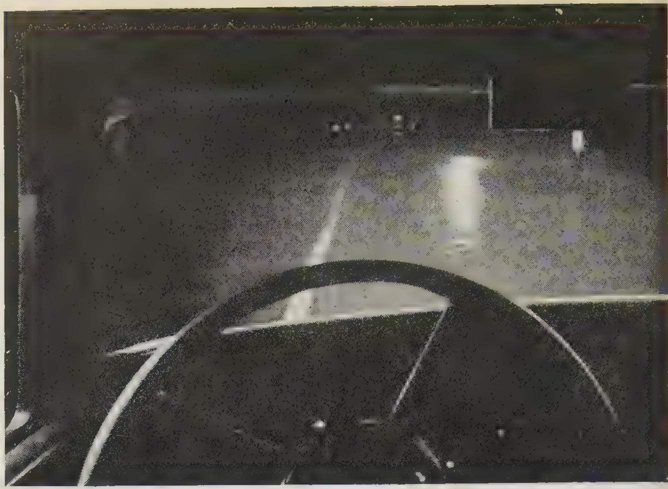
Headlights no Panacea

The view oftentimes expressed by automobile drivers and safety engineers that headlights provide safer and pleasanter driving conditions than can be furnished by highway lighting is an emphatically mistaken one and can be held only by citizens that never have seen a properly designed highway lighting installation. There are in the United States a number of so-called highway lighting systems; but



Figs. 1 and 2. Visibility as provided by highway lighting (left) and by automobile headlights (right) when pavement is wet, with pedestrian at a distance of 100 feet; note brick





Figs. 3 and 4. Visibility as provided by highway lighting (left) and by automobile headlights (right) when pavement is wet, with pedestrian at a distance of 200 feet; note brick

with few exceptions, these installations, covering short stretches of roadway, are compromises between good engineering practice, local conditions, and available funds. Such compromises have done more harm than good to the cause of highway lighting.

A few examples of properly designed highway lighting systems are scattered about the country, but it is doubtful whether these examples, if placed in a continuous line, would extend more than 30 miles. In other words, these installations are too limited in extent for the automobile driver thoroughly to appreciate their advantages. He has passed through the well lighted zone before his eyes have adapted themselves to the higher level of illumination.

Furthermore, the use of automobile headlights is not only ineffective in practice, but it is scientifically incorrect—a condition readily understood from a study of the methods by which objects on the highway are discerned at night. To a large extent, past experience has been confined to the field of urban street lighting where, with the exception of residential areas, illumination levels have been higher than are economically possible with highway lighting. Under these conditions, objects usually are "seen" by light falling on the object and thence reflected to the eyes of the observer, forming on the retina an image of the object exactly as such image would be formed on the ground glass screen of a camera. This is technically known as "discernment by surface detail."

In interurban highway lighting, where safety is the governing factor and ability to see and recognize obstructions on the road depends not on intensity of illumination, but on contrast, lower levels of illumination are permissible. This calls for a different method of discernment and the presence of an object on a well designed highway lighting system is discerned as a dark silhouette against the brighter background of the road surface. This is known technically as "discernment by silhouette."

When the pavement is concrete and *dry* the most modern type of automobile headlights furnish an approximation to safe and pleasant driving conditions, except for the blinding effect of oncoming headlights.

When the pavement is *wet*, however, its reflection characteristics are profoundly altered. Instead of diffusely reflecting the light in all directions, like white blotting paper, it reflects the light specularly, like a mirror. As a result, the light from an automobile headlamp is reflected in a direction away from the driver and, so far as the driver is concerned, produces no pavement brightness against which an object might be silhouetted. Objects on such wet pavement are seen only by the light they reflect back into the driver's eye and, since such objects are predominantly dark in color, their visibility is low.

Driving over wet pavements with no other light than that afforded by automobile headlights is inherently dangerous at speeds exceeding 30 miles an hour. A well designed highway lighting system, however, provides a bright pavement surface against which objects are clearly visible in silhouette equally well with wet pavements as with dry.

It will be noted that on a wet night headlights afford safe driving conditions if speeds are relatively low. On dry nights headlights afford safe driving conditions if the speeds are moderate and there are no approaching headlights, or if the speeds are low and there are approaching headlights. Experience has established conclusively that it is impossible to enforce speeds within the limits set by visibility conditions associated with automobile headlights. Under a proper system of highway lighting, however, the safety of night driving, regardless of speed, is as great as that by day.

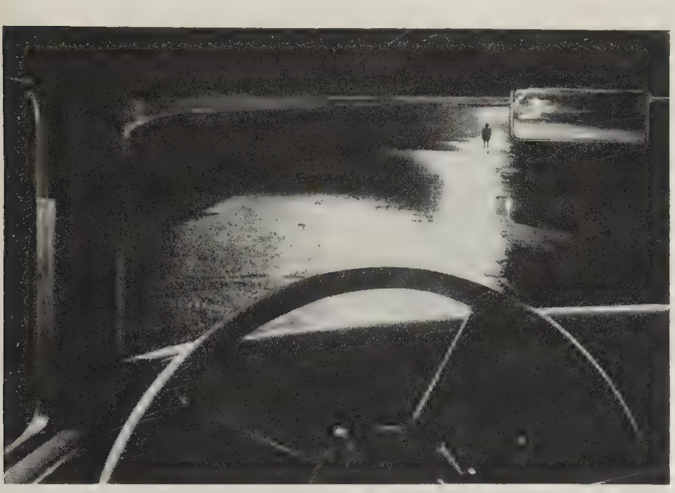
The accompanying illustrations, reproduced from photographs taken on a wet night at a model highway lighting installation, illustrate the relationship between headlighting and highway lighting. A pedestrian in dark clothes and a brick 10 feet in front of him furnish the objects to be seen.

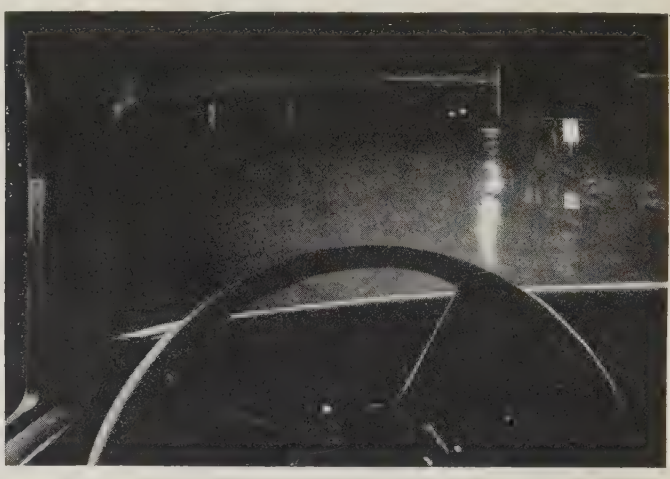
The headlights employed when these photographs were taken were of one of the best types of modern headlights and were in perfect adjustment for driving beam. The exposures and all photographical conditions were identical in all photographs. There

has been no retouching or other photographic manipulation, and the pictures correctly represent the visual conditions for a person of normal vision.

The highway lighting system used in these photographs is located at Pompton Lakes, N. J., installed through the co-operation of Westinghouse engineers and the Jersey Central Power and Light Company as a laboratory in which to study highway lighting under varying weather and other conditions. The roadway is dark bituminous macadam, and at the time these photographs were taken the pavement was *wet*. The lamps are of the 6,000-lumen sodium-vapor type, mounted in "butterfly" luminaires, spaced 240 feet apart, and suspended 30 feet above the pavement, 14 feet from the edge, on one side of the highway.

In figures 1 and 2 the pedestrian is 100 feet in front of the car. When figure 1 was taken the headlights of the car were turned out and the highway lights were on. In figure 2, the highway lights were turned out and the headlights turned on. Figures 3 and 4 were taken with pedestrian at 200 feet under the same conditions, and figures 5 and 6 with the pedestrian 400 feet distant. The difference in visibility of the objects is evident and the ineffectiveness of the headlights is beyond question. Thus, on a wet night, when traction is at its worst, automobile headlights will not reveal an obstruction on the highway in time for the driver to avoid a collision, while the highway lighting system provides adequate visibility.





Figs. 5 and 6. Visibility as provided by highway lighting (left) and by automobile headlights (right) when pavement is wet, with pedestrian at a distance of 400 feet; note that with highway lighting the brick is visible

States Responsible

A highway lighting system, to be effective, not only must be designed scientifically, but also must extend for a long distance and possess the same quality of illumination throughout its entire length, although not necessarily the same type of equipment. These conditions definitely assign highway lighting as a state function and not one to be engineered by county or township. Obviously, it would be difficult to design a uniform system extending from state line to state line if the responsibility for financing the service were divided between the various authorities along the highway.

With the responsibility for highway lighting vested in the states, financing of highway lighting service ceases to be a problem because it can be financed adequately through gasoline taxation. According to a recent statement before the American Bankers Association, automobile taxes in the form of license fees, gasoline taxes, and taxes on automobiles as personal property, totaled in 1934 more than $1,200,000,000—a sum slightly in excess of the price received by the automobile manufacturers for the cars they sold their dealers in the same period. From this it is evident that ample funds to finance highway lighting service throughout the United States are available through existing taxation in surpluses which now, with doubtful legality, often are diverted to other than highway purposes. The driving public, which apparently painlessly subscribes these huge sums, is entitled to the protection afforded by highway lighting; but so long as the public fails to recognize the relationship between darkness and death, the time of adequate highway lighting will be slow in coming.

The simple fact that state appropriations for grade crossing elimination are tremendous, while highway lighting goes begging, is ample proof that the public is not fully aware of the true solution.

New York State, for example, in response to public demand, authorized a bond issue of $300,000,000 for the elimination of grade crossings; and yet grade crossings in New York State were responsible for only 151 deaths in 1934, and only 1,440 deaths in the entire United States are ascribed to grade crossings in 1935. In the same period, 21,480 persons were killed by automobiles during the hours of darkness and only 14,600 were killed during daylight hours.

In the final analysis, it does seem deplorable that darkness and death should occupy the highways when the taxpayer can counteract them by insisting that all motor taxes be allocated to highway improvement and safety.

Conclusion

The comparatively recent compilation by the insurance companies of authentic data on night traffic accidents, the introduction of the gaseous conduction lamp as a light source for highway illumination, and the fact that there are large surpluses in funds collected from gasoline taxation that might be used for highway lighting service, presents highway illumination as an urgent major problem.

The facts of the situation are ably presented in an article[1] published in *Electrical World* on September 28, 1935, and in the conclusion established that the power companies have a public responsibility and a moral obligation to do something about it.

Reprints of the article were sent to prominent executives of the power companies asking for an expression of their attitude and company policy. Extracts from the replies were published[2] in *Electrical World* on March 14, 1936. These letters indicate marked differences in opinion on policy but "the majority are for action and agree that the power industry has a definite responsibility to push forward the wider use of modern lighting to reduce the present appalling loss through death in the night ride."

References

1. Highway Lighting—Face to Face. *Elec. World*, v. 105, Sept. 28, 1935, p. 2330–2.

2. Industry Opinion on Highway Lighting. *Elec. World*, v. 106, March 14, 1936, p. 762–3.

3. Live and Let Live, a pamphlet published by Travelers Insurance Company, Hartford, Conn., 1936.

4. Report on . . . Street and Highway Lighting. Published by Street and Highway Lighting Safety Bureau, New York.

Porcelain for High Voltage Insulators

In this paper the mechanical failure of porcelain is discussed with special reference to 3 seldom recognized characteristics: the effect of the porcelain surface on the strength, the apparent anomalous distribution of individual mechanical test values, and the effect of time on mechanical strength. A method of evaluating the result of these factors on electrical insulator performance is suggested.

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THERE CAN BE little argument as to what mechanical characteristics are desirable in high voltage insulators, but the factors which contribute to such characteristics are concealed, more often than not, by an extraordinary amount of sophistic data. Modern high voltage insulators on the average have improved enormously in the past decade. This improvement has been brought about in most cases by expensive and tedious cut and try tests conducted on assembled insulators. The procedure has been to assume that certain features of the completed units such as mechanical strength, ability to withstand immersion first in boiling water and then in ice water, imperviousness to moisture, and other qualities would insure insulators capable of standing up in service, which, after all, is the only real criterion of insulator worth. Finally specifications such as the A.I.E.E. Standards No. 41 for the testing of high voltage insulators were evolved and it may here be stated, unequivocally, that insulators which will successfully pass these specifications will give an unusually good account of themselves in the field. However, just because 2 groups of insulators of different manufacture successfully pass a certain specification it cannot be assumed that their maintenance cost in the field will be the same. How will they stand up under mechanical shock such as the impact from stones or bullets? What effect will time under load have upon their life? What is the maximum load that they will withstand? Is actual experience over a period of years the only way to find out? The purpose of this discussion is to provide engineers with a greater insight into some of the more recently investigated and rather peculiar mechanical characteristics of ceramic materials with the hope that an understanding of this work will help them to answer these questions and to point out that some of the characteristics which have been viewed with alarm by the uninitiated are really a true part of the whole scheme while other factors which have been unknown, or neglected, have been of major importance. A realization of this is bound to lead to still greater efficiency of systems dependent on the insulating qualities of such materials.

A paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 25, 1936; released for publication April 23, 1936.

10. For numbered references see list at end of paper.

The Failure of Nonductile Materials

Ceramic materials are nonductile and as such they behave in certain ways peculiar to such materials. In addition they reserve for themselves other individual characteristics which it is important to understand in order to make use of them in the most efficient manner. Cast iron, among the metals, comes closely enough to this class of materials in some respects so that a limited analogy may be drawn between them. A few of the elementary facts regarding cast iron may be considered. First of all, when it is pulled in tension its yield point and point of failure are practically coincident. It does not neck down before failure but breaks off squarely.

If cast iron tests rods as shown in figure 1*a* are pulled they will break at the abrupt change in section *cc* because the tension is concentrated in the cross section adjacent to the sharp corner. If, however, the material is ductile, necking will take place before failure and as the thick section to the left of *cc* will tend to prevent necking it will considerably strengthen the section to the right of *cc*. To obtain the greatest strength from nonductile materials sudden changes in section must be avoided and tension specimens should be shaped as in figure 1*b*.

The point to be emphasized between cast iron on the one hand and ductile material on the other is that the former will break suddenly as shown in figure 2*a* while the latter will first neck down and then break as shown in figure 2*b*.

Effect of Surface Conditions on Mechanical Strength

All this is well known to most engineers, but probably few of them realize the effect which surface characteristics exert upon the strength of such materials. It is a fact that polishing the surface of cast iron test specimens will considerably influence their strength and the phenomenon is, in general, more pronounced the less ductile the material.

Mechanical Failure of Ceramic Materials

The complete absence of ductility in ceramic materials means first of all that strength is to a great extent dependent upon surface conditions. In order to cut glass the surface is scratched, then it is tapped, and a crack goes through to the other surface from the bottom of the scratch. Porcelain, glass, and all other nonductile materials have somewhat this same characteristic to a greater or lesser degree. Failures start at the surface with an infinitesimal crack and proceed progressively through the body of the material until they reach another exterior surface.

If a porcelain test rod is subjected to a transverse load as in figure 3*a* it is obvious that the lower surface of the rod will be thrown into tension while the upper surface is compressed. The tensile strength of porcelain is much lower than the so-called compressive strength, which will be discussed later on, so that at the bottom of some slight indentation, where the stress is greatest, a tiny crack will occur at some value in the loading. When this happens a condition exists such as shown in figure 3*b*. A sharp crack projects into the porcelain. Because of the nonductility of the porcelain the angle at the crack remains acute and the stress approaches infinity in the minute localized zone at the apex of the crack. Thus the porcelain breaks down progressively fiber by fiber until the piece is broken. On the contrary, if the rod is made of a ductile material and for some

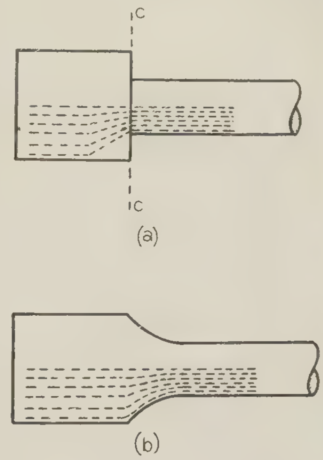


Fig. 1. Effect of shape on nonductile specimens in tension

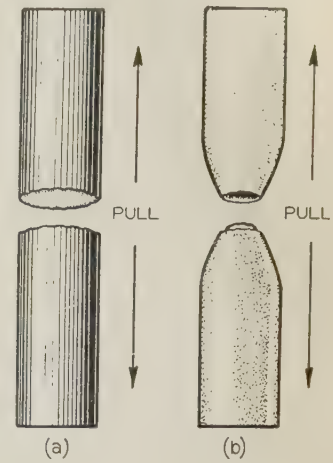


Fig. 2. Comparison of breaks in tension of nonductile and ductile materials

reason a crack occurs on the tension side as shown in figure 3*c* the ductility of the material allows the crack to widen and smooth out as shown by the dotted line, and this avoids a progressive breakdown.

Unglazed Porcelain

Unglazed porcelain necessarily has a slightly rough surface from the aggregate of ground silicon dioxide which it contains. The microscopic pits (figure 4*a*) in the surface offer focal points for incipient cracks. The fact that the surface of the material is one of the governing factors from the point of view of mechanical strength can be shown easily by giving a group of specimens a coat of varnish which smooths the surface as in figure 4*b*. Strange as it may seem this thin coat of varnish will raise the modulus of rupture of one inch rods from 10 to 15 per cent.

Glazed Porcelain

Because of the slight roughness of unglazed porcelain it is the practice to cover the surface with a glaze, as it is known to the pottery industry. Glaze has a composition close to that of porcelain but with additional fluxes which give it a melting point lower than that of the porcelain. At the firing temperature it flows over the surface in the molten state and on cooling creates a smooth glossy surface which prevents the adherence of dirt. From what has been said before it is obvious that this alteration of the surface must have an effect upon the physical strength of the porcelain. First of all the glaze smooths over the slight surface roughnesses. In

addition it forms a 2 ply structure made up of the glaze and the porcelain fused together. The glaze is only 0.005 inch thick, but it must be remembered that it is in a critical position so far as the breaking of the porcelain is concerned. All cracks must start at the surface and therefore the glaze must be the first to fail.

In addition to the smoothness created by the glaze and its effect on the mechanical strength, there is another factor which is vastly important. One aspect of what takes place when a piece of glazed porcelain is fired may be considered. As has been stated the glaze has a melting point considerably lower than that of the porcelain, and melts down and smooths out forming a thin film while the porcelain is in a slightly soft state. Soon after this condition is reached the temperature is reduced and the ware starts to cool and harden. At some point in this cooling process the porcelain and glaze assume, in a large measure, the physical characteristics which they have at ordinary temperatures. From there on as the material cools each part will shrink in accordance with its coefficient of expansion provided, of course, that no strains are set up through the structure. If the glaze has a greater coefficient of expansion than the porcelain it will try to shrink more than the porcelain body will allow and in doing so will put itself under a tensional stress. In fact it may be strained to the breaking point and show fine cracks or craze marks on the surface. Nearly everyone has seen these surface cracks on old sanitary ware. Frequently, however, the glaze may be under considerable stress but short of its yield point. Sometimes a piece of ware just out of the kiln may appear to be perfect but it will craze after a short period of time as a result of slight thermal or mechanical shocks.

Conversely, if the glaze has a lower coefficient of expansion than the porcelain the latter tends to shrink on cooling more than the glaze, but the glaze must come along with the great mass of porcelain so that at ordinary temperature the glaze has been pushed together and is in compression.

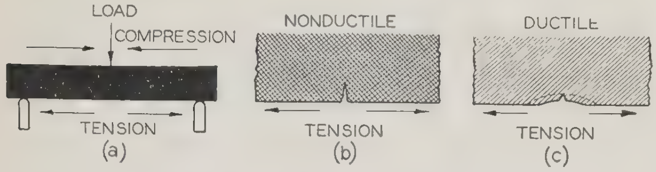


Fig. 3. Behavior of cracks in nonductile and ductile materials in tension

Glaze in Tension or Compression

In figure 5A is shown a transverse test specimen on supports ready to be subjected to a load. The outside surface is shown slightly shaded to represent an initial tension due to those factors which have just been described. Since action and reaction are equal and opposite the tension in the glaze must be balanced by slight compression in the porcelain. Now if a load is applied to the rod the zone above the neutral axis of the specimen is placed in compression while that below is placed in tension. In accordance with established beam theory the maximum stress in the bar induced by the load will be in the outermost fibers of the piece or in other words in the glaze. This stress on the underside will be added to that which is already present and if the load is great enough the glaze will fail (figure 5B) and a progressive crack will pass through the specimen at high velocity.

Where the glaze is in compression (figure 5a) and a load is placed on the rod (figure 5b) the tensile stress developed in the glaze will be that due to the load minus the initial compressive stress and so the load must be considerably greater than in the first case in order to start an incipient failure as indicated in figure 5c.

Effect of Compressive Stress

A question may arise regarding the effect of the compression components on mechanical failure. The apparent compressive strength of glass or porcelain is on the order of 8 times that of the apparent tensile strength. It may be stated here, categorically, that when such materials are placed under compression the initial failure occurs due to a concomitant tensile stress set up by the compression load. F. W. Preston[10] ably sums up this characteristic in the following words which pertain to glass but which are in a large measure applicable to all nonductile materials:

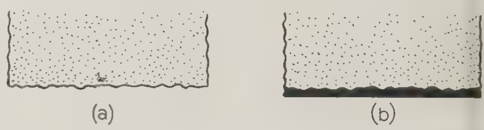


Fig. 4. Unglazed porcelain (a) and the effect of varnish (b)

"From what has been said before, it will be clear that the term 'compressive strength of glass' is a rather meaningless term. Compression tests may be made under strictly defined conditions, but up to date those conditions have not been defined. The failure of glass, when it occurs, will be due to minor tensile stresses developed incidentally 'and so to speak, accidentally,' and their amount depends, in general, on rather subtle factors, and on things that are difficult to specify or control accurately. From a practical point of view, the compressive strength of a glass may be almost anything. From a theoretical point of view it is infinite."

Method of Determining the Stress in a Glaze

It would be fair to ask the question, "How is it possible to tell whether the glaze is in compression or tension?" One of the most obvious methods would be to make up specimens of glaze and porcelain and measure the coefficient of expansion of each one separately by some suitable means. This method, however, does not work very well because when the ware is fired there is an interchange of ingredients between the glaze and porcelain while they are in the molten state and this alters the chemical make-up of the glaze and the adjacent porcelain.

A better method is that shown in figure 6. Sup-

pose a thin slab is cut from the porcelain in such a way that the glaze forms one surface and the porcelain the opposite one. Such slabs should be about $^1/_{16}$ inch thick and 5 inches long and perhaps $^1/_2$ inch wide. To all intents and purposes they resemble the familiar bimetallic strips used for thermostats except that in this case the materials are nonmetallic. If it is supposed that the glaze and porcelain are split apart into 2 separate strips as shown in figure 6*a* and that the 2 strips are heated by some convenient means, then, if the coefficient of expansion of the glaze is greater than that of the porcelain the strips will assume some such position as that shown by the dotted sections. If it is imagined that the strips are again fused together and heated it is evident that

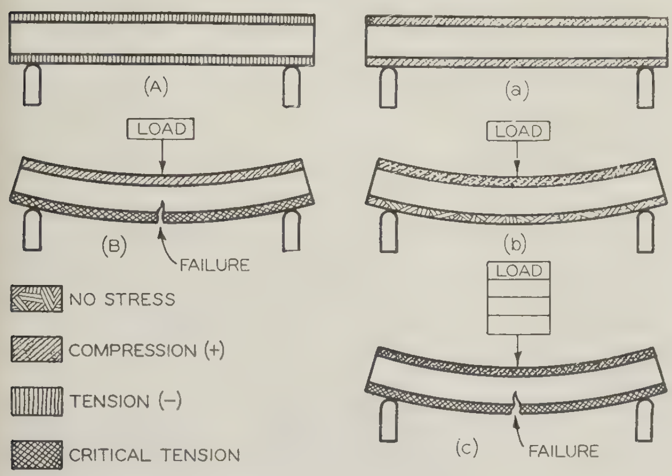


Fig. 5. Comparison of strengths of glazes initially in tension and initially in compression

the strip would bend as shown in figure 6*b*. If the converse is true and the porcelain coefficient is greater than that of the glaze, then under similar conditions the results will be as in parts *c* and *d* of the figure. This method gives a qualitative result as to the relative coefficients of expansion of the glaze and porcelain.

If it is desired to obtain a quantitative number for the differential coefficient of expansion of the glaze and porcelain the technique involved becomes somewhat more complicated. The method used in this work was briefly this: Strips of porcelain and glaze were cut to fixed dimensions and heated to 400 degrees Fahrenheit. The movement which took place at the end of the strips was measured in thousandths of an inch and designated as (+) when the glaze had a smaller coefficient of expansion than the porcelain and (−) when the opposite condition prevailed. Thus a glaze designated as +8 would be under approximately twice as much compression as one designated as +4. Conversely, −8 would signify tension in the glaze.

Mechanical Strength of Different Porcelains and Glazes

The mechanical strength of porcelain with its glaze in various stages of compression and tension can now be considered. Table I shows strength values obtained by breaking $^1/_2$ inch rods transversely on supports 4 inches apart. The average modulus of rupture of rods with the glaze under strong compression and tension are shown and also the strengths of unglazed porcelain together with several intermediate

Table I—Moduli of Rupture of $^1/_2$ Inch Rods Broken Transversely on Supports 4 Inches Apart

	Unglazed	Glaze +8	Glaze +1	Glaze −1	Glaze −2	Glaze −8	Glass
High value	12,900	20,950	15,500	12,340	11,100	5,380	18,950
Low value	4,860	12,850	6,000	4,470	4,230	3,075	7,660
Average	9,597	17,250	10,450	9,100	8,300	4,550	14,060

Load application 988 pounds per square inch per second in modulus of rupture. All values in pounds per square inch.

glazes. The numbers and signs at the tops of columns indicate the amount of initial compression or tension.

It must be conceded that any factor which can influence the strengths of test specimens by 300 or 400 per cent is something that must be considered when manufacturing high voltage insulators. Not only does the glaze influence the strength of porcelain under mechanical load but it may also add materially to the ability of the porcelain to withstand sudden thermal changes. Many theories have been advanced from time to time to explain the failure of certain insulators after a length of time in the field. The electrical failures in nearly all cases are preceded by mechanical cracks which start at the surface of the porcelain. It is believed that the evidence here presented is of sufficient weight to support another theory. Briefly it may be stated as follows: The mechanical cracking of porcelain, other things being equal, is caused by its surface characteristics and much of the deterioration of insulators in service is the result of an improper realization of the importance and adequate control of this factor.

Practical Application of the Glaze Principle

It might be interesting to touch briefly on some of the practical applications of what has already been discussed. Figure 7 shows the cross sections and silhouettes of 2 cement grips, *A* and *B* the ordinary type and *a* and *b* the effect of the reglazing process which consists of coating the porcelain surface, which is to be gripped by the cement, with a special +8 glaze, applying a layer of sized porcelain particles, and subsequently spraying or dipping a secondary coat of special +8 glaze over the particles. When insulators thus treated are fired the porcelain particles are completely covered with a glaze which is in compression and no sharp crevices exist between the particles to start progressive failure.

The effect which this treatment has upon the strength of the insulators is shown clearly in figure 8, which gives the results of a duration load test made on 16 insulators half of which were treated while the other half were made in the ordinary manner. The load on the units was started at 10,000 pounds and

increased 1,000 pounds per day. The units were tested daily and figure 8 shows the loads at which they failed, the S points representing untreated units and the R points treated units. Also, a comparison between the low values is given.

Figure 9 shows a machine in which insulators can be placed to determine their ability to withstand

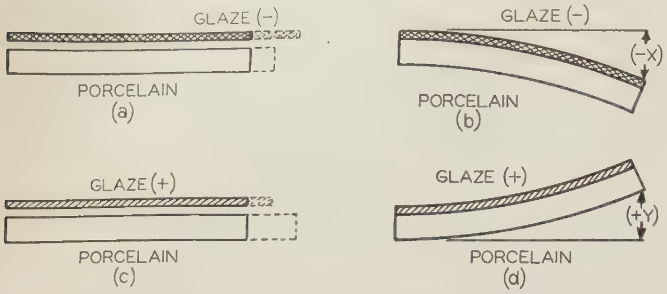


Fig. 6. Qualitative method of determining relative coefficients of expansion of porcelain and glaze

impact. The units are mounted as shown and given a blow on their outer edge by means of the pendulum. It must be apparent that other things being equal the ability of insulators to withstand such blows is some measure of the amount of breakage that will occur in service due to stone throwing or bullets. The effect that glaze has upon this ability is shown in figure 10. The white insulators were coated with a −8 glaze and the brown with a +8 glaze and each unit was then subjected to a single blow of exactly the same amount. The results are self-evident. It must be emphasized here that the color of the glaze has nothing to do with its mechanical characteristics and that the white color used here was to prevent any chance of weak insulators getting into the factory stock.

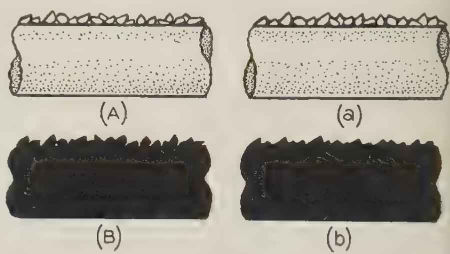
Variation in Mechanical Strength of Ceramic Material

The second important characteristic of ceramic materials and one which from time to time is the cause of considerable worry to the purchasers of insulators is the variation of mechanical strength between seemingly identical specimens. Such variations, while they may appear to be entirely anomalous to the uninitiated, are in reality predictable and form a part of the complete scheme of things. It is proposed here to give data pertaining to this subject in an effort to throw some light on the strength variations of insulators and to show that in properly manufactured insulators it is not an unknown but a known quantity and that by itself it cannot be used as a criterion of insulator quality.

Table I gives the high, low, and average moduli of rupture obtained by breaking a number of test rods. In the case of the +8 glazed and unglazed porcelain a large number of specimens were broken so that it was possible to get a good idea of the distribution of individual values which fell between the low and high values. Figure 11 shows distribution curves plotted for these 2 types of porcelain. The method used in plotting these curves was to indicate values of moduli of rupture along the abscissa and then to plot the percentage of the total number of rods which broke between fixed intervals along the abscissa as ordinates. The resulting curves are somewhat similar to sine curves and give a clear picture of the strength values which may be expected from a group of specimens. Relatively few specimens of glass were broken so that it was not feasible to plot a distribution curve, but it is evident from the high, low, and average values obtained from the small number of tests that its spread would have been greater than that of either of the porcelain curves.

The tests on the glass rods were introduced as check tests to answer any arguments which might be advanced that the reason for variations in the strength of ceramic materials lies in hidden flaws. Glass, being transparent, can be examined readily for flaws which in opaque materials cannot be so detected. The glass used was specially selected, homogeneous, smooth, and free from strains, and the results obtained indicate by comparison that the presence of flaws, strains or other concealed defects are not necessary to explain the variation in the individual test values of the porcelain specimens, but are the outcome of inherent properties of the material.

Fig. 7. Silhouettes of ordinary and specially glazed porcelain cement grips



Behavior of Insulators Made From Different Materials

If the characteristics of insulators made from these 3 materials are now considered it is possible to arrive at a few conclusions regarding their behavior.

Before proceeding, the statement made by F. W. Preston, quoted earlier in this paper, regarding the mechanics of failure of glass under load should again be called to mind. Briefly it was to the effect that regardless of how glass is loaded the initial failure develops as the result of an incidental tensile stress developed somewhere on the surface. It is believed that this theory applies to porcelain and to many other nonductile materials. Certainly all the available experimental data seem to support it and in the development which follows it will be considered as axiomatic. Having accepted this, it is permissible to go one step further and to say that since all failures start at the surface of porcelain, or glass, it may be assumed, other things being equal, that the modulus of rupture of the material is a fair criterion of its ability to withstand any type of loading. This follows because the modulus of rupture is the calculated tensile stress in the outermost fibers

of the test specimen at the instant of failure. In other words, it is a measure of the surface strength of the material and since we have assumed that this is the controlling factor governing the insulator strength it is possible to draw conclusions regarding the characteristic of insulators by considering the results obtained from laboratory specimens.

It is evident that for the same size and general construction insulators made with the material having the highest average modulus of rupture will have the highest average strength. This, of course, assumes that the surface of the insulators in the zone where the maximum surface tension is induced by the external stress is of the same nature as the laboratory specimens. By the same reasoning it may be concluded that the porcelain which shows the narrowest distribution curve on laboratory samples will when made into insulators show the greatest uniformity in strength values when tested at the same loading rate. Every type of material has its own particularly shaped distribution curve to which it will always adhere. Porcelain is no exception to this rule.

Since porcelain has such a strength distribution curve it at once removes the low and high test values obtained on insulators from the realm of unknown quantities. If the porcelain test rods have breaking strengths which vary in a certain way it is perfectly

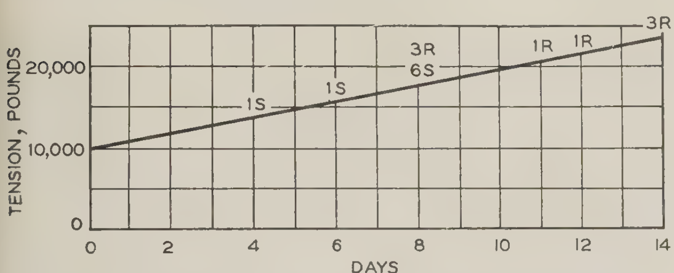


Fig. 8. Results of duration load test on porcelain insulators, showing effect of glazing treatment

S—Untreated, low value 14,000 pounds
R—Treated, low value 18,000 pounds

reasonable to suppose that insulators built of this same material will vary at least as much as the porcelain, and, if the assembly methods vary, a little bit more depending upon the manufacturing control. All this, of course, presupposes that the insulator design is such that under a mechanical load the porcelain is the first part to fail which is generally true. Now in order to find the possible low test value that would be obtained from a large quantity of insulators it is necessary only to find their average strength and to multiply this average by the ratio obtained by dividing the lowest rod value by the average rod value, or to obtain the maximum test figure to multiply by the ratio of the maximum rod value divided by the average. Since the testing of 10 insulators will give a figure very close to the true average it is apparent that this method of predicting the possible low and high test values of insulators is a very valuable tool to work with. In what follows the ratio of the low to the average values of test specimens will be referred to as the distribution factor F.

For glazed porcelain, based on 420 specimens, F is 0.74. These results were obtained at a rate of loading of 988 pounds per square inch per second in modulus of rupture and it would be natural to ask whether or not this ratio changes materially with different rates of load application. Table II shows F calculated for different loading rates. For comparison these factors were determined for 2 ceramic materials. At all but the fastest rate the data are obviously insufficient to establish fixed conclusions because of the small number of specimens tested. There is, however, an indication that the reduction in strength under time loading affects the high, low, and average values by the same percentage. In the calculations which follow the lowest values for F were taken for these 2 materials, 0.74 for material A, which is +8 glazed porcelain, and 0.54 for material B which is a ceramic material that has been advocated for making insulators.

Considering now groups of insulators made from these materials, what are the minimum test values to be expected if a large group of each type is tested each having a known average quick pull value of 15,000 pounds? Multiplying by their respective factors a possible low value is obtained for material A of 11,100 pounds and for material B of 8,100 pounds. A practical application of this is shown in table III, which gives a comparison of the predicted high and low values for porcelain insulators with the actual results obtained from testing 48 standard suspension units. It is indicated certainly by these results that if there are other variables, such as assembly differences, they cannot in this case be a very large factor since the observed results are accounted for by the expected inherent variation in the porcelain strength.

The Effect of Time and Load on Ceramic Materials

The third physical characteristic of porcelain to be discussed, is the effect of a duration load on mechanical strength. So far the strength of nonductile materials under relatively quick load applications has been considered. What happens with the in-

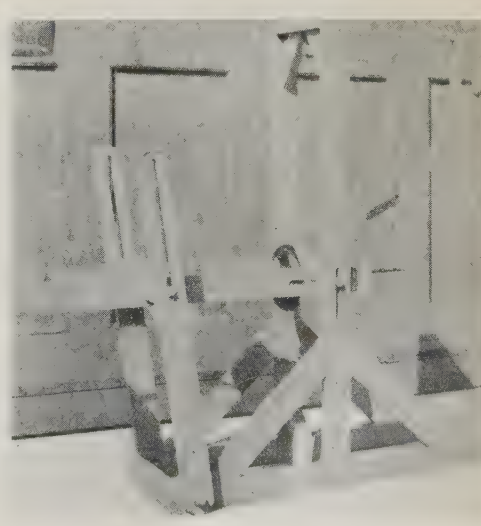


Fig. 9. Machine for testing impact resistance of insulators

Table II—Ratio of Low Transverse Value to Average Transverse Value

Rate of Loading in Modulus of Rupture, Pounds per Second	Distribution Factor F	
	For Material A +8 Glaze	For Material B
988	0.74 (420)	0.69 (21)
3.11	0.80 (10)	0.59 (10)
1.36	0.89 (10)	0.69 (10)
0.62	0.87 (10)	0.72 (10)
0.26	0.79 (10)	0.54 (10)

Figures in parentheses indicate number of tests.

troduction of a time factor? Unfortunately few data on the effect of time loading on the strength of nonductile materials have been published except by certain technologists in the glass industry. They have found that under long continued loads the strength of glass is materially reduced, not just perceptibly but a matter of from 30 to 60 per cent. It is reasonable to suppose that to some degree this characteristic is common to all similar materials.

Porcelain insulators in service are subjected to continued loads of more or less severity for prolonged periods of time and certainly the load time factor is of major importance.

Load Testing Procedure

In obtaining the present data the testing machine shown in figure 12 was devised. The test specimens used were 1/2 inch round rods placed on supports in the machine 4 inches apart and subjected to a transverse load at their center imposed by a weight carried on the upper beam. The gearing actuating this weight was variable so that it could be arranged to move the weight out quickly thus building up the load on the specimens at a rapid rate until failure, or the travel could be slowed down to the point where it required several days to break a rod. An automatic stop was provided to shut off the motor and apply a brake so that the breaking of the test rods and the recording of the strength values was automatic after each start. By running this machine day and night considerable progress could b
made even when the loading was very slow.

Figure 13 shows the results obtained on material A and B previously considered. The averag strength values of 10 samples for each point reduce to moduli of rupture are the ordinates while th abscissa values are plotted as the reciprocal of th rate of loading in modulus of rupture in pounds pe square inch per second. Thus the further to th right on the abscissa the slower the rate of loading Most of the loss in strength occurs with the first de crease in the rate of loading and the curves then flatten out. Probably the average load that th materials would support indefinitely is that which would be represented by the asymptotes of th curves. In the case of material B it would be ap proximately 60 per cent of the quick load averag while the value for material A would be about 8 per cent.

Although further data on this characteristic o ceramic materials are desirable, the foregoing i sufficient to emphasize the importance of studying the time load characteristics of porcelain which is t be subjected to continued loads. Certainly, other things being equal, the most desirable material to us in insulators is obviously the one which shows the lowest decrease in mechanical strength under a con tinued load.

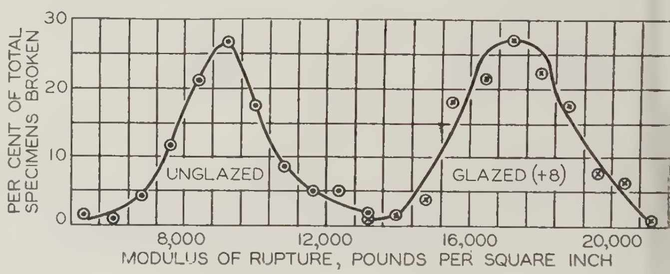


Fig. 11. Distribution curves for failure of unglazed and glazed specimens

Influence of Time on Service Life

In the earlier part of this discussion a theoretica method of arriving at the possible low strength values of insulators based on the strength of labora tory specimens was presented. The conclusions arrived at were the result of quick loading tests Now it is possible to go a step further and to conside the effect of time. It must be assumed that the decrease in strength shown in figure 13 is approxi mately the lower limit and that the curves will now flatten out, and a further slowing up in the rate o load application will result in no further decrease i strength. For material B this load is about 40 pe cent less than that found for the quick loading. The figure representing material A is 20 per cent lower thus, the previously given possible low strength value for insulators if made of the particular materi als in question, must be multiplied by 0.6 and 0. respectively. The figures thus obtained approach very closely to the load which any number of these insulators will sustain indefinitely without a single

Fig. 10. Comparison of impact resistance of insulators with glazes in tension (left) and in compression (right)

failure. As the load is increased above this figure an increasingly larger percentage of units will fail, depending upon the strength distribution curve and time factor of the dielectric used in the units.

Practical Use of Data

Of what value to the engineer are these figures? Most of the suspension insulators in service are working at values far below the theoretical point at which the first failure will occur. Even if they were working at a point slightly above this figure the number of failures as indicated by the distribution

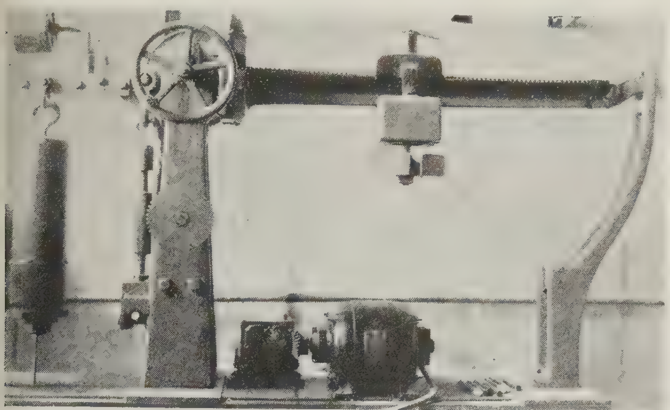


Fig. 12. Machine for time load testing

curve would be very few; thus, the data presented here may be of little practical value for the ordinary electrical porcelain installation. In recent years, however, the loading on the insulators has been increased in a number of cases. Certainly this is true for river crossings, radio work, and large switch insulator and bus support work. In arriving at a working load which a large number of such insulators will withstand indefinitely without the possibility of a failure the following expression may be used:

$$W_A = S_a \times F \times T$$

where

W_A = Working load at which no failure will occur in any number of insulators
S_a = Average quick strength of insulators
F = Distribution factor
T = Time loading factor

It should be borne in mind that the distribution factor and the time factor are 2 entirely separate and distinct entities, the former resulting from the inherent spread in the strength values obtained from breaking identical test specimens and the latter resulting from the effect of the time of loading on the strength of the material. For insulators which average 15,000 pounds made from the materials upon which the previous data were based breaking loads are obtained as follows:

$W_L = 15{,}000 \times 0.74 \times 0.8 = 8{,}900$ pounds for material A
$W_L = 15{,}000 \times 0.54 \times 0.6 = 4{,}860$ pounds for material B

In certain installations where a large percentage of

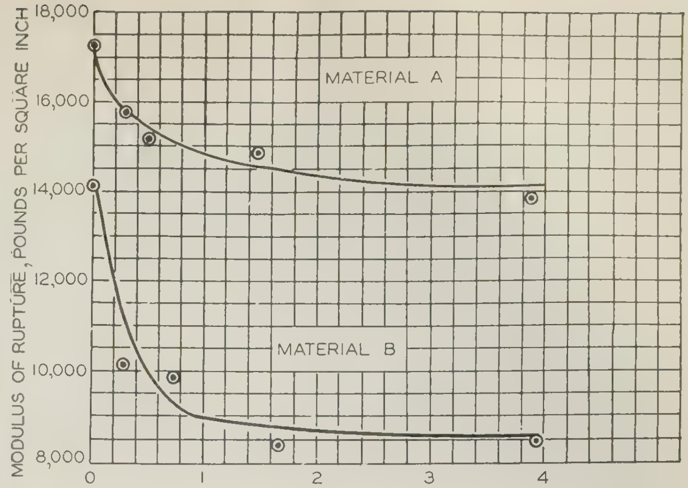


Fig. 13. Results obtained on time load tests; abscissa is the reciprocal of the rate of loading in pounds per square inch per second in modulus of rupture

the load is not of a constant nature, but occurs only for brief intervals, such as in the case of short circuits, or during switch operation, the time factor T in the above equations may be omitted.

Conversely, for a river crossing requiring insulators which must hold a constant 10,000 pound load the average strength is found as follows:

$10{,}000 = S_A \times 0.74 \times 0.8$
$S_A = 16{,}900$ pounds for material A

$10{,}000 = S_A \times 0.54 \times 0.6$
$S_A = 30{,}800$ pounds for material B

Hence, for such work insulators made of the particular materials in question should have these average strengths in a quick pull test.

It is conceded readily that the above rules are not infallible and that certain minor factors have been neglected; also, the accuracy of the figures is limited to some extent by the relatively small number of specimens tested. Nevertheless, it is believed that 3 of the basic factors affecting the life of ceramic insulators have been differentiated for the first time and an effort made to evaluate them in an orderly manner.

Sources of Error

It is apparent that there are several possible sources of error in the calculations that have been given. In the first place, it must be assumed that

Table III—Mechanical and Electrical Strength Values on Suspension Insulators Obtained With A.I.E.E. Standard Loading Rate

	Actual Test Values, 48 Tests	Predicted Values Using Rod Strengths With +8 Glaze. Load Rate 988 Pounds per Square Inch per Second (Modulus of Rupture)
Average	16,835	
High	20,400	20,450
Low	12,900	12,450

the material in the test specimens has the same mechanical characteristics as that in the insulators. Since ceramic materials may vary widely in strength and uniformity it is obvious that the values of the factors used in this paper cannot be applied indiscriminately to several classes of material; also, in some cases the number of specimens tested was inadequate. Then too, the time loading factor may be found to be considerably lower when the loading is applied for a much longer time. Further research will be conducted to find this out.

It is believed, however, that even with the possibilities of discrepancies as the result of these factors, the conclusions drawn are surprisingly accurate for the dielectrics that have been dealt with.

Summary

Based on the study reported in this paper, the following statements may be made:

1. The present general method used for arriving at guaranteed mechanical values for insulators using ceramic dielectrics is insufficient.

2. Porcelain is an ideal insulating medium when the factors affecting its physical characteristics are properly controlled.

3. All indications point to surface strength as being of fundamental importance to the proper performance of an insulator dielectric.

4. Even with perfect porcelain one must expect an inherent variation in individual strength values when tests are conducted on a group of specimens. This is a characteristic of brittle nonductile materials. Since, regardless of the method of loading, the initial failure of a piece of this class of ceramic material is the result of a tension crack in the surface it is possible to predict the probable variations in quick strength of any insulator design by a simple calculation based upon the moduli of rupture obtained from a number of laboratory samples.

5. Uniformity of test values is not, by itself, a criterion of good porcelain.

6. Distribution curves of ultimate strength may be obtained by breaking a considerable number of test specimens and plotting the results so as to indicate the percentages of the total number which break within fixed limits. Since, regardless of the method of loading, the initial failure of a piece of glass or porcelain is caused by excessive localized tensile stress set up incidentally at some point in the surface, the distribution curves obtained from test specimens may be used to predict the range of strength values which may be expected from any insulator design.

7. Tests made with variable speeds of load application indicate a considerable drop in strength as the loading rate is decreased.

8. By evaluating the strength distribution factor and time factor a simple formula may be obtained for calculating safe insulator loads.

9. The testing of relatively few specimens is a source of error in arriving at the correct values to be used in the formula.

10. It is readily conceded that this work is not absolutely conclusive, but it is believed that for the first time the important factors which materially influence insulator life are evaluated and a simple method of approach outlined for the selection of insulators.

11. Since ceramic materials vary widely in their characteristics the factor values here used are only applicable to the particular materials studied.

References

1. Electrical Properties of Glass (a book), J. T. Littleton and G. W. Morey. John Wiley and Sons, Inc., New York, N. Y., 1933.

2. The Influence of Glaze on Insulator Strength, D. H. Rowland. *Gen. Elec. Rev.*, v. 32, March 1929, p. 136–8.

3. The Angle of Forking of Glass Cracks as an Indicator of the Stress System, F. W. Preston. American Ceramic Society *Jl.*, v. 18, June 1935, p. 175.

4. The Time Factor in the Testing of Glassware, F. W. Preston. American Ceramic Society *Jl.*, v. 18, July 1935, p. 220–4.

5. Recent Improvements in High Voltage Insulator Design, D. H. Rowland. *Gen. Elec. Rev.*, v. 33, July 1930, p. 384–7.

6. Mechanical and Thermal Shock Tests on Ceramic Insulating Materials, H. M. Kraner and R. A. Snyder. American Ceramic Society *Jl.*, v. 14, Sept. 1931, p. 617.

7. Mechanical Electrical Stress Studies of Porcelain Insulator Bodies, Cullen W. Parmelee and John O. Kraehenbuehl. Bulletin No. 273, Engineering Experiment Stations, University of Illinois, Urbana.

8. Influence of Glaze Composition on the Mechanical Strength of Electrical Porcelain, L. E. Theiss. American Ceramic Society *Jl.*, v. 19, March 1936, p. 70–3.

9. Statistical Methods of Research Workers (a book), R. A. Fisher. Oliver and Boyd, London, England, 1925.

10. Glass As a Structural and Stress Resisting Material, American Ceramic Society *Jl.*, v. 12, April 1933, p. 163–86.

Circuit Breakers for Boulder Dam Line

The trend in recent years toward large concentrations of electric power at important centers and the necessity for stable transmission over ever-increasing distances have resulted in a demand for high voltage switching equipment with unusually short operating time. This paper describes the development along conventional lines of a circuit breaker of the deionizing grid type capable of interrupting within 3 cycles a short circuit on a 287.5-kv 60-cycle line.

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SOME YEARS AGO a paper[1] was presented describing a high speed circuit breaker for single-phase 11,000-volt 25-cycle railway service which was capable of detecting and interrupting a short circuit in 0.04 second. It was then stated

A paper recommended for publication by the A.I.E.E. committee on protective devices and scheduled for oral discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 26, 1936; released for publication April 13, 1936.

The authors wish to express appreciation of the splendid co-operation offered by the representatives of both the Bureau of Reclamation, U.S. Department of the Interior, and the Bureau of Light and Power of the City of Los Angeles.

1. For all numbered references see list at end of paper.

that application of such a breaker to high tension circuits, or to multiple-phase circuits of any voltage, presented problems requiring for their solution a very substantial amount of development work. In a later paper[2] discussing new developments in arc rupture it was mentioned that in the transmission class of voltages circuit interruption within 7 to 8 cycles (0.117 to 0.134 second) was then feasible, but it was intimated that material decreases in operating times below this point would entail further intensive development. The purpose of this paper is to present the results of such further development work, which now makes possible the application to multiple phase circuits in the upper range of transmission voltages of oil circuit breakers built along conventional lines with an operating time of 0.04 to 0.05 second, substantially the same as times obtained on single phase circuits in the lower voltage range a few years ago.

Fundamental Principles of High Speed Arc Rupture

These advances in the art of high speed arc rupture are the logical extension of those principles of rapid circuit interruption previously discussed[1,2] and proved effective in more limited fields. Past experience with high speed tripping and rapid acceleration of movable contact members proved applicable in securing the necessary prompt separation of contacts, although new problems were encountered in the form of larger masses to be accelerated, increased travel of contacts, and the applications to multiple phase service. Thus the principles inherent in the earlier structure became effective in the new application in securing the requisite mechanical time, which is that period of the circuit-interrupting operation

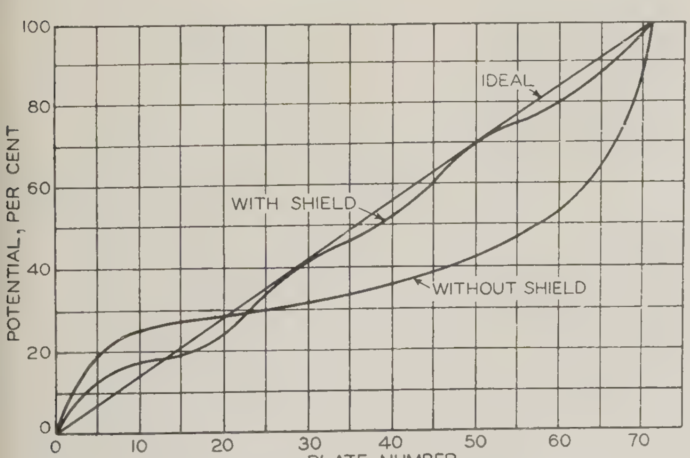


Fig. 1. Potential distribution along a stack of 72 deionizing air circuit breaker plates with and without a shield

extending from the start of rise of tripping current to the separation of the arc-drawing contacts and the initiation of the arc.

To obtain a substantial reduction in the time required for rupturing a high voltage arc, however, necessitated the extension of former principles into newer fields. A review of some fundamentals of arc phenomena is essential to a clearer understanding of the problems involved.

The interruption of a high voltage a-c arc is accomplished by the action of a suitable medium, in this case oil, on the arc path in such a way that the high conductivity of this space during the period in which current is flowing is transformed into an insulating medium of relatively high dielectric strength in the few microseconds necessary for voltage to build up across the space after a zero in the current wave. The greater the voltage available to break down this space the higher must be the dielectric strength, or the longer must be the path, if the arc is not to reestablish and current to flow again. There are various means for accomplishing this transformation from high conductivity to high dielectric strength, and the effectiveness of each may be measured by the maximum gradient in volts per inch which can be sustained across the arcing space. Given this measure of effectiveness, it is obvious that the length to which an arc must be drawn before consistent extinction can be assured is determined by the voltage available to break down the space; in other words, by the voltage to be interrupted. Although the interrupting effectiveness is modified somewhat by the recovery rate, or perhaps more correctly by the recovery time to reach the first voltage peak, it has been pointed out by Slepian[3] that modern interrupting devices capable of handling the highest available

Fig. 2. Single pole of 287.5 kv oil circuit breaker set up for test

recovery rates corresponding to recovery times of 50 microseconds or less, are not particularly sensitive to fairly large changes in this recovery time.

There have been many discussions of the theory of a-c arc rupture in oil, and extensive research testing has developed a variety of arc rupturing devices, es-

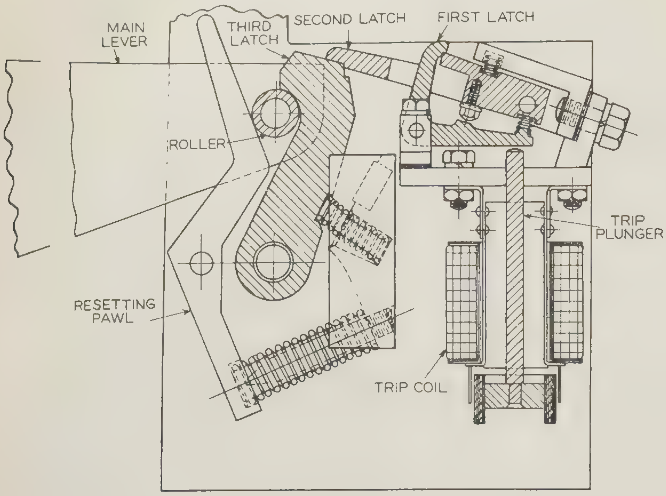


Fig. 3. Triple latch high speed tripping mechanism, which releases the main breaker pull rod in less than a half cycle

tablishing the several general design principles recognized today. A comparison of the most effective of these interrupting devices discloses a feature common to all, namely, the provision of some means for maintaining the most intimate contact possible between the arc and the oil to bring about final extinction. It is in the manner of providing such means that general design principles differ. In certain devices previously described[2,4,5] it has proved effective to confine a portion of the oil in a relatively narrow slot as small as is practicable from other design considerations, and to move the main body of the arc into this confined slot by magnetic means as it is lengthened on the parting contacts, thus maintaining the necessary intimate contact between the arc and the oil and creating in the arc path a turbulent, gaseous medium capable of a high rate of deionization in the period immediately following a current zero.

Experience in the lower range of voltages indicated that even though contact separations required for arc rupture were comparatively short, high speed operation was not to be obtained by waiting until this break distance was reached, and early high speed breakers in this class were provided with means for transferring the arc from the contacts on which it was drawn to adjacent diverging arcing horns. These devices were developed to the point of moving an arc on such horns at speeds which resulted in little or no burning,[1] and extending it to a length suitable for extinction in appreciably less time than would be required to obtain the same amount of contact separation. The use of a magnetic blowout field in moving the arc from the contact members to arcing horns has other advantages as well in high speed circuit interruption. As the arc moves away from the contact tips under the influence of the magnetic field, the most highly ionized gas areas will move away with it, leaving a relatively cool un-ionized medium in the contact gap. In fact, by proper attention to details, some fresh oil may be moved into this gap by the time the arc, traveling on the diverging horns, has reached a length suitable for extinction. Thus it becomes possible to work at higher voltage gradients across the contact gap than those across the arc path at its final current zero, and the arc may be extinguished while the contact gap is appreciably shorter than would have been required had the arc terminals been maintained on the contacts.

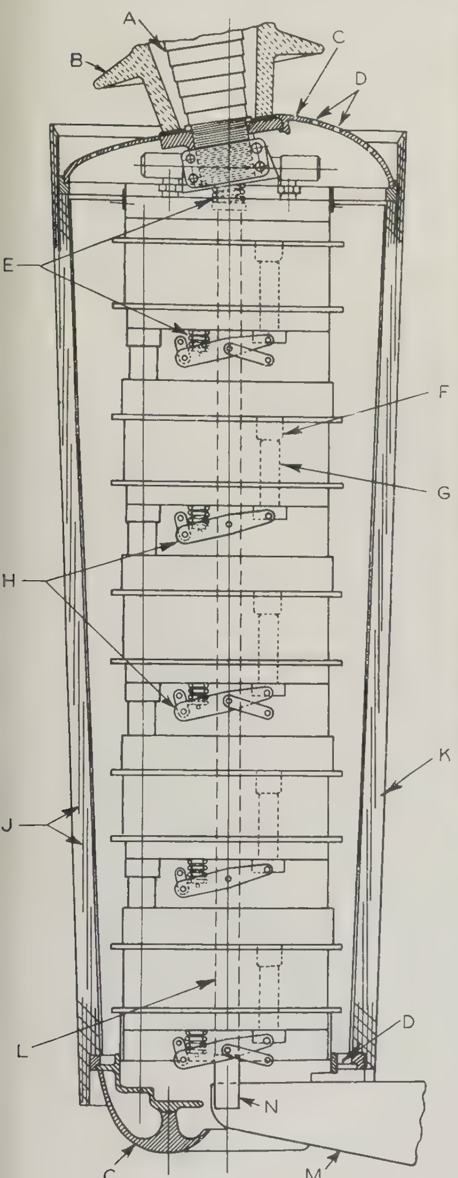
Another advantage from the use of a magnetic field is that once the arc is removed from the contacts, it may be manipulated almost at will to meet the requirements of a large variety of interrupting mediums. Thus, with a conventional design of contacts, the single arc may be divided into a multiplicity of short arcs as in the case of the deionizing air-break circuit breaker;[7] it may be passed in air into a narrow slot with walls of gas-evolving material;[4] it may be moved into relatively open oil as in early high speed oil breakers;[1] or it may be driven into a relatively narrow oil-filled slot as in the deionizing grid type[2,5] where the magnetic energy from the short-circuit current itself is utilized to force the arc into intimate contact with the oil, acting not only to lengthen the arc but to assist in the interrupting function as well.

Further development in arc phenomena has indicated, however, that there is a limit to the speed at which an arc may be lengthened in this manner if the rate of dissipation of arc energy is to be maintained at a reasonably low level within a given interrupting unit. Furthermore, the rapidity with which contact separations required for high voltage interruption can be attained has definite mechanical limitations. It appears, then, that a point in high speed applications at the higher voltages may be reached at which it becomes desirable to employ a larger number of breaks opening simultaneously, thus confining critical arc lengths per break within a practical range for high speed operation. With suitable shielding to divide the voltage evenly, the volts per break can be reduced roughly in proportion to the number of breaks employed.

Multibreak Application to High Voltage Oil Circuit Breakers

The practical application of multibreak contacts to high voltage oil circuit breakers depends to a large degree on the division of voltage between the several breaks. This problem of voltage distribution or electrostatic balance is by no means a new one. In general it appears wherever a series of conductors insulated from one another is interposed between 2 potentials. A problem in electrostatic shielding involving a stack of plates in the deionizing chamber of a circuit breaker was discussed some time ago.[6] The effectiveness of the method of solution is apparent from the voltage distribution curves reproduced in figure 1 for both shielded and unshielded condi-

tions which show that the end gap on the unshielded stack is subjected to 9 times the average voltage per gap, but when shielded the maximum gap potential is only a little over twice the mean value. The subject was discussed again[4] in connection with conventional oil circuit breakers, and it was pointed out that even in a double break circuit breaker the division of voltage between the 2 breaks might be far from equal under certain circuit conditions; also that the resulting electrostatic distortion along the arc path may diminish very greatly the gradient in volts per inch which can be sustained during the interrupting period. The opinion was expressed there that if the charged conducting parts of an open breaker are arranged so that the electrostatic field they produce gives a uniform gradient along the arc path, then there will be no distortion of the voltage gradient in the arc stream during the extinction period, and there will be no lowering of the average volts per inch for the arc length which can be interrupted.



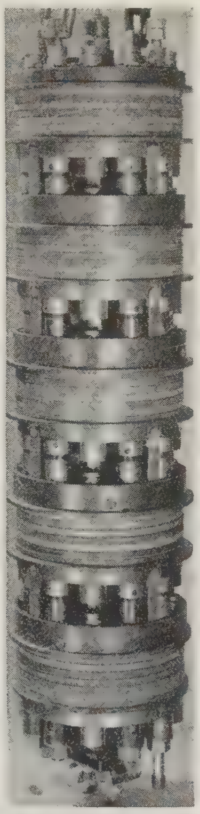


Fig. 5 (above). Five grid contact assembly with shield removed

Fig. 4 (left). Contact assembly and shield for voltage distribution

The main moving crossbar bridges a pair of these assemblies in each pole and opens and closes 10 interrupting breaks simultaneously

A—Capacitor bushing
B—Porcelain arc shield
C—Aluminum shield
D—Gas vents
E—Accelerating springs
F—Stationary contact
G—Moving contact
H—Contact levers
J—Foil layers
K—Shield for voltage distribution
L—Operating rod
M—Main crossbar
N—Contact fingers

As it is well known that the relative positions of the electrodes and the enclosing steel tank of the conventional oil circuit breaker do not in themselves produce an electrostatic field giving a uniform gradient along the arc path under the grounded short-circuit conditions common in high voltage transmission service, it follows that some form of electrostatic balancing is necessary for multibreak circuit breakers if the greatest effectiveness in volts interrupted per inch of break is to be obtained. The obvious method of preventing distortion would seem to be the interposition of other electrically charged surfaces in the electrostatic field in such manner as to neutralize the distorting effect of the tank on the contacts. One such arrangement consists of the provision of capacitance connected in parallel with each break and of sufficiently greater magnitude than the capacitance between the contacts and the tank wall to be the controlling influence determining the voltage across that break. This can be accomplished conveniently by the use of an electrostatic shield of high capacitance constructed by embedding layers of metal foil in some insulating material, this shield to be extended across the several breaks or across each group of breaks if they are arranged in more than one group. Theoretically it would seem desirable to tap the shield for a connection to each break but this was found unnecessary in practice, a single shield having given satisfactory distribution for as many as 5 breaks in series with connections to the end breaks only.

Factors other than voltage distribution should, however, be given careful consideration before a decision is reached as to the number of breaks to be employed in any high voltage circuit breaker. Practically all 220 kv switching service in this country up to the present time has been done with conventional double break oil breakers and in recent years many breakers of this type using modern interrupting devices have been operating well within 8 cycles. A review of test data obtained from these breakers leads to the conclusion that, given the 8 cycle operating time, modern arc rupturing devices are capable of extension to 287.5 kv service with the double break arrangement. Reduction of operating time to 3 cycles, however, requires consideration of whether additional breaks may not prove more advantageous for such special applications. This problem in oil circuit breakers is quite different from that of the air break type[6] previously referred to, which is dependent for operation on dielectric established in a thin sheath at the cold cathodes of a multiplicity of short arcs. These arcs are interrupted at a well defined voltage per gap capable of little or no increase, whatever the other characteristics, and the number of gaps used must, therefore, increase directly with the voltage to be interrupted regardless of contact speed or other considerations. As opposed to this, experience with oil breakers indicates the possiblity of designing arc-rupturing de-

vices with the 2 break arrangement capable of interrupting any voltage within the present service range by increasing the contact break in some ratio such as the $^1/_2$ power of the voltage. If, then, additional breaks are to be used they must be justified by other requirements such as the need for higher speed of operation.

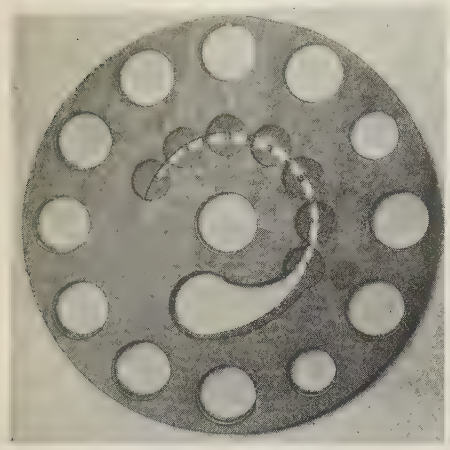


Fig. 6. A group of fiber plates from one grid unit

The slot is circular for compactness, and the oil pockets are staggered in successive plates so as to obtain the maximum creepage distance in the slot at right angles to the plates

In general it is to be expected that the minimum operating time will be obtained by the use of as many breaks as possible, and therefore consideration must be given to those factors which determine in practice the most advantageous number of breaks. In the first place, tests indicate that after the arcing time has been reduced to about one cycle the use of additional breaks results in very little further reduction in arcing time. Furthermore, the size of mechanism required to open simultaneously at high speed an unnecessarily large number of breaks would be out of all proportion to the rupturing capacity of the breaker. Other facts, however, also should be considered.

When several interrupting devices are placed in series and operated at voltages per break approaching the limit of their interrupting ability, certain irregularities in performance begin to appear. The flow of current may persist for one or more half cycles after the normal interrupting point has been reached, or there may be an occasional re-establishment of current flow after one or more half cycles of interruption which, although interrupted in turn, results in an increase in the time during which a faulty circuit may still be connected to the system. This tendency toward occasional longer operations in the region near the limit of voltage interrupting ability becomes more pronounced in devices designed to operate at voltages per inch of contact separation of 20,000 or more, but it evidently cannot be tolerated when the allowable over-all operating time is 3 cycles. The use of additional breaks beyond the minimum number indicated on the basis of single break tests may be desirable so that these variations in performance will tend to be eliminated not only because of the lower voltage per grid but also because of the reduced burden shifted to the remaining breaks in the event of some one functioning improperly.

The actual determination of the number of breaks to be applied to any given breaker in high-speed high-voltage service then requires, first, a knowledge of the operating times required and whether or not they may be met economically with the conventional double-break arrangement; second, a rather definite knowledge of the interrupting effectiveness of available single break interrupting devices; third, a knowledge of the degree of uniformity obtainable in the distribution of voltage between the individual breaks under all conditions to be encountered in service; and fourth, a decision as to the factor of safety which is desired over and above actual service voltages. The minimum number of breaks necessary will be that giving consistent and reliable interruption within the specified operating time limit up to the maximum voltage. It has been found that the results of investigations made in one type of breaker structure to determine the upper limit of interrupting ability of a single break device may not always be capable of extrapolation directly to a number of these interrupters in series in some other breaker structure, but may require a reducing factor dependent on differences in heads of oil, relationship of various parts, and other characteristics of the 2 structures. The number of breaks actually used for any given application will depend on the weight

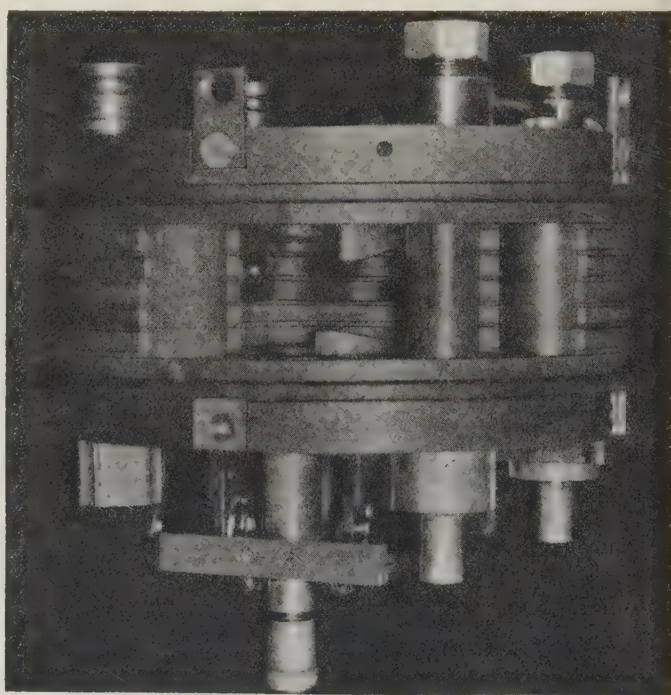


Fig. 7. Single grid unit from a 5 break assembly

The fiber plates have been cut away to show the position of the arcing horns relative to the moving and stationary contacts

given these various factors by the designer and the characteristics of the interrupting device used.

One advantage of the use of several breaks in high voltage breakers should be noted although it has no influence in deciding the proper number of breaks to be used for a given application, nor does it affect the rupturing performance of the breaker. This advan-

tage lies in the testing possibilities arising from the increased number of breaks. Even though laboratory testing facilities have been increased by the addition of relatively large generator and transformer units in recent years, the rated interrupting capacities of the larger high voltage breakers are still considerably beyond the ability of any testing laboratory to prove by direct tests at rated current and voltage. Accordingly it has become general practice to demonstrate the rupturing capacity of such breakers by testing a single pole unit, first, at the highest voltage available, 87 per cent of line-to-line voltage if possible, with whatever current may be available at that voltage and, second, with currents up to or perhaps 25 to 50 per cent above the interrupting rating at the highest voltage at which such currents are available. Supplementary tests are sometimes made at intermediate voltages and currents as a further demonstration of the sufficiency of the design to withstand high voltage stresses as well as mechanical forces resulting from current interruption. Power testing transformers in this country are insulated for 132 kv to ground and testing voltages above this value ordinarily are obtained by interposing ground between 2 such transformers, in effect placing the potential of one transformer above ground and the other below it. By connecting the test breaker across these 2 transformers in series, test voltages up to 264 kv become available, the intermediate voltages being obtained by underexcitation of the test generator. Under these conditions the maximum current values obtainable at normal rated voltages sometimes have fallen below 25 per

Fig. 8. Top end plate assembly

A complete arcing horn and the vents through the end plate between supporting stud holes are shown

cent of the breaker's current interrupting rating and, while accepted as the best demonstration possible under the circumstances, have left a question as to how adequately the rupturing ability of the breaker has been proved.

When a single pole unit is supplied with several breaks and the division of voltage among these several breaks is known for the various conditions to be encountered in service, it becomes possible to test smaller subdivisions of breakers than the complete pole unit and to test these subdivisions at voltages and currents to which they will be subjected in actual service. This is accomplished by short-circuiting any desired number of breaks and subjecting the remaining units to interrupting tests within the limits of laboratory capacity, actually imposing on these units the stresses which would be present if the complete circuit breaker were being tested to its maxi-

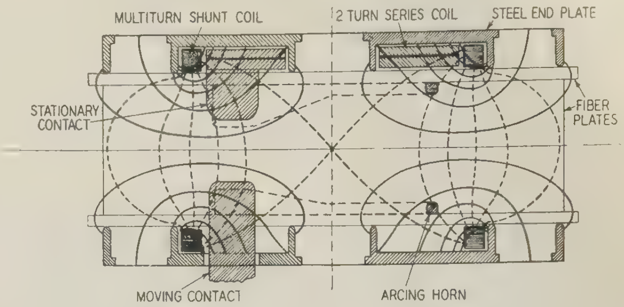


Fig. 9. Magnetic field set up by multiturn coils in grid end plates

Top and bottom coils are wound in opposite directions so that flux will take a radial direction in the plane between the contacts and arcing horns

mum rating in current and voltage. For instance, 9 of the 10 breaks in a 10 break unit may be short-circuited and the remaining break tested at or above the voltage imposed on that break under the worst service conditions, presumably a 3 phase ungrounded fault, with current equal to or above the current interrupting capacity of the breaker. It will, of course, be apparent that this method of demonstrating the sufficiency of the design for rated service applies only when the interrupting device for each individual break is complete in itself and is unaffected by operation of the remaining devices. The interrupting capacity of the 287.5 kv oil circuit breakers for the Boulder Dam lines has been proved in this manner, and thus stands as the highest ever demonstrated by actual test.

Oil Circuit Breaker for 287.5 Kv

An outstanding example of the successful application of these principles of high speed arc rupture is the 287.5-kv 2,500,000-kva capacity oil circuit breaker designed for service on the new high voltage transmission line at Boulder Dam. The specifications called for a total operating time from the moment of energizing the trip coil until the arc is extinguished of not to exceed 3 cycles or 0.05 second. A view of one pole unit of this breaker is shown in figure 2; 4 3 pole breakers of this type will be used at the Boulder Dam end of the transmission line to Los Angeles.

Each steel tank is 10 feet in diameter and contains approximately 8,500 gallons of oil. The over-all height of the breaker is 29 feet to the top of the terminals. The high voltage bushings are of the capacitor type equipped with a tap for use with a potential device. Through type current transformers are provided on each terminal for relaying purposes. One solenoid operated mechanism opens and closes

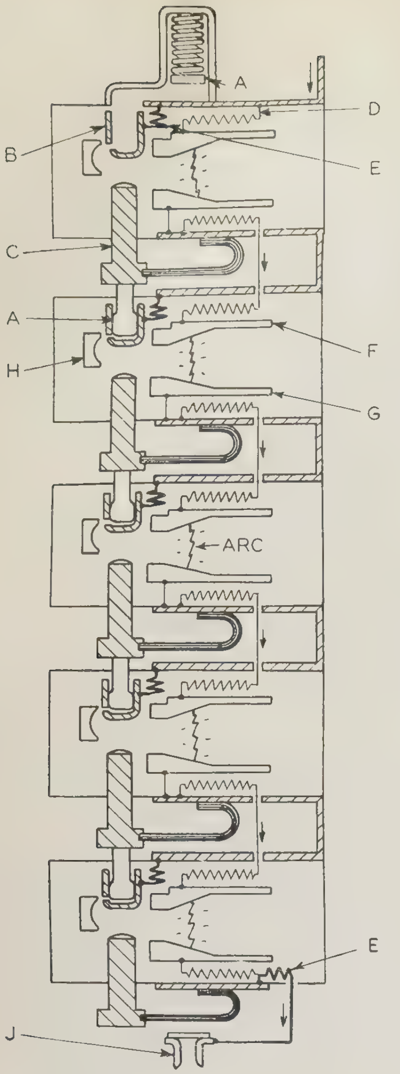


Fig. 10. Schematic diagram of coil connection in 5 break contact assembly

A—Piston
B—Hollow stationary contact
C—Moving contact
D—Multiturn shunt coil
E—Two turn series coil
F—Upper arcing horn
G—Lower arcing horn
H—Oil deflector
J—Contact fingers on operating rod

the 3 poles simultaneously, providing a definite mechanical interlock to prevent one pole unit from opening ahead of another. This mechanism is trip free in all positions, and is capable of instantaneous reclosure if necessary. A special high speed tripping device consisting of 3 latches tripped in series, shown in figure 3, permits the main breaker lever to start to move within a half cycle after the trip coil is energized. The contacts in the interrupting units part $^3/_4$ cycle later and require less than $1^1/_2$ cycles to open their full break distance.

The general arrangement of the contacts and shield is shown in figures 4 and 5. There are 10 interrupting breaks in 2 groups of 5 per pole, and 2 disconnecting breaks which open after the circuit has been cleared and serve to remove potential from the lower end of the contact assemblies attached to each terminal. Each group of 5 breaks is surrounded by a cylindrical molded insulating shield containing layers of metal foil to provide sufficient capacitance to distribute properly the voltage across the several contacts.

In closing the breaker, the crossbar attached to the main lift rod first closes the disconnect gaps and then engages a pair of fingers at the bottom of the operating rod passing through the axis of each contact assembly. Levers carrying the individual moving contacts are attached at 5 points to these operating rods so that simultaneous closing of all 10 breaks is accomplished by a further movement of the crossbar of approximately $1^1/_2$ inches. In tripping, it is necessary for the main operating rod to move only this short distance to open all 10 breaks. The crossbar then pulls out of the fingers and opens the 2 disconnect breaks while moving to the full open position.

The deionizing grid units, although modified structurally to obtain a stronger magnetic field, utilize the same principles of operation as the conventional type.[2,5] The slot is circular in section instead of straight and the arc is driven around this slot on copper arcing horns by a radial magnetic field produced by multiturn coils wound in opposite directions in each pair of end plates. The recesses or oil pockets containing oil used to rupture the arc are shown in figure 6 along the sides of the slot in one of the groups of fiber plates making up each grid structure. Figure 7 shows a single grid unit with the fiber plates cut away to show the contacts and arcing horns. Another view of an arcing horn is given in figure 8 showing an upper end plate assembly. Vents to allow for the escape of gas formed during the interrupting process also may be seen between the supporting stud holes in this figure.

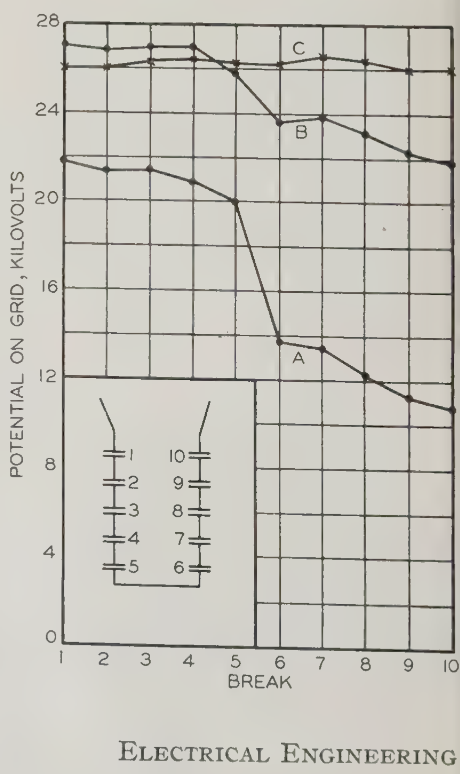
In the cross-sectional view of figure 9, the radial magnetic field is shown plotted in relation to the contacts, horns, coils, and steel end plates. The multiturn radial field coils are not permanently in the circuit, but are connected to the arcing horns and carry current only while the arc is actually on the horns. The arc is transferred from the contacts to the horns by the magnetic effect of a 2 turn series coil in the upper end of each grid. When interrupting low currents, arc transfer is assisted by the movement of oil discharged from a small dashpot through an opening in the stationary contact and deflected across the

Fig. 11. Voltage distribution across 10 breaks of one pole of breaker

A—Grounded fault on 287 kv system with 167 kv across the pole and one terminal grounded

B—First pole to clear a 3 phase ungrounded fault on 287 kv system with 249 kv across the pole (one terminal + 166 kv, other −83 kv)

C—Laboratory test at 264 kv with midpoint of transformers grounded



contact tips by a block of fiber. By making the area of the dashpot piston approximately equal to the cross section of the moving contact, the pressure in one grid tends to balance the pressure in the next, and allows oil to flow in to take the place evacuated by the contact as it moves out.

Figure 10 is a schematic diagram of all of the coil connections in a 5 grid assembly. The upper arc terminal transfers to the arcing horns first, and advantage has been taken of this by interconnecting the coils so that both the bottom coil of one grid and the top coil of the grid below are connected in the circuit by the transfer of one upper arc terminal. The bottom grid of each assembly is provided with a separate 2 turn series coil to assure the transfer of the bottom arc terminal which cuts in the lower shunt field coil. The arc is ruptured by the strong deionizing action produced by the turbulent gas blast as the arc is forced magnetically against the oil trapped in the pockets along the slot. At usually the first current zero after the arc has transferred to the arcing horns, dielectric is built up so rapidly by this intense turbulence that the breakdown value always exceeds that of the rising recovering voltage and the circuit remains interrupted.

The effectiveness of the static shields in improving the voltage distribution across the separate breaks is shown by the curves in figure 11. Under the worst condition of a 3 phase ungrounded fault on the 287 kv system, the maximum voltage on any one break of the first pole to clear would be only 27.1 kv, which is 10.9 per cent of the voltage appearing across that pole. When interrupting grounded faults, the voltage across the 10 breaks of a single pole is 167 kv, and the grid with maximum stress according to the distribution measurements would be the first break on the high side with a voltage of 21.9 kv. The voltage distribution for the laboratory test conditions of 264 kv applied to a single pole unit with the midpoint of the testing transformers grounded is also given. For this case the maximum voltage across one grid is 26.9 kv, practically the same as for the 3 phase ungrounded condition at 287.5 kv.

The rather complete investigation of the problem of accurate measurement of the voltage distribution across a series of very small capacitances, which was undertaken during the development of this circuit breaker, will probably form the subject of another paper. However, it may be of interest to outline briefly the method by which these measurements were obtained.

A small neon tube having a definite breakdown voltage was connected across each of the deionizing grids in turn. With the circuit breaker crossbar raised far enough to bridge the disconnect gaps but leaving the 10 interrupting breaks open, voltage was applied to the breaker terminals and increased slowly to the value at which the neon tube across the particular break being measured was observed visually to break down. The voltage across this break evidently was then equal to the breakdown voltage of the tube, and could be expressed in per cent of the terminal voltage by dividing by the known applied voltage. After the per cent of voltage across each break was measured separately, a proportional correction factor was added to each so that the sum of the 10 percentages would equal exactly 100. The fact that leads which might distort the electrostatic field do not have to be brought out of the circuit breaker from the different grids contributes greatly to the accuracy of this method of measurement.

Insulation and Interrupting Tests

The standard A.I.E.E. 60 cycle insulation test of 650 kv for one minute has been applied successfully to the circuit breaker both from terminal to terminal with the contacts open, and from terminal to ground with the contacts closed.

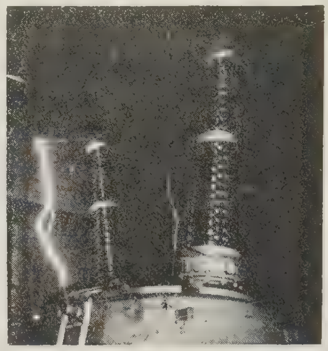


Fig. 12. One pole of circuit breaker on surge test

Circuit breaker contacts closed; surge applied to both terminals. A suppressed discharge may be noted in the gap which did not flash

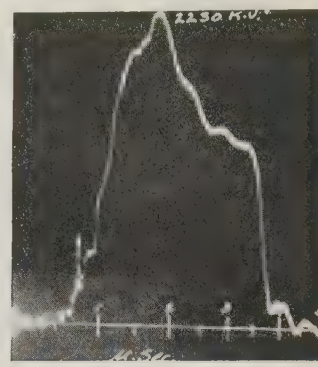


Fig. 13. Impulse wave used in surge test

The wave reached its peak of slightly over 2,000 kv in approximately $1^1/_2$ microseconds, and then discharged across the gap about $1^1/_2$ microseconds later

Impulse tests at over 2,000 kv also were made on the breaker in both the open and closed positions. Every discharge took place over the outside of the breaker clear of the bushings, as shown in figure 12, the internal insulation not being affected in any way. The steep wave front of the applied surges which reached crest value in approximately $1^1/_2$ microseconds is indicated by the oscillogram in figure 13.

In table I are given the results of representative short-circuit tests made in the high power laboratory on a single-pole unit with different numbers of active breaks in series. Many hundreds of verification tests have been made, and it is of course possible to refer to only a few of them. To indicate the consistency of performance, however, groups of 3 consecutive tests have been included for each important test condition.

The first group of 9 tests was made with all 10 breaks functioning in the single breaker pole unit. As indicated in the curves of voltage distribution, 264 kv across one pole subjects the maximum stressed units to approximately the same voltage they would receive as part of the first pole to clear on a 287-kv 3-phase ungrounded short circuit. Furthermore, the rate of rise of recovery voltage of 6,000 volts per microsecond is probably much more severe than would ever be encountered in service.

The average breaker operating time on the 3 consecutive interruptions at 264 kv was 2.53 cycles. Since the available short-circuit current at this voltage is limited to approximately 25 per cent of the breaker rating, tests with half of the pole unit shunted out at proportionately lower voltage give a still better check on the interrupting capacity of the whole circuit breaker.

The voltage to ground on the 287 kv system is 167 kv, so tests with only 5 active breaks in series were made at a little more than half this voltage, or 88 kv, to demonstrate the breaker performance under grounded fault conditions. As may be seen in the table, short-circuit currents up to and above the breaker rating of 5,000 amperes were interrupted at this voltage. As is to be expected with an arc rupturing device utilizing the energy of the short-circuit current to interrupt the arc, the stronger magnetic field at the higher currents moves the arc more rapidly and thus reduces the interrupting time. The average breaker operating time on the 3 consecutive high-current 88-kv tests was only 2.33 cycles.

With 50 per cent more voltage, or 132 kv, to check the operation under 3 phase ungrounded short-circuit conditions, tests were made up to the limit of the laboratory equipment. The average current on the 3 consecutive tests was 3,100 amperes and the breaker

Table I—287.5 Kv Oil Circuit Breaker Interrupting Tests

Test Voltage, Kilovolts	Interrupted Current, Amperes	Arcing Time After Transfer, Cycles	Total Breaker Time, Cycles	Circuit Recovery Rate, Volts per Microsecond
Ten Breaks in Series				
132	660	0.70	2.85	2,100
176	870	0.45	2.60	3,100
220	1,220	0.60	2.50	4,500
264	480	0.70	2.85	3,600
264	800	0.60	2.70	4,500
264	1,100	0.60	2.70	5,100
264	1,360	0.35	2.35	5,600
264	1,380	0.55	2.40	5,600
264	1,440	0.85	2.85	5,800
Five Breaks in Series				
88	1,330	0.45	2.60	1,900
88	2,050	0.40	2.60	2,300
88	2,710	0.50	2.50	2,600
88	4,200	0.60	2.50	3,200
88	5,800	0.30	2.15	3,700
88	5,100	0.40	2.35	3,500
132	1,160	0.60	2.70	1,900
132	1,880	0.35	2.55	2,400
132	2,170	0.35	2.45	2,600
132	3,500	0.50	2.40	3,200
132	2,900	0.45	2.45	3,000
132	3,000	0.60	2.45	3,000
176	1,090	0.60	2.90	3,300
220	1,520	0.40	2.60	4,900
264	1,850	0.30	2.35	6,200
One Break Only				
22	1,040	0.60	2.85	450
22	3,300	0.25	2.60	750
22	6,400	0.35	2.35	1,050
22	6,800	0.20	2.10	1,100
22	7,200	0.25	2.25	1,150
22	8,900	0.35	2.20	1,250
36	1,600	0.50	2.75	900
36	3,100	0.60	2.60	1,250
36	4,200	0.30	2.40	1,450
38	6,600	0.50	2.25	1,900
38	6,400	0.45	2.20	1,900
38	6,600	0.60	2.55	1,900
44	1,000	0.60	2.85	600
66	1,130	0.60	2.95	950
72	1,500	0.60	3.00	1,150

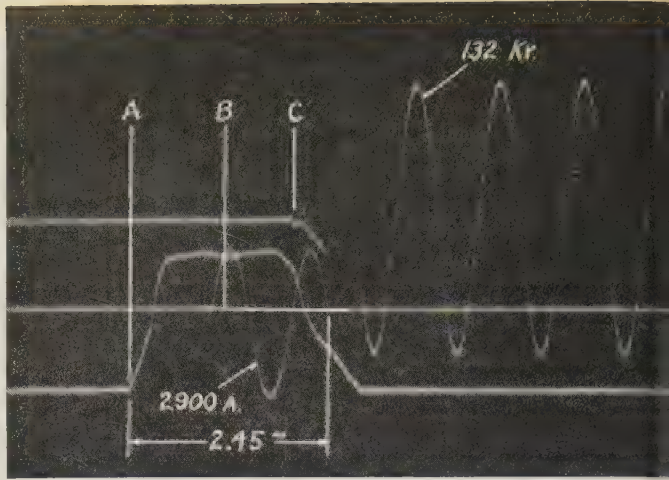


Fig. 14. Oscillogram of 5 break test at 132 kv

This test represents the duty on the first pole to clear a 3 phase ungrounded fault of 1,500,000 kva on a 287 kv system. The breaker was tripped by cam switch control just before the short circuit began. *A*, *B*, and *C* indicate, respectively, energizing of trip coil, parting of contacts, and transferring of arc to horns

operating time 2.43 cycles. The oscillogram for one of these tests is shown in figure 14.

A demonstration of the factor of safety in voltage interrupting ability is given by tests on the half pole unit up to the maximum available voltage of 264 kv. These tests represent conditions far beyond practical requirements, but give further assurance of reliable operation in service.

The last group of tests was made on a single grid only, all of the other 9 breaks being shunted out of the circuit. Referring to the voltage distribution curves in figure 11 again, it will be noted that the unit under maximum stress during a grounded fault is subjected to 21.9 kv. The series of tests at 22 kv across one break thus should represent operation to be expected under these conditions. Sufficient current was available to carry the tests to more than 8,000 amperes, or well beyond the breaker rating, thus effectively demonstrating a high factor of safety in current interrupting ability at actual service voltage. The 3 interrupting tests at the highest currents given in the table averaged 7,600 amperes, or 50 per cent above the breaker rating, and the circuit was cleared with an average breaker time of only 2.18 cycles. Figure 15 shows an oscillogram of the interruption of 8,900 amperes.

A number of high current tests were also made at 36 kv, or more than 60 per cent higher voltage across a single break. The average breaker time on 3 consecutive tests at currents averaging 6,500 amperes was 2.33 cycles. As a further check on the voltage interrupting ability of a single grid, still higher voltages up to 72 kv were interrupted satisfactorily.

The interrupting characteristic curve in figure 1[illegible] has been plotted using data taken from table I. It is evident that the total breaker operating time consists of, first, the mechanical time to part contacts after the trip coil is energized; second, the time for the arc to transfer to the arcing horns; and, finally, the time during which the arc is actually interrupted

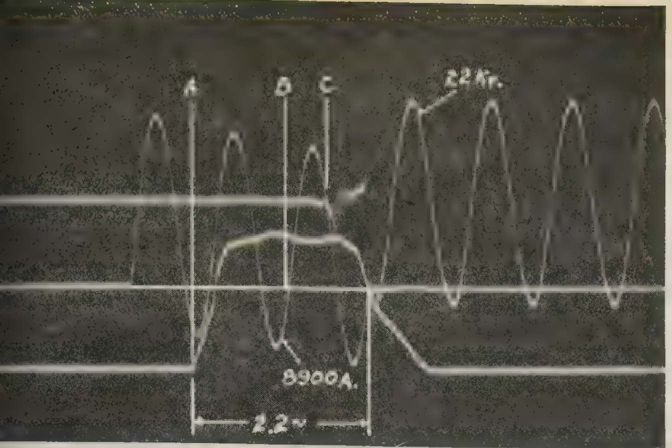


Fig. 15. Oscillogram of single break test at 22 kv

This test represents the duty on the unit having maximum stress in one pole clearing a grounded fault of 4,500,000 kva on a 287 kv system

Arc transfer has been purposely delayed for at least a half cycle, so that there will be a gap between the contacts of an inch or more at the time when the arc is ready to interrupt on the arcing horns. This eliminates unnecessary arc energy generation in vain attempts to interrupt the arc as soon as the contacts part. Referring to the oscillograms in figures 14 and 15, it may be seen that the arc voltage during this period, between points B and C, is so low as to be hardly detectable.

The characteristic curve indicates that the operating time is practically independent of the test voltage, since the squares and circles, representing tests differing by more than 50 per cent in the interrupted voltage per grid, are both distributed fairly uniformly about the mean curve. On the contrary, increasing current very definitely reduces the time both to transfer and to rupture the arc, because of the increased magnetic effect of the radial field coils in the deionizing grid interrupters. Thus the forces used to interrupt the circuit are automatically adjusted to the duty to be performed, avoiding unnecessary strains on the breaker during switching or low-power arc rupturing service, but having instantly available full interrupting ability to clear the most severe faults in the minimum time.

The few tests referred to are hardly sufficient to give an adequate picture of the ruggedness and reliability of this type of circuit breaker. As an indication of the repeated interrupting duty of which this breaker is capable without requiring maintenance of any kind, it should be pointed out that on a number of different occasions the test breaker pole unit was subjected in one day to between 75 and 100 short circuits representing duty from 10 per cent to 150 per cent of the breaker rating. Since short circuits on a high voltage system such as the one for which this circuit breaker is designed occur at very infrequent intervals and those faults that do occur are divided among a number of different breakers, it may be realized that a schedule of repeated tests such as that just described amounts to crowding a life time of service into a single day. The oil volume in these circuit breakers is considerable, and the arc energy during interruption low, so that the oil deterioration from arcing has proved to be practically negligible. These tests carried out on the highest service voltage rating of conventional oil circuit breakers throw added assurance on the dependability of large numbers of high voltage breakers now in service incorporating the same fundamental principles of arc rupture but not adapted to proof of their sufficiency by the same test methods.

High speed arc rupture of the type described in this paper is applicable to other voltage ratings as well. However, the considerable additional complication involved in the multibreak design should be justified by definite economic advantages that may be gained by the use of ultra high speed switching. A reduction in breaker time from 8 to 6 cycles made possible by moderate changes in the tripping mechanism will probably take care of most applications.

The cylindrical shape of the new deionizing grids lends itself to use in a porcelain-clad breaker design, and enough tests have been made to indicate satisfactory performance if such an application should prove desirable from other considerations. At the present time, there does not appear to be indication

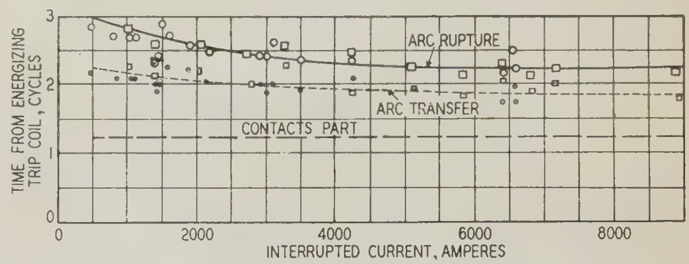


Fig. 16. Operating time characteristic curve of 287.5 kv oil circuit breaker

The average total time of $2^1/_2$ cycles includes approximately $1^1/_4$ cycles to part contacts, $^3/_4$ cycle to open the contacts and transfer the arc to the horns, and $^1/_2$ cycle to interrupt the arc

○—10 break tests at 264 kv; 5 break tests at 132 kv; single break tests at 36 kv

□—5 break tests at 88 kv; single break tests at 22 kv

of sufficient economic advantage or improvement from the point of view of safety to justify a complete swing away from the ruggedness and reliability of the conventional steel tank design.

References

1. High Speed Circuit Breakers for Railway Electrification, H. M. Wilcox. A.I.E.E. Trans., v. 47, Oct. 1928, p. 1285–92.

2. The Use of Oil in Arc Rupture, B. P. Baker and H. M. Wilcox. A.I.E.E. Trans., v. 49, Apr. 1930, p. 431–46.

3. Discussion by J. Slepian. Elec. Engg., v. 55, Feb. 1936, p. 191–2.

4. Extinction of a Long A-C Arc, J. Slepian. A.I.E.E. Trans., v. 49, Apr. 1930, p. 421–30.

5. Recent Developments in Arc Rupturing Devices, R. C. Van Sickle and W. M. Leeds. A.I.E.E. Trans., v. 51, Apr. 1932, p. 177–84.

6. Theory of the Deion Circuit Breaker, J. Slepian. A.I.E.E. Trans., v. 48, Apr. 1929, p. 523–7.

7. The Structural Development of the Deion Circuit Breaker, R. C. Dickinson and B. P. Baker. A.I.E.E. Trans., v. 48, Apr. 1929, p. 528–34.

Salient Pole Motors Out of Synchronism

An extension of the development and application of methods of tensor analysis is given here, applied to the predetermination of the subsynchronous performance characteristics of salient pole synchronous motors.

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THAT salient pole synchronous machines when operated out of step with the field excited are subjected to very high pulsating torques that may cause mechanical damage has long been known. Also, it is known that when the field is excited a motor's ability to start from rest or to pull into step is markedly affected; the precise results in any given case being affected by the relative values of applied armature voltage, the field excitation, and especially the resistance of the armature circuit. Even when the field is unexcited, and either open or short-circuited, the motor torque pulsates through a wide range during operation out of synchronism. To design machines properly to withstand these abnormal forces, and to determine the starting and synchronizing ability of a machine under all circuit conditions, it is therefore necessary to be able to predetermine the instantaneous torque and current relationships while the motor is operating out of synchronism.

The object of this paper is to present a more complete theory of this subject than heretofore has been available. The paper extends analyses of starting performance by Putman,[1] Park,[2] Linville,[3] Shutt,[4] and other writers to include continuous operation out of synchronism with the field excited. Figure 11, showing the average torque and the maximum and minimum instantaneous values developed over the range from zero to full speed by a typical low speed synchronous motor, with and without field excitation illustrates the results obtainable by this analysis. The value of torque pulsation of more than 300 per cent of normal developed at standstill with full excitation on this motor shows the importance of taking account of the stresses that occur during operation out of synchronism.

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted March 14, 1936; released for publication Apr. 3, 1936.

The author is grateful for the stimulating interest of Gabriel Kron in the general subject treated by this paper, and is indebted to D. R. Shoults for assistance in obtaining test data, to S. B. Crary for an independent check of certain calculations, and to Charles Concordia for suggestions concerning Appendix II.

1. For all numbered references, see list at end of paper.

Conclusions

The continued extension of power systems and installation of larger generating units makes increasingly important the control and protection of synchronous machines in a manner that will prevent operation out of synchronism and thus minimize the possibility of damage to connected apparatus or to the machines themselves.

It is obvious that the machines should be so designed that they will withstand operation out of step for sufficient time to permit protective devices to function, or some correspondingly reasonable period.

Moreover, careful consideration should be given to the use of directly connected exciters on large units. When a machine equipped with a directly connected exciter accidentally is operated out of synchronism, the excitation decreases with the speed and the resulting torque pulsations are greatly reduced at low speeds and at standstill.

Method of Analysis

Several of the characteristics associated with the starting performance, and the synchronous operation of a motor occur simultaneously when it runs out of step with the field excited. A consideration of this phenomenon therefore must include a means of determining all these characteristics. The synchronous action may be expressed by simple algebraic equations. The starting performance presents a more difficult problem, but may be determined in any manner that takes account of the most important factors. For example, one may use the generalized equations and impedance operators of Park,[2] calculating the operators directly from the fundamental circuit equations[7] or evaluating them by means of equivalent circuits.[3]

Identical results may be obtained from the tensor method of Kron[8] by employing an impedance matrix which contains all the circuit constants. In general, such a matrix would be comprised of $2n + 3$ rows and columns where there were n amortisseur circuits in both the direct and quadrature axes. It can readily be reduced to a minimum of 2 rows and columns where the various components contain Park's operators $x_d(p)$ and $x_q(p)$. The final equations will be in the most concise form when they are expressed in terms of these functions. However, from the standpoint of design or calculation, it is not always advisable to use these operators exclusively. They contain armature and field terms in combination with those for the amortisseur and do not permit the various windings to be classified to the best advantage.

This obstacle may be overcome by reducing the matrix of order $(2n + 3)$ to one having 3 rows and columns and containing analogous operators $x_{do}(p)$, $x_f(p)$, and $x_{af}(p)$ in addition to $x_q(p)$. These operators may be separated into terms which apply exclusively to the armature, field, and amortisseur windings.

In view of these facts the following procedure was adopted in preparing this paper.

1. The performance characteristics of a machine operating continuously out of synchronism, and the methods of calculating them, are discussed and illustrated in the body of the paper. Wherever possible, the terms of the several equations are arranged to show how the familiar synchronous and induction characteristics are combined when the motor operates out of step. This material is of general interest and should be readily understood and appliable by anyone familiar with synchronous machines.

2. The several appendices constitute a discussion of the theory and equations involved in the operation of a synchronous motor at any constant speed. They are intended to appeal to the reader who does not possess a very complete understanding of the "2-reaction theory" of machine analysis. Further, inasmuch as several of the basic equations are more general in some respects than those previously presented, it is believed that the appendices will have value as references for future work. The tensor method of Kron,[8] which he uses in treating 2 phase motors, has been adapted to 3 phase machines by transformation matrices which are essentially the foundation of the 2-reaction theory and were first given in scalar form by Park.[2] The tensor method has been followed throughout the appendices primarily because it leads to a clear understanding of the problem at hand together with associated problems. It also affords the simplest method of defining operators for the case of n amortisseur circuits, and demonstrating the methods of separating and recombining the field, armature, and amortisseur terms. A reader familiar with the theory of determinants will be able to obtain in a few hours a knowledge of tensor analysis sufficient to follow the derivation of any of the equations.

Results of Analysis

A few of the more important points developed in the paper are as follows.

1. A general expression is derived in Appendix I for the armature voltage impressed along the direct and quadrature axes together with the appropriate modifications to be used in steady state and transient problems. The angle defining the phase voltage at the instant the switch is closed is shown to be the initial mechanical displacement plus the angular displacement of the direct axis from the reference phase.

2. The time vector j, used in the complex number system, is frequently confused with the space vectors used to identify various reference axes. Therefore, Appendix II has been included to show how the instantaneous values of current, etc., are obtained when the solution appears in complex form.

3. A method is given in Appendix III for reducing any matrix to one of lower order by inspection. The method is applied in deriving the operational impedance for the case of n amortisseur circuits in each axis.

4. The transient performance of a motor when switched on the line may be obtained by following the procedure outlined in Appendix IV.

5. The instantaneous power and torque of a motor running at any constant speed with or without excitation may be calculated from the equations developed in Appendix V.

6. Several characteristic curves are included which, to the author's knowledge, have never been published before. The most interesting of these is the total instantaneous torque developed out of synchronism (figure 11) as it emphasizes the seriousness of stalling an excited machine.

Assumptions

The performance of a synchronous machine is expressed mathematically by a set of simultaneous differential equations. Particular solutions of these equations can be obtained by means of mechanical integration, or step-by-step analysis for a limited number of circuit constants. More general solutions, in the sense that they are valid for an arbitrary number of circuits, assume constant angular velocity for a given value of slip. The expressions contained in this paper are evaluated in this manner. It has been found that the steady component of torque calculated on the basis of constant slip agrees reasonably well with the actual value.[5] A knowledge of the oscillating torque is desired merely to indicate the magnitude and frequency of the disturbing forces. Thus, the assumption is justifiable from a practical standpoint.

The analysis neglects saturation and hysteresis, but assumes that the effect of eddy currents in the copper will be taken into account when the circuit constants are evaluated. In all other respects the machine is considered "ideal" as defined by Park.[6]

While the material presented in this paper applies to any polyphase machine, within the limit of the assumptions, particular attention is devoted to 3 phase salient pole motors of the conventional type.

Power and torque have positive signs during motor operation. The displacement angle δ is a small positive quantity during synchronous motor operation at light loads.

Determination of Subsynchronous Performance

Equations for determining the characteristic behavior of a machine when it operates out of synchronism are derived in the several appendices. Because of the length of these expressions only a few of them are repeated here, the remainder being referred to by number. In deriving the equations the component currents in the various circuits were determined separately for each applied voltage, and then combined vectorially to obtain the total currents. The flux density vectors resulting from each applied voltage also were evaluated separately and combined in a similar manner. This procedure was followed because it permitted the component of torque due to induction motor action to be sepa-

rated from the total instantaneous value. The resultant flux density vector has the same value regardless of whether it is obtained in the described manner or evaluated directly from the total current.

The steady state power and torque equations were determined from the complete expressions for current and flux density. They are expressed in terms of the real and imaginary components of the several current and flux density vectors due to the various applied voltages. Hence, in applying the equations, the current and flux density vectors first are calculated for the case of alternating voltage applied to the machine with the field short-circuited through the impedance of the excitation circuit. The process then is repeated for the condition of excitation voltage being applied to the motor when its armature is short-circuited through the impedance of the power bus. These results then are substituted directly in the equations for instantaneous power and torque. The following discussion of the various characteristic curves, for a machine operating continuously out of step with the field excited, illustrates the proper use of the equations.

The curves apply to a motor that would develop a per-unit torque of approximately 0.40 at standstill and at 95 per cent speed when running as an induction motor with the field closed on the starting or discharge resistor. Motors of this type are used for direct connection to reciprocating compressors and all manner of drives that start unloaded. They probably represent a class of synchronous motors familiar to everyone connected with the electrical industry.

Part of the machine constants selected were as follows:

r	$= 0.030$	
R_{fd}	$= 0.004$	Field closed on excitation source
	$= 0.025$	Field closed on discharge resistor
x_d	$= 1.0$	
x_q	$= 0.75$	
x_{afd}	$= 0.75$	
e	$= 1.0$	
e_d	$= 1.38$	

The remaining constants are represented by the steady state impedance operators shown on the curves of figure 1. The data were calculated by the methods of Appendix III using $R_{fd} = 0.004$. These particular operators represent the impedances of the direct and quadrature armature circuits when the field and all amortisseur circuits are closed. They appear in the reduced impedance matrix having 2

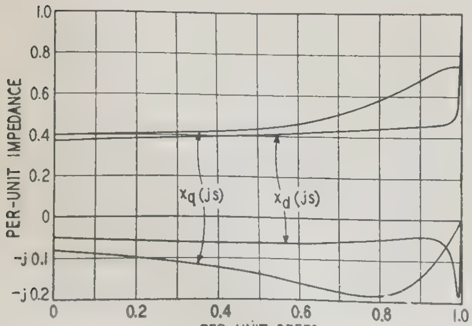


Fig. 1. Steady state impedance functions; field short-circuited and amortisseur circuits closed

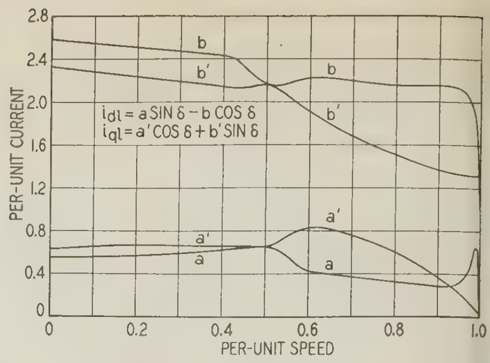


Fig. 2. Armature currents caused by impressed alternating voltages

rows and columns (equation 64) and are expressed in complex form in figure 1. At standstill, $x_q\ (js)$ would be read $0.40 - j0.0772$.

Armature currents resulting from the application of balanced sinusoidal phase voltages to the machine are shown in figure 2. These currents were obtained by substituting values from figure 1 in equation 68. The solution reduces to the following form,

$$i_{d1} = e(a - jb) \sin \delta \tag{86}$$

$$i_{q1} = e(a' - jb') \cos \delta \tag{87}$$

where i_{d1} and i_{q1} are defined by equation 69.

These curves are characteristic of salient pole motors in general in that i_{d1} and i_{q1} are equal in magnitude at 50 per cent speed. The influence of the short-circuited field is shown by the sudden increase in the real component of i_{d1} near synchronous speed.

The armature flux density vector due to applied phase voltages was calculated from equation 79. The results were obtained in the form,

$$\psi_{d1} = e(c - jd) \cos \delta \tag{93}$$

$$\psi_{q1} = -e(c' - jd') \sin \delta \tag{94}$$

where ψ_{d1} and ψ_{d2} are defined by equation 80. The coefficients c, d, c', d' are plotted in figure 3. The scale of c and c' has been increased 10 times in order to illustrate the characteristic shape of these curves near half speed.

The armature currents that appear when the machine is excited are shown by figure 4. The current vector was evaluated from equation 75, and the components i_{d2}, i_{q2} are defined by equation 76. Figure 4 also shows the flux density produced by the currents i_{d2} and i_{q2}. The components of the vector are defined by equation 83 and were calculated in accordance with equation 81.

The information contained in figures 2, 3, and 4 is sufficient for the calculation of power and torque at any steady value of slip between zero and unity. The equation for the total instantaneous power transferred between the line and the machine is derived in Appendix V. It is written in terms of the various components of armature current shown by figures 2 and 4, as follows:

$$P = e^2\frac{(a + a')}{2} - e^2\frac{(b - b')}{2}\sin 2\delta - e^2\frac{(a - a')}{2}\times \cos 2\,\delta + e\,i_{d2}\sin\delta + e\,i_{q2}\cos\delta \tag{90}$$

The first 3 terms are independent of excitation, and represent the power that would be supplied to the

machine if it operated as an induction motor with the field short-circuited. The first term is constant for a given value of slip, and represents the average or useful power supplied to the machine over a slip cycle. The second and third terms pulsate at twice slip frequency and represent components of a second harmonic in the total power pulsation. The second harmonic is due to the saliency of the rotor and any inequality of the windings in the 2 axes, such as the presence of a closed field winding. This component

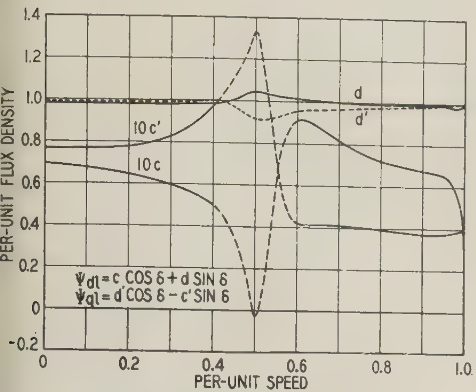


Fig. 3. Components of flux density vector resulting from impressed alternating voltages

of power is always zero at half speed in a salient pole machine. It does not appear at all in the induction motor. The first 3 terms of the power equation are plotted in figure 5 with the total magnitude of the second harmonic indicated by a dotted line.

The fourth and fifth terms of equation 90 pulsate at slip frequency and vary directly with excitation. They are plotted in figure 6. The first harmonic of pulsating power is shown by a broken line, and is only slightly greater than the *sin* δ term over most of the speed range.

The total instantaneous power may be obtained by test. It appears on oscillographic records as a function of time. For that reason an attempt sometimes is made to interpret the power trace in accordance with the power angle characteristic obtained at synchronous speed. This would place the maximum points at from approximately 70 to 90 degrees and 270 to 290 degrees, which seldom is the case. Figure 7 shows a typical record taken on a 350 horsepower, 400 rpm synchronous motor, fully excited, and operating at an average slip of 5.25 per cent. The large notches in the trace indicating angular displacement are 90 electrical degrees apart while the small notches occur every 10 degrees. The instantaneous power has been plotted as a function of displacement angle in figure 8. The smooth curve was obtained by the methods of this paper, using the equation

$$P = 0.4765e^2 - 0.535e^2 \sin 2\delta - 0.0175e^2 \cos 2\delta + 1.546e \sin \delta - 0.0848e \cos \delta$$

The test points taken from figure 7 are indicated by the small circles.

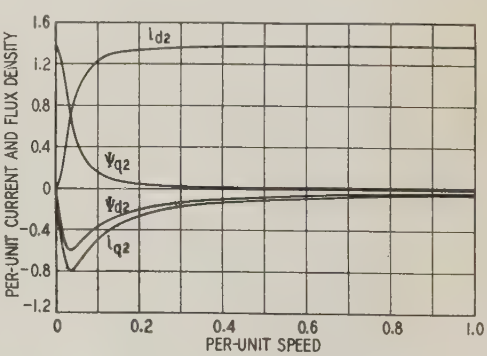
In order to have a direct comparison between the calculated and test points the actual values of e from figure 7 were used in equation 100. The voltage varied from 0.905 per-unit at $\delta = 0$ degrees to 0.855 at $\delta = 180$ degrees. This accounts for the variation in the average component of torque shown in figure 8. Most of the divergence between the calculated and test curves probably is due to the effect of the rate of change of slip on the electrical torque. Saturation modifies the lower parts of the curve, but has little effect on the maximum values. The accuracy with which the circuit constants can be predetermined also is a factor. All things considered, the agreement between the 2 curves is quite satisfactory.

The total instantaneous torque developed by the motor when running continuously out of step was determined by substituting values from figures 2, 3, and 4 in the following equation:

$$T = \frac{e^2}{2}(ad + a'd' - bc - b'c') + \frac{e^2}{2}(ac + b'd' - bd - a'c') \sin 2\delta - \frac{e^2}{2}(ad + bc - b'c' - a'd') \cos 2\delta + (i_{d2}\psi_{d2} + i_{q2}\psi_{q2}) + e(a\psi_{d2} + di_{d2} + b'\psi_{q2} - c'i_{q2}) \sin \delta - e(b\psi_{d2} - ci_{d2} - a'\psi_{q2} - d'i_{q2}) \cos \delta \quad (97)$$

The first 4 terms of the torque equation are shown in figure 9. The first term represents the steady or average component of torque that carries the load and produces continuous acceleration of the rotor. The second and third terms comprise the second harmonic component of the total instantaneous torque. The total magnitude of this harmonic is indicated by a dotted line in figure 9. All 3 of these terms are independent of excitation, and together represent the torque that the machine would develop when running as an induction motor with the field winding short-circuited. The steady component of torque is approximately equal to the average power minus the armature circuit losses. The second harmonic torque and power pulsations are about equal in magnitude near standstill and synchronous speed. At half speed the power pulsation disappears while the torque pulsations continue to increase. The

Fig. 4. Components of armature current and flux density vectors that appear when the field is excited



fourth term of equation 97 is a function of speed, and varies directly as the square of the excitation. This component has been referred to in Appendix V as the braking torque. It results from the circulation of current through the armature and power source due to the presence of direct current in the field winding.

The last 2 terms of equation 97 pulsate at slip frequency, and comprise the first harmonic of the oscillating torque. They are shown together with their total magnitude in figure 10. The difference

between the first harmonic of pulsating power, shown in figure 6, and the corresponding component of torque in the region from 0 to 20 per cent speed is very important. One might expect that since the pulsating power decreased to zero when the machine slows down to rest, the torque might do likewise. This is not the case, as the pulsating torque increases enormously when the machine stalls with its field excited. The oscillating torque near standstill is due almost entirely to the action of the component of armature current produced by alternating voltages, on the field produced by the exciting current. The magnitude of this torque at zero speed is equal to the product of the quadrature axis armature current and the quadrature axis flux density vector, ψ_{q2}. The vector ψ_{q2} is equal in magnitude to the direct axis armature flux linkages or $(e_d - i_{d2}x_d)$. As the machine decreases in speed, i_{d2} and its armature reaction approach zero. This allows the flux in the pole to build up to a value where all the field magnetomotive force is consumed in overcoming the reluctance of the magnetic circuit. ψ_{d2} approaches the value of e_d and the limiting value of the first harmonic of pulsating torque becomes $e_d i_{q1}$. The first harmonic of the power pulsation approaches zero since both i_{d2} and i_{q2} are the result of generated voltages and must be zero at standstill.

Curve A of figure 11 shows the net average torque developed by the motor over a complete slip cycle at any particular speed. The extent of the torque pulsation is represented by the distance from curve B to curve C. At some point in the slip cycle the torque will increase in a positive direction to the value indicated by curve B. It then will decrease to zero, and finally increase negatively to the value indicated by curve C. The average component of torque developed during the normal accelerating cycle with the field closed through its discharge resistor is shown

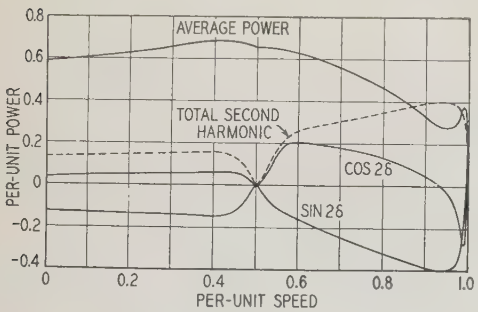
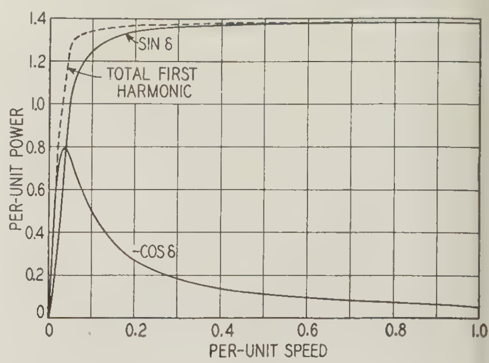


Fig. 5. Components of total instantaneous power produced by induction motor action

by curve D. The magnitude of the corresponding second harmonic pulsation is represented by the distance from curve E to curve F.

A consideration of figure 11 shows that stalling the average low torque synchronous motor without removing excitation may easily result in as much (or more) damage than any other condition to which it might be subjected. As the machine pulls out of step and slows down to rest, the principal component of pulsating torque passes through the entire frequency range from zero to the rated value. The pulsating torque has an appreciable magnitude throughout this range and, being continuous, constitutes a serious hazard to the mechanical system. The magnitude of these pulsations at standstill usually exceeds the short-circuit torque, and with machines having a power factor of 0.8 or less it frequently exceeds the maximum value occasioned by synchronizing 180 degrees out of phase. Once the machine reaches standstill it may not be able to accelerate, even though the load is reduced

Fig. 6. Components of total instantaneous power produced by synchronous motor action



to zero. Under these conditions the amortisseur will overheat rapidly and ultimately destroy itself and other windings unless power is removed within a matter of seconds.

Nomenclature

All machine constants defined in the following paragraphs have per-unit values expressed in terms of normal kilovolt-amperes, voltage, or flux linkages. The reactances are based upon fundamental frequency, and are numerically equal to the per-unit inductances. They are both denoted by the symbol, x. The reader is referred to Doherty and Nickle[10,11] for a discussion of the per-unit system of notation and to Linville[3] for the method of converting the constants to per-unit values.

Lower case letters denote armature terms; upper case letters refer to the field and amortisseur windings. Resistance terms, except that for the armature, have 2 subscripts, the first referring to the circuit and the second referring to the axis in which the resistance is located. Reactance or inductance terms have 3 subscripts, synchronous reactances excepted. Where the first 2 subscripts are identical they indicate the self-inductance of a particular circuit. When the first 2 subscripts are different they denote the mutual inductance of the 2 circuits. The last subscript is always d or q and identifies the axis in which the terms are located; a refers to the armature, f to the field; numbers 1 to n, including k, refer to the amortisseur.

Thus, a few of the individual circuit constants are:

r = armature circuit resistance
x_d = synchronous reactance or total self-inductance of direct axis armature circuit
x_q = synchronous reactance or total self-inductance of quadrature axis armature circuit
R_{fd} = direct axis field circuit resistance
X_{ffd} = total self inductance or self-inductive reactance of direct axis field circuit

x_{afd} = mutual inductance or reactance between the direct axis armature and field circuits

R_{kq} = resistance of the kth amortisseur circuit in the quadrature axis

X_{knd} = mutual inductance or reactance between circuits k and n located in the direct axis

The principal operators are:

$p = \dfrac{d}{dt}$

1 = Heaviside's unit function

$Z_d(p) = r + x_d(p)\,p$ operational impedance of the direct axis armature circuit when the field and amortisseur circuits are closed

$Z_q(p) = r + x_q(p)\,p$ operational impedance of the quadrature axis armature circuit when the amortisseur circuits are closed

$Z_{do}(p) = r + x_{do}(p)\,p$ operational impedance of the direct axis armature circuit with the amortisseur circuits closed and the field circuit open

$Z_f(p) = R_{fd} + x_f(p)\,p$ operational impedance of the direct axis field circuit when the amortisseur circuits are closed and the armature circuit is open

$x_{af}(p)$ = mutual inductance operator between the armature and field circuits when the amortisseur circuits are closed

Other quantities are:

t = time in electrical radians

θ = displacement of phase a from the direct axis (see Appendix I)

δ = mechanical displacement angle (see Appendix I)

ϕ = switching angle of phase voltage

v = $p\theta$ velocity of rotor

s = $p\delta$ slip

E_{fd} = field excitation voltage

e_d = direct axis voltage corresponding to field excitation

A particular value of the following terms is indicated by an appropriate subscript:

e = voltage

i = current

P = power

T = torque

ψ = flux density or flux linkages

Appendix I—Resolution of Phase Quantities Along the Direct and Quadrature Axes

The 2-reaction theory of synchronous machines is built around the relationship existing between the known or measurable phase quantities and the hypothetical ones that, for purposes of analysis, are assumed to exist along the axis of the poles and that of the interpolar space. In the diagram of figure 12, which shows a revolving armature machine, let a, b, c, be the axes of the respective phases rotating in a clockwise direction with instantaneous angular velocity, $p\theta$. The stationary direct and quadrature axes are represented by d and q, respectively. d is along the axis of the poles and denotes the direct axis, while q is directed along the center line between poles and indicates the quadrature axis. The positive direction of current in all cases will be taken along the axes away from the center of the armature.

Let:

i_a, i_b, i_c = instantaneous phase currents

i_d, i_q = instantaneous currents acting along the direct and quadrature axes

$\boldsymbol{e}', \boldsymbol{i}'$ = the phase voltage and current vectors

$\boldsymbol{e}, \boldsymbol{i}$ = the voltage and current vectors along the stationary axes

e_0, i_0 = zero phase sequence quantities

$\boldsymbol{C}$ = a transformation matrix relating $\boldsymbol{i}'$ with $\boldsymbol{i}$ and $\boldsymbol{e}'$ with $\boldsymbol{e}$

θ = angle between phase a and the direct axis having the value θ_0 at $t = 0$

δ = mechanical displacement angle or angle between the rotating voltage vector and the quadrature axis. Its value is δ_0 when $t = 0$

In the per-unit system of notation, unit phase currents must produce unit currents along the stationary and neutral axes and *vice versa*. These relations are shown by the following scalar equations which may be written directly from figure 12:

$$i_a = i_d \cos\theta + i_q \sin\theta + i_0$$

$$i_b = i_d \cos\left(\theta - \frac{2\pi}{3}\right) + i_q \sin\left(\theta - \frac{2\pi}{3}\right) + i_0$$

$$i_c = i_d \cos\left(\theta + \frac{2\pi}{3}\right) + i_q \sin\left(\theta + \frac{2\pi}{3}\right) + i_0 \qquad (1)$$

$$i_d = \frac{2}{3} i_a \cos\theta + \frac{2}{3} i_b \cos\left(\theta - \frac{2\pi}{3}\right) + \frac{2}{3} i_c \cos\left(\theta + \frac{2\pi}{3}\right)$$

$$i_q = \frac{2}{3} i_a \sin\theta + \frac{2}{3} i_b \sin\left(\theta - \frac{2\pi}{3}\right) + \frac{2}{3} i_c \sin\left(\theta + \frac{2\pi}{3}\right)$$

$$i_0 = \frac{1}{3} i_a + \frac{1}{3} i_b + \frac{1}{3} i_c \qquad (2)$$

Let

$$\boldsymbol{i}' = i_a\boldsymbol{a} + i_b\boldsymbol{b} + i_c\boldsymbol{c} \qquad (3)$$

$$\boldsymbol{i} = i_d\boldsymbol{d} + i_q\boldsymbol{q} + i_0\boldsymbol{o} \qquad (4)$$

where $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, $\boldsymbol{d}$, $\boldsymbol{q}$, $\boldsymbol{o}$ are unit space vectors.

Equation 1 may then be written in tensor form as

$$\boldsymbol{i}' = \boldsymbol{C}\cdot\boldsymbol{i} \qquad (5)$$

Where $\boldsymbol{C}$ is a transformation matrix containing the coefficients of the currents on the right hand side of equation 1

$$\boldsymbol{C} = \begin{array}{c|ccc} & d & q & o \\ \hline a & \cos\theta & \sin\theta & 1 \\ b & \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ c & \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{array} \qquad (6)$$

Likewise, equation 2 may be written

$$\boldsymbol{i} = \boldsymbol{T}\cdot\boldsymbol{i}'$$

Where $\boldsymbol{T}$ is a transformation matrix containing the coefficients of the currents on the right hand side of equation 2. But from equation 5

$$\boldsymbol{i} = \boldsymbol{C}^{-1}\cdot\boldsymbol{i}' \qquad (7)$$

Matrix $\boldsymbol{T}$ must therefore equal $\boldsymbol{C}^{-1}$ which is the inverse of matrix $\boldsymbol{C}$ in order for the equations to be consistent. These conditions are fulfilled by the following matrix:

$$\boldsymbol{C}^{-1} = \begin{array}{c|ccc} & a & b & c \\ \hline d & \frac{2}{3}\cos\theta & \frac{2}{3}\cos\left(\theta - \frac{2\pi}{3}\right) & \frac{2}{3}\cos\left(\theta + \frac{2\pi}{3}\right) \\ q & \frac{2}{3}\sin\theta & \frac{2}{3}\sin\left(\theta - \frac{2\pi}{3}\right) & \frac{2}{3}\sin\left(\theta + \frac{2\pi}{3}\right) \\ o & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array} = \boldsymbol{T} \qquad (8)$$

Let

$$\boldsymbol{e}' = e_a\boldsymbol{a} + e_b\boldsymbol{b} + e_c\boldsymbol{c} \qquad (9)$$

$$\boldsymbol{e} = e_d\boldsymbol{d} = e_q\boldsymbol{q} + 2e_o\boldsymbol{o} \qquad (10)$$

The per-unit notation results in an unusual transformation for voltage. This is due to the definition of power which is expressed as the ratio of the total power to the rated or base power. In terms of phase quantities the per unit power is,

$$P = \frac{2}{3}\,\boldsymbol{e}'\cdot\boldsymbol{i}' = \frac{2}{3}\,(e_a i_a + e_b i_b + e_c i_c) \tag{11}$$

which includes a zero phase sequence component

$$P_o = 2e_o i_o$$

where

$$e_o = \frac{1}{3}\,(e_a + e_b + e_c)$$

$$i_o = \frac{1}{3}\,(i_a + i_b + i_c) \tag{12}$$

The power along the direct and quadrature axes is $e_d i_d + e_q i_q$ to which the zero phase sequence power must be added to obtain the total per-unit power. Thus, from equations 4, 10, and 12

$$P = \boldsymbol{e}\cdot\boldsymbol{i} = e_d i_d + e_q i_q + 2e_o i_o \tag{13}$$

The magnitude of the per-unit power must not change with the transfer from phase to direct and quadrature quantities. This may be insured by transforming the voltage vectors in accordance with the identity obtained from equations 12 and 13.

$$\boldsymbol{e}\cdot\boldsymbol{i} = \frac{2}{3}\,\boldsymbol{e}'\cdot\boldsymbol{i}' \tag{14}$$

Substituting the value of $\boldsymbol{i}'$ from equation 5 and solving for $\boldsymbol{e}$ and $\boldsymbol{e}'$

$$\boldsymbol{e} = \frac{2}{3}\,\boldsymbol{C_t}\cdot\boldsymbol{e}' \tag{15}$$

$$\boldsymbol{e}' = \frac{3}{2}\,\boldsymbol{C_t}^{-1}\cdot\boldsymbol{e} \tag{16}$$

$\boldsymbol{C_t}$ and $\boldsymbol{C_t}^{-1}$ may be written directly from equations 6 and 8, respectively, by interchanging unit vectors thus:

$$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{a} & \boldsymbol{b} & \boldsymbol{c} \end{array} \\ \frac{2}{3}\boldsymbol{C_t} = \begin{array}{c} \boldsymbol{d} \\ \boldsymbol{q} \\ \boldsymbol{o} \end{array} & \begin{bmatrix} \frac{2}{3}\cos\theta & \frac{2}{3}\cos\left(\theta - \frac{2\pi}{3}\right) & \frac{2}{3}\cos\left(\theta + \frac{2\pi}{3}\right) \\ \frac{2}{3}\sin\theta & \frac{2}{3}\sin\left(\theta - \frac{2\pi}{3}\right) & \frac{2}{3}\sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix} \end{array} \tag{17}$$

$$\begin{array}{cc} & \begin{array}{ccc} \boldsymbol{d} & \boldsymbol{q} & \boldsymbol{o} \end{array} \\ \frac{3}{2}\boldsymbol{C_t}^{-1} = \begin{array}{c} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \end{array} & \begin{bmatrix} \cos\theta & \sin\theta & \frac{1}{2} \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & \frac{1}{2} \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & \frac{1}{2} \end{bmatrix} \end{array} \tag{18}$$

Let balanced sinusoidal phase voltages be applied to the machine. If ϕ is the switching angle in electrical radians and the phase rotation is counterclockwise the voltage vector will be:

$$\boldsymbol{e}' = e\sin(t+\phi)\,\boldsymbol{a} + e\sin\left(t+\phi-\frac{2\pi}{3}\right)\boldsymbol{b} + e\sin\left(t+\phi+\frac{2\pi}{3}\right)\boldsymbol{c} \tag{19}$$

By combining equations 15, 17, and 19, the per-unit voltages impressed along the $\boldsymbol{d}$ and $\boldsymbol{q}$ axes are found to be

$$\boldsymbol{e} = e\sin(t+\phi-\theta)\,\boldsymbol{d} + e\cos(t+\phi-\theta)\,\boldsymbol{q} \tag{20}$$

It is apparent from figure 12 that the voltage vector may also be written

$$\boldsymbol{e} = e\sin\delta\boldsymbol{d} + e\cos\delta\boldsymbol{q} \tag{21}$$

Hence,

$$\delta = t + \phi - \theta \tag{22}$$

Referring again to figure 12, the voltage vector rotates at normal or unit speed with respect to the armature. The angular velocity of the armature with respect to the poles or stationary axes is $p\theta$ while the slip or velocity of e with respect to the stationary axes is $p\delta$. For subsynchronous operation,

$$\theta = \theta_0 + \int_0^t p\theta dt = \theta_0 + \int_0^t (1 - p\delta)dt$$

$$\delta = \delta_0 + \int_0^t p\delta dt$$

$$\theta + \delta = \theta_0 + \delta_0 + t \tag{23}$$

Combining equations 22 and 23

$$\phi = \theta_0 + \delta_0 \tag{24}$$

When the slip remains constant and motor operation is assumed,

$$\begin{aligned} p\delta &= s \\ p\theta &= 1 - s \\ \theta &= \theta_0 + (1-s)t \\ \delta &= \delta_0 + st \end{aligned} \tag{25}$$

and

$$\boldsymbol{e} = e\sin(\delta_0 + st)\,\boldsymbol{d} + e\cos(\delta_0 + st)\boldsymbol{q} \tag{26}$$

Hence the components of voltage applied to the stationary armature axes pulsate at slip frequency. If the voltage is suddenly applied the sinusoidal waves are converted into operators on the unit function[9] and the voltage vector becomes

$$\boldsymbol{e} = e\,\frac{(ps\cos\delta_0 + p^2\sin\delta_0)}{p^2+s^2}\,\mathbf{1}\boldsymbol{d} + e\,\frac{(p^2\cos\delta_0 - ps\sin\delta_0)}{p^2+s^2}\,\mathbf{1}\boldsymbol{q} \tag{27}$$

Appendix II—Significance of the Time Operator, j

When a voltage

$$E\cos\omega t = E\,\frac{\epsilon^{j\omega t} + \epsilon^{-j\omega t}}{2}$$

exists across an inductive impedance, the steady current has the form

$$I\cos(\omega t - \alpha) = I\,\frac{\epsilon^{j(\omega t-\alpha)} + \epsilon^{-j(\omega t-\alpha)}}{2}$$

In order to employ complex notation in the solution of alternating problems, a convention has been established such that

$E\cos\omega t$ is represented by $E\epsilon^{j\omega t}$ and $I\cos(\omega t - \alpha)$ is represented by $I\epsilon^{j(\omega t-\alpha)}$ (28)

The actual voltages or currents are equal to the real parts of the complex expressions. Thus,

$$I\epsilon^{j(\omega t-\alpha)} = I[\cos(\omega t - \alpha) + j\sin(\omega t - \alpha)]$$

and

$$I\cos(\omega t - \alpha) = \text{Real of } I\epsilon^{(\omega t-\alpha)} \tag{29}$$

Impedance is the ratio of the actual voltage to the actual current, and may be regarded as the ratio of the real parts of the complex expressions.

$$Z = \frac{E\cos\omega t}{I\cos(\omega t - \alpha)} \text{ is represented by } \frac{E}{I}\,\epsilon^{j\alpha} \tag{30}$$

where

$$\frac{E}{I}\epsilon^{j\alpha} = \frac{E}{I}(\cos\alpha + j\sin\alpha) = R + jX$$

Let both sides of equation 29 be multiplied by j.

$$\begin{aligned} Ij\cos(\omega t - \alpha) &= \text{Real of } Ij\epsilon^{(\omega t - \alpha)} \\ &= \text{Real of } I[-\sin(\omega t - \alpha) + j\cos(\omega t - \alpha)] \end{aligned}$$

In a similar manner

$$\begin{aligned} Ij\sin(\omega t - \alpha) &= \text{Real of } I[j(-j)\epsilon^{(\omega t - \alpha)}] \\ &= \text{Real of } I[\cos(\omega t - \alpha) + j\sin(\omega t - \alpha)] \end{aligned}$$

Hence in accordance with established convention, j may be regarded as a time operator such that

$$\begin{aligned} &j\cos(\omega t \pm \alpha) \text{ may be replaced by } -\sin(\omega t \pm \alpha) \\ &\sin(\omega t \pm \alpha) \text{ may be replaced by } \cos(\omega t \pm \alpha) \end{aligned} \qquad (31)$$

Multiplication of the complex expressions by j represents a phase displacement of the real component 90 degrees ahead in time. When impedance is expressed in complex form, the instantaneous values of current may be found by interpreting the operator j in accordance with the relations of equation 31.

Appendix III—Equivalent Impedance Dyadics of Machines Having Any Number of Amortisseur Circuits

In the assumed ideal case the impedance encountered by a voltage acting along the stationary direct or quadrature axis, is constant. This condition is approached in modern polyphase machines inasmuch as the windings are balanced and arranged to minimize the effect of imbedding the armature conductors in slots. It is permissible, therefore, to express a machine's performance by a set of simultaneous differential equations, each having the form of the general expression;

$$\boldsymbol{e} = \boldsymbol{R}\cdot\boldsymbol{i} + p\boldsymbol{L}\cdot\boldsymbol{i} + \boldsymbol{G}\cdot\boldsymbol{i}p\theta \qquad (32)$$

If there is an odd number of bars per pole, $(2n + 1)$ representing n amortisseur circuits in the direct axis and $n + 1$ in the quadrature axis, the total number of individual voltage equations will be $2n + 4$.

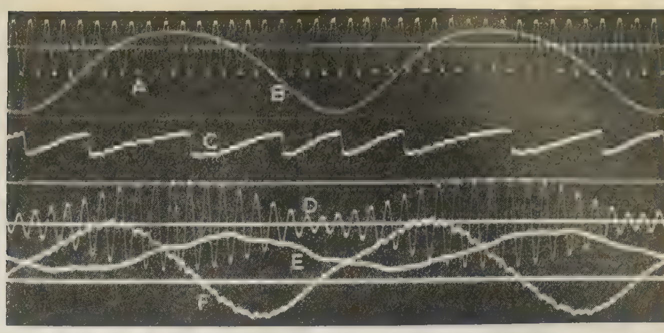


Fig. 7. Oscillogram of 350-horsepower 400-rpm 2,200-volt synchronous motor operating out of synchronism

A—Line voltage
B—Field current
C—Angular displacement
D—Line current
E—Slip
F—Power

Of this number $2n + 1$ pertain to the amortisseur windings, 1 to the direct axis field, and the remainder to the direct and quadrature axis armature circuits. Zero phase sequence currents are assumed to be zero when sinusoidal voltages are impressed on the armature.

Kron[8] has shown that the $2n + 4$ scalar equation can be represented by equation 32 where:

(1) $\boldsymbol{e}$ is a vector which denotes all applied voltages
(2) $\boldsymbol{R}\cdot\boldsymbol{i}$ represents all resistance drops
(3) $p\boldsymbol{L}\cdot\boldsymbol{i}$ represents all induced voltages
(4) $\boldsymbol{G}\cdot\boldsymbol{i}\,p\theta$ denotes all generated voltages

Equation 32 may be written

$$\boldsymbol{e} = (\boldsymbol{R} + p\boldsymbol{L} + \boldsymbol{G}p\theta)\cdot\boldsymbol{i}$$

or simply

$$\boldsymbol{e} = \boldsymbol{Z}\cdot\boldsymbol{i} \qquad (33)$$

$\boldsymbol{Z}$ is composed of the coefficients of the currents in the several voltage equations and may be written in matrix form as in equation 34.

$\boldsymbol{Z} =$ (34)

	d_n		d_2	d_1	d_f	d_a	q_a	q_0	q_1	q_2		q_n
d_n	$R_{nd} + X_{nnd}p$	—	$X_{2nd}p$	$X_{1nd}p$	$X_{fnd}p$	$x_{and}p$	0	0	0	0	—	0
	—	—	—	—	—	—	—	—	—	—	—	—
d_2	$X_{2nd}p$	—	$R_{2d} + X_{22d}p$	$X_{12d}p$	$X_{f2d}p$	$x_{a2d}p$	0	0	0	0	—	0
d_1	$X_{1nd}p$	—	$X_{12d}p$	$R_{1d} + X_{11d}p$	$X_{f1d}p$	$x_{a1d}p$	0	0	0	0	—	0
d_f	$X_{fnd}p$	—	$X_{f2d}p$	$X_{f1d}p$	$R_{fd} + X_{ffd}p$	$x_{afd}p$	0	0	0	0	—	0
d_a	$x_{and}p$	—	$x_{a2d}p$	$x_{a1d}p$	$x_{afd}p$	$r + x_dp$	$x_qp\theta$	$x_{a0q}p\theta$	$x_{a1q}p\theta$	$x_{a1q}p\theta$	—	$x_{anq}p\theta$
q_a	$-x_{and}p\theta$	—	$-x_{a2d}p\theta$	$-x_{a1d}p\theta$	$-x_{afd}p\theta$	$-x_dp\theta$	$r + x_qp$	$x_{a0q}p$	$x_{a1q}p$	$x_{a2q}p$	—	$x_{anq}p$
q_0	0	—	0	0	0	0	$x_{a0q}p$	$R_{0q} + X_{00q}p$	$X_{01q}p$	$X_{02q}p$	—	$X_{0nq}p$
q_1	0	—	0	0	0	0	$x_{a1q}p$	$X_{01q}p$	$R_{1q} + X_{11q}p$	$X_{12q}p$	—	$X_{1nq}p$
q_2	0	—	0	0	0	0	$x_{a2q}p$	$X_{02q}p$	$X_{12q}p$	$R_{2q} + X_{22q}p$	—	$X_{2nq}p$
	—	—	—	—	—	—	—	—	—	—	—	—
q_n	0	—	0	0	0	0	$x_{2nq}p$	$X_{0nq}p$	$X_{1nq}p$	$X_{2nq}p$	—	$R_{nq} + X_{nnq}p$

An inspection of equation 34 shows that it is the sum of the following matrices:

1. A resistance matrix containing the resistances of all the windings. The resistance terms appear only in the diagonal elements. Actually there is a small amount of mutual resistance between the amortisseur circuits when the bars are connected by a common end ring. In extreme cases where the effect of mutual resistance is important it may be included with the elements containing mutual inductance terms.

2. An inductance matrix containing all self- and mutual inductance of the various circuits. This matrix is multiplied by $p = d/dt$ where p refers only to current. The self inductances appear in the diagonal elements. The total self inductance of the nth, quadrature axis amortisseur circuit is denoted by X_{nnq}. The mutual inductance of the direct axis armature circuit with amortisseur circuit equation 2 is xa_2d, etc.

3. A matrix $\boldsymbol{G}p\theta$ composed of armature terms multiplied by $p\theta$ or per-unit velocity. Kron[8] refers to the $\boldsymbol{G}$ matrix as the torque tensor and to the vector $\psi = \boldsymbol{G}\cdot\boldsymbol{i}$ as the armature flux density, since torque is obtained from the scalar product $\boldsymbol{i}\cdot\psi = \boldsymbol{i}\cdot\boldsymbol{G}\cdot\boldsymbol{i}$. Torque also may be obtained from the cross product of the current and armature flux linkage vectors as shown by Park.[2] The $\boldsymbol{G}$ matrix is written as follows:

$$\boldsymbol{G} = \begin{array}{c|cccccccccccc|} & d_n & & d_2 & d_1 & d_f & d_a & q_a & q_0 & q_1 & q_2 & & q_n \\ \hline d_a & 0 & - & 0 & 0 & 0 & 0 & x_q & x_{aoq} & x_{a1q} & x_{a2q} & - & x_{anq} \\ q_a & -x_{and} & - & -x_{a2d} & -x_{a1d} & -x_{afd} & -x_d & 0 & 0 & 0 & 0 & - & 0 \\ \hline \end{array} \qquad (35)$$

The impedance matrix and the torque tensor may be put into a more suitable form for tabulation by eliminating all axes except the ones along which voltages are most frequently impressed. All the amortisseur circuits are permanently short-circuited, and should be eliminated by reducing the matrices to equivalent ones containing 3 axes. One axis will represent the field circuit while the other 2 will pertain to the direct and quadrature armature circuits. The resulting impedance matrix will contain elements which correspond to the impedances of the armature or field circuits as measured at their terminals with all the amortisseur circuits closed. In some instances it may be advisable to eliminate the field axis. When this is done, the matrices contain the 2 operations $x_d(p)$, $x_q(p)$, defined by Park.[2]

Any number of co-ordinate axes may be eliminated and the equivalent matrices written directly in their reduced form from the following considerations:

1. Each element of the reduced impedance matrix will be a fraction composed of two determinants.

2. The determinant comprising the denominator will be of order n and will be made up of such elements as are common to the n rows and columns to be eliminated from the original matrix.

3. The numerator of each element will be a determinant of order $(n + 1)$ and will contain the denominator as a minor.

4. The numerator of the element in column (c) and row (r) will be composed of such elements as are common to row (r) and column (c) of the original matrix together with those of the n rows and columns to be eliminated.

5. The reduced torque tensor will bear the same relationship to the reduced impedance matrix as that existing between the 2 original matrices. The $\boldsymbol{G}$ matrix is composed of all terms in the impedance matrix that are multiplied by $p\theta$.

The correctness of these statements may be verified by expanding the determinants and comparing the results with those obtained when the co-ordinate axes are eliminated by means of short-circuit matrices or the more general short-circuit theorem.[8] All final results are identical with those obtained from the simultaneous solution of the $2n + 4$ individual circuit equations.

When all amortisseur axes are eliminated matrices 34 and 35 may be expressed in terms of impedance operators as follows:

$$\boldsymbol{Z} = \begin{array}{c|c|c|c|} & d_f & d_a & q_a \\ \hline d_f & R_{fd} + x_f(p)p & x_{af}(p)p & 0 \\ \hline d_a & x_{af}(p)p & r + x_{do}(p)p & x_q(p)p\theta \\ \hline q_a & -x_{af}(p)p\theta & -x_{do}(p)p\theta & r + x_q(p)p \\ \hline \end{array} \qquad (36)$$

$$\boldsymbol{G} = \begin{array}{c|c|c|c|} & d_f & d_a & q_a \\ \hline d_a & 0 & 0 & x_q(p) \\ \hline q_a & -x_{af}(p) & -x_{do}(p) & 0 \\ \hline \end{array} \qquad (37)$$

The principal advantage of expressing $\boldsymbol{Z}$ in terms of 3 axes instead of 2 is that it permits the field, armature, and amortisseur terms to be separated. The several operators may be written in the following form:

$$Z_f(p) = R_{fd} + x_f(p)p$$
$$= R_{fd} + X_{ffd}p + D_f(p)p \qquad (38)$$

$$x_f(p) = X_{ffd} + D_f(p) \qquad (39)$$

$$x_{af}(p) = x_{afd} + D_{af}(p) \qquad (40)$$

$$Z_{do}(p) = r + x_{do}(p)p$$
$$= r + x_dp + D_{do}(p)p \qquad (41)$$

$$x_{do}(p) = x_d + D_{do}(p) \qquad (42)$$

$$Z_q(p) = r + x_q(p)p$$
$$x_q(p) = x_q + D_q(p) \qquad (43)$$

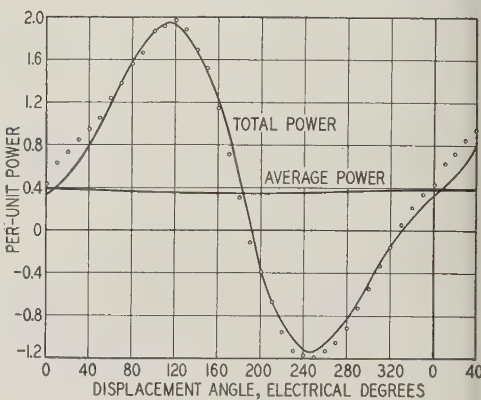


Fig. 8. Instantaneous armature power of a machine operating out of synchronism

Solid curve calculated; test points indicated by circles

The functions $D_f(p)$, $D_{af}(p)$, $D_{do}(p)$, $D_q(p)$ contain terms which depend only upon the material, size, and number of amortisseur bars, and the configuration of the air gap. To evaluate these functions, the elements of the reduced impedance matrix must be determined. From the previous discussion it follows that the dyad $Z_f(p)$ may be expressed as the ratio of 2 determinants which are functions of p. All quadrature axis terms cancel and the final expression becomes:

$$Z_f(p) = \frac{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p & X_{fnd}p \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p & X_{f2d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p & X_{f1d}p \\ X_{fnd}p & \cdot & X_{f2d} & X_{f1d}p & R_{fd}+X_{ffd}p \end{vmatrix}}{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p \\ \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p \end{vmatrix}} \qquad (44)$$

Let the determinant in the denominator of equation 44 be identified as $D_{ds}(p)$. It is composed of the elements common to the rows and columns eliminated from the direct axis.

$$D_{ds}(p) = \begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p \\ \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22nd}p & X_{12d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p \end{vmatrix} \qquad (45)$$

In the numerator of equation 44 $D_{ds}(p)$ is a minor complimentary to $R_{fd} + X_{ffd}p$. Hence from equation 38

$$D_f(p) = \frac{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p & X_{fnd}p \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p & X_{f2d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p & X_{f1d}p \\ X_{fnd}p & \cdot & X_{f2d}p & X_{f1d}p & 0 \end{vmatrix}}{D_{ds}(p)p} \tag{46}$$

Similar reasoning shows that the remaining direct axis operators may be expressed as follows:

$$D_{af}(p) = \frac{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p & X_{fnd}p \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p & X_{f2d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p & X_{f1d}p \\ x_{and}p & \cdot & x_{a2d}p & x_{a1d}p & 0 \end{vmatrix}}{D_{ds}(p)p} \tag{47}$$

$$D_{do}(p) = \frac{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p & x_{and}p \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p & x_{a2d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p & x_{a1d}p \\ x_{and}p & \cdot & x_{a2d}p & x_{a1d}p & 0 \end{vmatrix}}{D_{ds}(p)p} \tag{48}$$

The impedance of the quadrature axis armature circuit with the amortisseur circuits closed is

$$Z_q(p) = \frac{\begin{vmatrix} r+x_qp & x_{aoq}p & x_{a1q}p & x_{a2q}p & \cdot & x_{anq}p \\ x_{aoq}p & R_{oq}+X_{ooq}p & X_{o1q}p & X_{o2q}p & \cdot & X_{onq}p \\ x_{a1q}p & X_{o1q}p & R_{1q}+X_{11q}p & X_{12q}p & \cdot & X_{1nq}p \\ x_{a2q}p & X_{o2q}p & X_{12q}p & R_{2q}+X_{22q}p & \cdot & X_{2nq}p \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{anq}p & X_{onq}p & X_{1nq}p & X_{2nq}p & \cdot & R_{nq}+X_{nnq}p \end{vmatrix}}{D_{qs}(p)} \tag{49}$$

From equations 43 and 49

$$D_q(p) = \frac{\begin{vmatrix} 0 & x_{aoq}p & x_{a1q}p & x_{a2q}p & \cdot & x_{anq}p \\ x_{aoq}p & R_{oq}+X_{ooq}p & X_{o1q}p & X_{oq2}p & \cdot & X_{onq}p \\ x_{a1q}p & X_{o1q}p & R_{1q}+X_{11q}p & X_{12q}p & \cdot & X_{1nq}p \\ x_{a2q}p & X_{o2q}p & X_{12q}p & R_{2q}+X_{22q}p & \cdot & X_{2nq}p \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x_{anq}p & X_{onq}p & X_{1nq}p & X_{2nq}p & \cdot & R_{nq}+X_{nnq}p \end{vmatrix}}{D_{qs}(p)p} \tag{50}$$

Where $D_{qs}(p)$ is composed of the quadrature axis amortisseur terms:

$$D_{qs}(p) = \begin{vmatrix} R_{oq}+X_{ooq}p & X_{o1q}p & X_{oq2}p & \cdot & X_{onq}p \\ X_{o1q}p & R_{1q}+X_{11q}p & X_{12q}p & \cdot & X_{1nq}p \\ X_{o2q}p & X_{12q}p & R_{2q}+X_{22q}p & \cdot & X_{2nq}p \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{onq}p & X_{1nq}p & X_{2nq}p & \cdot & X_{nnq}p \end{vmatrix} \tag{51}$$

When the field axis is also eliminated, the impedance and torque matrices may be written in the following form.

$$\boldsymbol{Z} = \begin{array}{c|cc} & d_a & q_a \\ \hline d_a & r + x_d(p)p & x_q(p)p\theta \\ q_a & -x_d(p)p\theta & r + x_q(p)p \end{array} \tag{52}$$

$$\boldsymbol{G} = \begin{array}{c|cc} & d_a & q_a \\ \hline d_a & 0 & x_q(p) \\ q_a & -x_d(p) & 0 \end{array} \tag{53}$$

$$\boldsymbol{Y} = \begin{array}{c|ccc} & d_f & d_a & q_a \\ \hline d_f & \dfrac{Z_{do}(p)Z_q(p) + x_{do}(p)x_q(p)(p\theta)^2}{D} & \dfrac{-Z_q(p)x_{af}(p)p}{D} & \dfrac{x_q(p)x_{af}(p)pp\theta}{D} \\ d_a & \dfrac{-x_{af}(p)[Z_q(p)p + x_q(p)(p\theta)^2]}{D} & \dfrac{Z_q(p)Z_f(p)}{D} & \dfrac{-Z_f(p)x_q(p)p\theta}{D} \\ q_a & \dfrac{x_{af}(p)p\theta[Z_{do}(p) - x_d(p)p]}{D} & \dfrac{(Z_f(p)x_{do}(p) - x_{af}^2(p)p)p\theta}{D} & \dfrac{Z_f(p)Z_{do}(p) - x_{af}^2(p)p^2}{D} \end{array} \tag{58}$$

The operators $x_d(p)$, $x_q(p)$ are those used by Park.[2] $x_q(p)$ may be evaluated from equations 43 and 50. $x_d(p)$ may be determined from matrix 36 by eliminating the field axis, or from the original matrix (equation 34) by eliminating both field and amortisseur axes.

In the case of matrix 36:

$$Z_d(p) = r + x_d(p)p$$

$$= r + x_{do}(p)p - \frac{x_{af}^2(p)p^2}{R_{fd} + x_f(p)p} \tag{54}$$

$$x_d(p) = x_{do}(p) - \frac{x_{af}^2(p)p}{R_{fd} + x_f(p)p} \tag{55}$$

From matrix 34 the determinant form of $x_d(p)$ is

$$x_d(p) = \frac{1}{p}\,\frac{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p & X_{fnd}p & X_{and}p \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p & X_{f2d}p & x_{a2d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p & X_{f1d}p & x_{a1d}p \\ X_{fnd}p & \cdot & X_{f2d}p & X_{f1d}p & R_{fd}+X_{ffd}p & x_{afd}p \\ x_{and}p & \cdot & x_{a2d}p & x_{a1d}p & x_{afd}p & x_dp \end{vmatrix}}{\begin{vmatrix} R_{nd}+X_{nnd}p & \cdot & X_{2nd}p & X_{1nd}p & X_{fnd}p \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ X_{2nd}p & \cdot & R_{2d}+X_{22d}p & X_{12d}p & X_{f2d}p \\ X_{1nd}p & \cdot & X_{12d}p & R_{1d}+X_{11d}p & X_{f1d}p \\ X_{fnd}p & \cdot & X_{f2d}p & X_{f1d}p & R_{fd}+X_{ffd}p \end{vmatrix}} \tag{56}$$

Appendix IV—Transient and Steady-State Solutions

The basic equation for determining a machine's performance was given in Appendix III as

$$\boldsymbol{e} = \boldsymbol{Z}\cdot\boldsymbol{i} \tag{33}$$

where $\boldsymbol{e}$ represents all applied voltages, $\boldsymbol{i}$ all currents resulting from the applied voltages, and $\boldsymbol{Z}$ the impedance matrix. When solved for the currents, equation 33 becomes

$$\boldsymbol{i} = \boldsymbol{Z}^{-1}\cdot\boldsymbol{e}$$

or,

$$\boldsymbol{i} = \boldsymbol{Y}\cdot\boldsymbol{e} \tag{57}$$

$\boldsymbol{Y}$ is an admittance matrix and is determined from the impedance matrix $\boldsymbol{Z}$ by calculating its inverse $\boldsymbol{Z}^{-1}$.[8] The most convenient form of $\boldsymbol{Z}$ or $\boldsymbol{Y}$ will depend upon the number of currents to be evaluated. Suppose that $\boldsymbol{Z}$ is given by equation 36; then its inverse would be as given in equation 58 where

$$D = Z_f(p)[Z_{do}(p)]Z_q(p) + x_d(p)x_q(p)(p\theta)^2 - x_{af}^2(p)(Z_q(p)p^2 + x_q(p)p(p\theta)^2$$

Matrices 36 and 58 are referred to as transient matrices since they contain the operator p. The transient currents are obtained in operational form when the transient admittance matrix is used in conjunction with equation 57. For example, let balanced sinusoidal voltages suddenly be applied to the armature of a synchronous machine that is operating at a constant speed ($p\theta = 1 - s$) with the field winding closed. The applied voltage vector is given in Appendix I as,

$$\boldsymbol{e} = e\frac{(ps\cos\delta_0 + p^2\sin\delta_0)}{p^2 + s^2}\mathbf{1d} + e\frac{(p^2\cos\delta_0 - ps\sin\delta_0)}{p^2 + s^2}\mathbf{1q} \tag{27}$$

Hence from equations 27, 57, and 58 the transient currents will be

$$i = \frac{e x_{af}(p) p^2}{(p^2 + s^2)D} [-Z_q(p)(s \cos \delta_o + p \sin \delta_o) + (1 - s)x_q(p) \times (p \cos \delta_o - s \sin \delta_o)] \mathbf{1} d_f + \frac{eZ_f(p)p}{(p^2 + s^2)D} [Z_q(p)(s \cos \delta_o + p \sin \delta_o) - (1 - s)x_q(p)(p \cos \delta_o - s \sin \delta_o)] \mathbf{1} d_a + \frac{ep}{(p^2 + s^2)D} [(1 - s) \times (Z_f(p)x_{do}(p) - x_{af}{}^2(p)p)(s \cos \delta_o + p \sin \delta_o) + (Z_f(p)Z_{do}(p) - x_{af}{}^2(p)p^2)(p \cos \delta_o - s \sin \delta_o)] \mathbf{1} q_a \tag{59}$$

By combining equations 54 and 55 with equation 59 the current may be written as follows:

$$i = \frac{-e x_{af}(p) p^2}{Z_f(p)(p^2+s^2)} \times \left[\frac{Z_q(p)(s \cos \delta_o + p \sin \delta_o) - (1 - s)x_q(p)(p \cos \delta_o - s \sin \delta_o)}{Z_d(p)Z_q(p) + x_d(p)x_q(p)(1 - s)^2}\right] \mathbf{1} d_f + \frac{ep}{p^2+s^2}\left[\frac{Z_q(p)(s\cos\delta_o + p\sin\delta_o) - (1-s)x_q(p)(p\cos\delta_o - s\sin\delta_o)}{Z_d(p)Z_q(p) + x_d(p)x_q(p)(1-s)^2}\right] \mathbf{1} d_a + \frac{ep}{p^2+s^2}\left[\frac{(1-s)x_d(p)(s\cos\delta_o + p\sin\delta_o) + Z_d(p)(p\cos\delta_o - s\sin\delta_o)}{Z_d(p)Z_q(p) + x_d(p)x_q(p)(1-s)^2}\right] \mathbf{1} q_a \tag{60}$$

The last 2 terms would have been obtained in exactly the same form had the current been calculated from the inverse of the matrix of equation 52 where the field axis was eliminated. Field current would not have appeared. However, from equation 60 it is apparent that the field current bears the following relationship to the direct axis armature current,

$$i_f d_f = -i_d \frac{x_{af}(p)p}{Z_f(p)} d_f \tag{61}$$

A similar relationship exists for any amortisseur circuit in either the direct or quadrature axis.

$$i_{kd} d_k = \pm i_d \frac{x_{akd}(p)p}{Z_{kd}(p)} d_k \tag{62}$$

$$i_{kq} q_k = \pm i_q \frac{x_{akq}(p)p}{Z_{kq}(p)} d_q \tag{63}$$

When the field winding is closed or there is a circuit q_o the sign is positive when k is odd and negative when k is even or zero. In case the field is open-circuited or a circuit q_o does not exist the sign is negative when k is odd and positive when it is even. The operators in equations 62 and 63 are determined in the same way as was done for the field circuit.

The actual solution of the operational expression for current may be accomplished by means of Heaviside's "expansion theorem." Before the theorem is applied, numerical values must be assigned to the circuit constants and the terms arranged in powers of p. The reduced impedance matrices therefore are of no particular value in calculating the transient performance, as their use does not reduce the number of terms in the final equation. They are of some

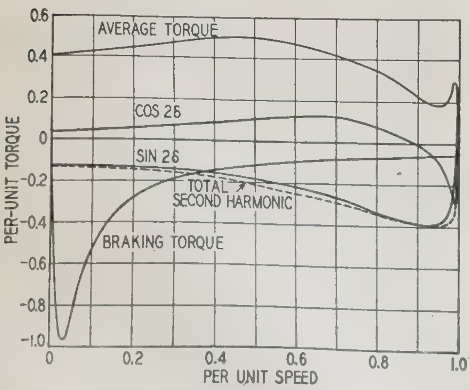


Fig. 9. Components of total instantaneous torque

Average torque and second harmonic components are produced by induction motor action

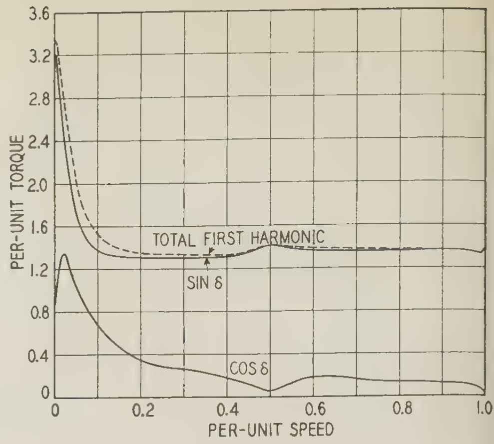


Fig. 10. First harmonic component of instantaneous torque

assistance in grouping terms when calculating the inverse of an impedance matrix.

The elimination of co-ordinate axes is decidedly worth-while in steady state calculations. The operators reduce from functions of p to single complex numbers. The performance may be expressed by relatively simple algebraic equations in terms of the impedance operators. The operators then can be evaluated in the most convenient manner, and the problem reduced to one of substitution.

The steady state currents may be obtained from the operational solution by replacing p by js where s is the per-unit frequency of the steady current. It is customary, however, to replace p directly in the impedance matrix. The voltage impressed along any axis is the sum of the impedance drops acting along that axis. If the impressed voltage is constant,

$$c = f(p,i)$$

differentiating both sides

$$0 = pf(p,i)$$
$$p = 0$$

When a sinusoidal voltage is impressed

$$e \sin (st + \delta_o) = f(p,i)$$
$$-s^2 e \sin (st + \delta_o) = p^2 f(p,i)$$
$$p = js$$

If no voltage is impressed

$$0 = f(p,i)$$
$$0 = pf(p,i)$$

and p may have any value.

Hence when a direct voltage is impressed on the field, or when voltage is impressed on the armature at synchronous speed, p is made zero throughout the matrix. When an alternating voltage is impressed below synchronism, p is replaced by js. Alternating and direct voltages must be impressed separately when the speed is not synchronous. The 2 resulting matrices are solved independently and the currents of slip and zero frequency are superposed to find the total current. Since under the assumptions the relation between current and voltage is linear for a given value of slip, the flux linkage and flux density vectors also are added in order to determine the resultant values.

All matrices in which p has been replaced by zero or js will be referred to as steady state matrices. The impedance operators are expressed as functions of (js) when p is replaced by js. The correct procedure will be illustrated by determining the steady state currents flowing in a machine that is operating at a constant slip with the field excited.

In order to have the final results appear in the most concise form, the component currents produced by the application of alternating voltages will be derived from matrix 52. When p is replaced by (js) and $(p\theta)$ is written as v or $(1 - s)$ the steady state impedance matrix becomes

$$Z_1 = \begin{array}{c|c|c} & d_a & q_a \\ \hline d_a & Z_d(js) & x_q(js)v \\ q_a & -x_d(js)v & Z_q(js) \end{array} \tag{64}$$

The torque tensor is from equation 52

$$G_1 = \begin{array}{c|cc} & d_a & q_a \\ \hline d_a & 0 & x_q(js) \\ q_a & -x_d(js) & 0 \end{array} \tag{65}$$

The steady state admittance matrix is the inverse of matrix 64:

$$Y_1 = \begin{array}{c|cc} & d_a & q_a \\ \hline d_a & \dfrac{Z_q(js)}{Z_d(js)Z_q(js)+x_d(js)x_q(js)v^2} & \dfrac{-x_q(js)v}{Z_d(js)Z_q(js)+x_d(js)x_q(js)v^2} \\ q_a & \dfrac{x_d(js)r}{Z_d(js)Z_q(js)+x_d(js)x_q(js)v^2} & \dfrac{Z_d(js)}{Z_d(js)Z_q(js)+x_d(js)x_q(js)v^2} \end{array} \tag{66}$$

The applied alternating voltage vector will be taken as

$$\boldsymbol{e}_1 = e \sin \delta \boldsymbol{d}_a + e \cos \delta \boldsymbol{q}_a \tag{21}$$

The currents produced by the alternating voltages are found from the product

$$\boldsymbol{i}_1 = \boldsymbol{Y}_1 \cdot \boldsymbol{e}_1 \tag{57}$$

to be

$$\boldsymbol{i}_1 = \frac{e[Z_q(js) \sin \delta - x_q(js)(1-s) \cos \delta]}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \boldsymbol{d}_a + \frac{e[Z_d(js) \cos \delta + x_d(js)(1-s) \sin \delta]}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \boldsymbol{q}_a \tag{67}$$

Making use of the relations of equations 31 from Appendix II and 61 from Appendix IV:

$$\boldsymbol{i}_1 = \frac{e[r - j(1-2s)x_q(js)] \sin \delta}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \boldsymbol{d}_a + \frac{e[r - j(1-2s)x_d(js)] \cos \delta}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \boldsymbol{q}_a - \frac{e[r - j(1-2s)x_q(js)] \sin \delta}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \cdot \frac{js\, x_{af}(js)}{Z_f(js)} \boldsymbol{d}_f \tag{68}$$

These currents will be designated as follows:

$$\boldsymbol{i}_1 = i_{d1}\boldsymbol{d}_a + i_{q1}\boldsymbol{q}_a + i_{f1}\boldsymbol{d}_f \tag{69}$$

The per-unit field current maintained by the excitation voltage usually is defined as the ratio of the actual field current to that required to overcome normal armature reaction. It is expressed most conveniently in terms of the "nominal" voltage in the direct axis, e_d

$$i_{fd}\boldsymbol{d}_f = -\frac{e_d}{x_{afd}} \boldsymbol{d}_f \tag{70}$$

The per-unit excitation voltage therefore is

$$\boldsymbol{e}_2 = E_{fd}\boldsymbol{d}_f = -\frac{e_d R_{fd}}{x_{afd}} \boldsymbol{d}_f \tag{71}$$

When the signs of $e \sin \delta \boldsymbol{d}_a + e \cos \delta \boldsymbol{q}_a$ are positive that of E_{fd} must be negative and *vice versa* in order for δ to be approximately zero when the motor operates in synchronism without load.

The steady components of current resulting from the application of field voltage may be determined from matrix 36. When p is replaced by zero and $p\theta$ by v or $(1 - s)$, the impedance matrix and torque tensor reduce to the following:

$$Z_2 = \begin{array}{c|ccc} & d_f & d_a & q_a \\ \hline d_f & R_{fd} & 0 & 0 \\ d_a & 0 & r & x_q v \\ q_a & -x_{afd}v & -x_d v & r \end{array} \tag{72}$$

$$G_2 = \begin{array}{c|ccc} & d_f & d_a & q_a \\ \hline d_a & 0 & 0 & x_q \\ q_a & -x_{afd} & -x_d & 0 \end{array} \tag{73}$$

The admittance matrix is the inverse of that of equation 72

$$Y_2 = \begin{array}{c|ccc} & d_f & d_a & q_a \\ \hline d_f & \dfrac{1}{R_{fd}} & 0 & 0 \\ d_a & \dfrac{-x_{afd}x_q(1-s)^2}{R_{fd}(r^2+x_dx_q(1-s)^2)} & \dfrac{r}{r^2+x_dx_q(1-s)^2} & \dfrac{-x_q(1-s)}{r^2+x_dx_q(1-s)^2} \\ q_a & \dfrac{rx_{afd}(1-s)}{R_{fd}(r^2+x_dx_q(1-s)^2)} & \dfrac{x_d(1-s)}{r^2+x_dx_q(1-s)^2} & \dfrac{r}{r^2+x_dx_q(1-s)^2} \end{array} \tag{74}$$

The currents resulting from the excitation voltage are found by taking the product of equations 71 and 74

$$\boldsymbol{i}_2 = \boldsymbol{Y}_2 \cdot \boldsymbol{e}_2 \tag{57}$$

$$\boldsymbol{i}_2 = \frac{e_d x_q(1-s)^2}{r^2 + x_d x_q(1-s)^2} \boldsymbol{d}_a - \frac{e_d\, r(1-s)}{r^2 + x_d x_q(1-s)^2} \boldsymbol{q}_a - \frac{e_d}{x_{afd}} \boldsymbol{d}_f \tag{75}$$

These currents also will be referred to as

$$\boldsymbol{i}_2 = i_{d2}\boldsymbol{d}_a + i_{q2}\boldsymbol{q}_a + i_{f2}\boldsymbol{d}_f \tag{76}$$

The total steady state current flowing in the armature and field circuits of an excited machine operating below synchronism is the vector sum of equations 69 and 76

$$\boldsymbol{i} = (i_{d1} + i_{d2})\boldsymbol{d}_a + (i_{q1} + i_{q2})\boldsymbol{q}_a + (i_{f1} + i_{f2})\boldsymbol{d}_f \tag{77}$$

The flux density vector resulting from currents $\boldsymbol{i}_1$ is found from equations 67 or 68 and 65 by the product

$$\boldsymbol{\psi}_1 = \boldsymbol{G}_1 \cdot \boldsymbol{i}_1$$

$$\boldsymbol{\psi}_1 = \frac{e[r - j(1-2s)x_d(js)]x_q(js) \cos \delta}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \boldsymbol{d}_a - \frac{e[r - j(1-2s)x_q(js)]x_d(js) \sin \delta}{Z_d(js)Z_q(js) + x_d(js)x_q(js)(1-s)^2} \boldsymbol{q}_a \tag{78}$$

which is equivalent to

$$\boldsymbol{\psi}_1 = i_{q1}x_q(js)\boldsymbol{d}_a - i_{d1}x_d(js)\boldsymbol{q}_a \tag{79}$$

Thus the component of the armature flux density vector acting along the direct axis is composed of the quadrature axis armature flux linkages, whereas the quadrature axis flux density vector is composed of the armature flux linkages in the direct axis. (When the voltages due to the rate of change of the armature phase linkages are resolved along the direct and quadrature axes, components of voltage appear along these axes that are proportional to the product of velocity and the quadrature and direct axis armature flux linkages, respectively.[2] Mathematically, these voltages may be regarded as generated voltages proportional to the product of velocity and flux density.[8] A physical analogy between flux density and flux linkages cannot be extended to include the flux in the leakage paths or that associated with inductance external to the machine.) $\boldsymbol{\psi}_1$ also will be designated as follows,

$$\boldsymbol{\psi}_1 = \psi_{d1}\boldsymbol{d}_a + \psi_{q1}\boldsymbol{q}_a \tag{80}$$

From equations 73 and 75 the flux density vector produced by currents $\boldsymbol{i}_2$ is

$$\boldsymbol{\psi}_2 = \frac{-e_d r x_q(1-s)}{r^2 + x_d x_q(1-s)^2} \boldsymbol{d}_a + \frac{e_d\, r^2}{r^2 + x_d x_q(1-s)^2} \boldsymbol{q}_a \tag{81}$$

The usual, but less accurate form is

$$\boldsymbol{\psi}_2 = i_{q2}x_q\boldsymbol{d}_a - (i_{f2}x_{afd} + i_{d2}x_d)\boldsymbol{q}_a \tag{82}$$

Referring to the direct axis component of $\boldsymbol{\psi}_2$ as ψ_{d2} and the quadrature component as ψ_{q2},

$$\boldsymbol{\psi}_2 = \psi_{d2}\boldsymbol{d}_a + \psi_{q2}\boldsymbol{q}_a \tag{83}$$

The total armature flux density vector produced by all currents flowing in the machine is the vector sum of equations 80 and 83.

$$\boldsymbol{\psi} = (\psi_{d1} + \psi_{d2})\boldsymbol{d}_a + (\psi_{q1} + \psi_{q2})\boldsymbol{q}_a \tag{84}$$

Appendix V—The Form of the Steady State Power and Torque Equations

The armature power of a machine operating continuously out of step with the field excited is obtained from the product

$$P = \mathbf{e} \cdot \mathbf{i} \tag{13}$$

where,

$$\mathbf{e} = e \sin \delta \mathbf{d}_a + e \cos \delta \mathbf{q}_a \tag{21}$$

$$\mathbf{i} = (i_{d1} + i_{d2})\mathbf{d}_a + (i_{q1} + i_{q2})\mathbf{q}_a + (i_{f1} + i_{f2})\mathbf{d}_f \tag{77}$$

Hence,

$$P = e\,(i_{d1} + i_{d2}) \sin \delta + e(i_{q1} + i_{q2}) \cos \delta \tag{85}$$

The components of current due to applied alternating voltages will be obtained from equation 68 in the following form:

$$i_{d1} = e(a - jb) \sin \delta \tag{86}$$

$$i_{q1} = e\,(a' - jb') \cos \delta \tag{87}$$

Substituting the relations of equation 31,

$$i_{d1} = e\,(a \sin \delta - b \cos \delta) \tag{88}$$

$$i_{q1} = e\,(a' \cos \delta + b' \sin \delta) \tag{89}$$

The total instantaneous per-unit armature power may be written in the following form by substituting equations 88 and 89 in equation 85

$$P = e^2 \frac{(a + a')}{2} - e^2 \frac{(b - b')}{2} \sin 2\delta - e^2 \frac{(a - a')}{2} \cos 2\delta + e i_{d2} \sin \delta + e i_{q2} \cos \delta \tag{90}$$

Instantaneous torque is obtained similarly from the product.

$$T = \mathbf{i} \cdot \boldsymbol{\psi} = \mathbf{i} \cdot \mathbf{G} \cdot \mathbf{i} \tag{91}$$

Where $\mathbf{i}$ is given by equation 77 and

$$\psi = (\psi_{d1} + \psi_{d2})\mathbf{d}_a + (\psi_{q1} + \psi_{q2})\mathbf{q}_a \tag{84}$$

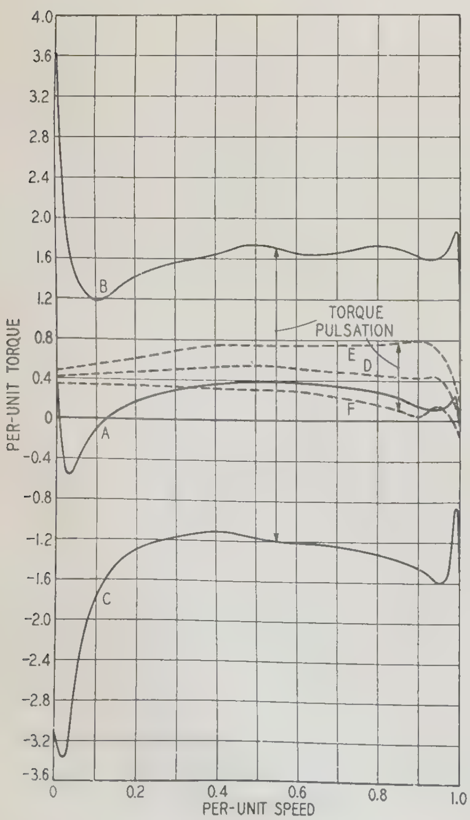


Fig. 11. Comparison of torques developed out of synchronism, with normal accelerating cycle

A—Net average torque out of synchronism

B—Maximum positive torque out of synchronism

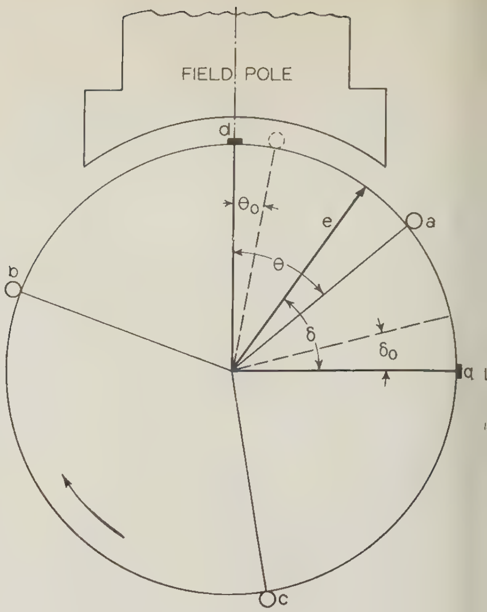
C—Maximum negative torque out of synchronism

D—Average torque during starting cycle

E—Maximum positive torque during starting cycle

F—Maximum negative torque during starting cycle

Fig. 12. Diagram of synchronous machine



Hence,

$$T = (i_{d1}\psi_{d1} + i_{q1}\psi_{q1}) + (i_{d2}\psi_{d2} + i_{q2}\psi_{q2}) + (i_{d1}\psi_{d2} + i_{d2}\psi_{d1}) + (i_{q1}\psi_{q2} + i_{q2}\psi_{q1}) \tag{92}$$

The armature flux linkages due to impressed alternating voltages obtained from equation 78 will have the form:

$$\psi_{d1} = e(c - jd) \cos \delta \tag{93}$$

$$\psi_{q1} = -e(c' - jd') \sin \delta \tag{94}$$

From equation 31

$$\psi_{d1} = e(c \cos \delta + d \sin \delta) \tag{95}$$

$$\psi_{q1} = e(d' \cos \delta - c' \sin \delta) \tag{96}$$

By substituting equations 88, 89, 95, and 96 in equation 92, the total instantaneous per-unit torque of a motor operating out of synchronism is obtained in the following form:

$$\begin{aligned} T = {} & \frac{e^2}{2}(ad + a'd' - bc - b'c') + \frac{e^2}{2}(ac + b'd' - bd - a'c') \sin 2\delta \\ & - \frac{e^2}{2}(ad + bc - b'c' - a'd') \cos 2\delta + (i_{d2}\psi_{d2} + i_{q2}\psi_{q2}) + \\ & e(a\,\psi_{d2} + d i_{d2} + b'\,\psi_{q2} - c' i_{q2}) \sin \delta - \\ & e(b\psi_{d2} - c i_{d2} - a'\psi_{q2} - d' i_{q2}) \cos \delta \end{aligned} \tag{97}$$

The component of torque $i_{d2}\psi_{d2} + i_{q2}\psi_{q2}$ will be referred to as braking torque. It may be written as follows by substituting values from equations 75 and 81

$$T_b = -\frac{e_d^2 r\,(1 - s)[r^2 + x_q^2(1 - s)^2]}{[r^2 + x_d x_q (1 - s)^2]^2} \tag{98}$$

Near synchronous speed, when r is small,

$$T_b \cong \left(\frac{e_d}{x_d}\right)^2 \frac{r}{(1 - s)} \tag{99}$$

The value of T_b is a maximum when

$$v = \pm \frac{r}{x_q}\left[\frac{(9x_d^2 - 14x_d x_q + 9x_q^2)^{1/2} - 3(x_d - x_q)}{2x_d}\right]^{1/2} \tag{100}$$

References

1. Starting Performance of Synchronous Motors, H. V. Putman. A.I.E.E. Trans., v. 46, 1927, p. 39–57.

2. Two-Reaction Theory of Synchronous Machines—Generalized

Method of Analysis—Part I, R. H. Park. A.I.E.E. Trans., v. 48, July 1929, p. 716–27.

3. Starting Performance of Salient-Pole Synchronous Motors, T. M. Linville. A.I.E.E. Trans., v. 49, April 1930, p. 531–45.

4. Some Considerations in the Design of Damper Windings for Synchronous Machines, C. C. Shutt. A.I.E.E. Trans., v. 51, June 1932, p. 424–30.

5. Pull-In Characteristics of Synchronous Motors, D. R. Shoults, S. B. Crary, A. H. Lauder. Elec. Engg., v. 54, Dec. 1935, p. 1385–95.

6. Definition of an Ideal Synchronous Machine and Formula for the Armature Flux Linkages, R. H. Park. *Gen. Elec. Rev.*, v. 31, June 1928, p. 332–4.

7. The Operational Impedance of a Synchronous Machine, M. L Waring and S. B. Crary. *Gen. Elec. Rev.*, v. 35, Nov. 1932, p. 578–82.

8. The Application of Tensors to the Analysis of Rotating Electrical Machinery, Gabriel Kron, *Gen. Elec. Rev.*, v. 38, Apr. 1935, p. 181–91, and v. 39, Feb. 1936, p. 108–16.

9. Heaviside's Operational Calculus (a book), E. J. Berg. McGraw-Hill Book Company, New York, N. Y., 1929, p. 29.

10. Synchronous Machines. I—An Extension of Blondel's Two-Reaction Theory, R. E. Doherty and C. A. Nickle. A.I.E.E. Trans., v. 45, 1926, p. 912–26.

11. Synchronous Machines, IV, R. E. Doherty and C. A. Nickle. A.I.E.E. Trans., v. 47, 1928, p. 457–92.

Power Transformers With Concentric Windings

The influence of basic engineering principles on the evolution of the concentric winding type of high voltage power transformer during the past 2 decades is discussed in this paper with particular reference to 55,000 kva water cooled transformers which will step up 16.3 kv to 287.5 kv at the Boulder Dam power plant.

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THE influence of basic engineering principles on the evolution of the high voltage power transformer structure may be illustrated by recently built single-phase 16.3 to 287.5 kv 55,000-kva transformers, described by figure 1 and table III, which are good examples of modern design. Except for their unusually high rated voltage, these transformers are typical of present power transformer practice. These transformers are provided with electrostatic shields to increase their transient voltage strength, concentric coil windings, electrically brazed conductor joints, adjustable clamps, double-weld bottle-tight tanks, and other essential features.

The transformers, which are now installed at the power plant end of the Boulder Dam-Los Angeles 287.5 kv transmission line, were shipped, with a special tracer, across the United States in specially built tank cars filled with nitrogen. A large part of the trip was made at 60 miles an hour. The first unit was installed and made ready for service in 30 hours after arrival of the car. (figures 12, 13, 14 and 15.)

The transformers installed at the other end of the line are described in a companion paper by W. G. James and F. J. Vogel (Elec. Engg., v. 55, May 1936, p. 438–44).

The modern power transformer is superior to its predecessor in so many respects that it is quite impossible to express the degree of over-all improvement by a single quantity or a statement. For this reason the progress is illustrated by comparison of a number of essential characteristics (table I). The comparison is made between single-phase 110-kv modern power transformers and transformers of the type shown in figure 2 having the same ratings but built some 20 years ago.

Improvement in transformers has been achieved through correlated progress in 5 basic fields of endeavor:

I. *Calculation.* Accurate analytical determination of all important electric, magnetic, thermal, and mechanical phenomena taking place in a transformer.

II. *Structural Harmony.* Choice of a type of transformer structure where requirements imposed on the configuration, position, and proportion of various structural elements by various forces and other operating factors harmonize with one another.

III. *Stress Distribution.* Devising a structure where mechanical, electrical, magnetic, and thermal stresses are distributed more uniformly and high local stresses are reduced to a minimum, with corresponding increase of safety factor.

IV. *Materials.* Improvements in magnetic steel and insulating materials and their testing.

V. *Manufacturing.* Improvement of methods and facilities.

Calculation

Accurate analytic determination of all the essential effects of physical phenomena taking place in a transformer is of prime practical importance, because factory tests prescribed by A.I.E.E. standards determine only over-all or average performance of the apparatus. The tests give no indication of the distribution of stresses throughout the structure or of their local effects, whereas damage and even complete failure of apparatus in service commonly start by local overstress developing into more general damage. Local stresses in the old type have often been many times greater than the average stress.

To determine and insure the uniformity of magnetic, dielectric, thermal, and mechanical stress dis-

A paper recommended for publication by the A.I.E.E. committee on electrical machinery, and scheduled for discussion at the A.I.E.E. summer convention; Pasadena, Calif., June 22–26, 1936. Manuscript submitted April 3, 1936. released for publication May 4, 1936.

tribution, an engineer designing a modern large high voltage transformer is obliged to perform up to 1,300 separate calculations. This is in spite of the fact that he is provided with full engineering data, accumulated through years of research and service experience, and covering in systematic order every detail of the structure.

Table I—Comparison of Modern Transformers With Transformers of 1912–15; Capacity Range 2,500 to 6,667 Kva

	Per Cent Reduction		Object of Improvement
Losses			
Load loss per kilovolt-ampere	22		Economy of operation
No load loss per kilovolt-ampere	44		
Total loss per kilovolt-ampere	32		
Load or copper loss per pound of copper at current densities actually employed	42		Increase in reliability
No load or iron loss per pound of iron at flux densities actually employed	25		Increase in reliability
Insulation			
Operating frequency voltage between adjacent coils of the same winding	90 or more		Reliability
Maximum impulse voltage between turns	97 or more		Reliability
Maximum impulse voltage between coils	99 or more		Reliability
	Modern	**Old**	
Deterioration of insulation, during tests due to corona under oil	Corona absent; more uniform and low gradient	Corona present	Elimination of deterioration of insulation (aging)
A.I.E.E. impulse test	Required	Not considered	Reliability
Impulse insulation strengths between turns, coils, and to ground are co-ordinated with bushing impulse strength	Yes	No	Reliability
Impulse voltage distribution	Practically uniform due to shielding	Extremely nonuniform	
Over-all impulse or transient voltage strength	From 2 to 10 times higher than old		Reliability
Resistance to unavoidable short-circuit force	Ample	Insufficient	Reliability
Resistance to unavoidable forces due to rush of magnetizing current	Ample	Insufficient	Reliability
Avoidable forces due to short-circuit or magnetizing current rush	Avoided	Present	Reliability
Distribution of magnetic forces	Practically uniform	Very nonuniform	Reliability
Coil temperature	Practically uniform	Essentially nonuniform	Reliability
Hot spot in winding	Within 10 deg C (or less) of average	Within 25 deg C of average	Reliability
Local high copper loss in windings	Absent	Present	Reliability
Current distribution between parallel conductors	Uniform due to thorough transpositions	Not uniform	Reliability
Core temperature	Uniform due to dividing the core into large number of small parallel sections	Less uniform due to smaller number of sections	Reliability
Tank	Bottle - tight double-welded	Not bottle tight	

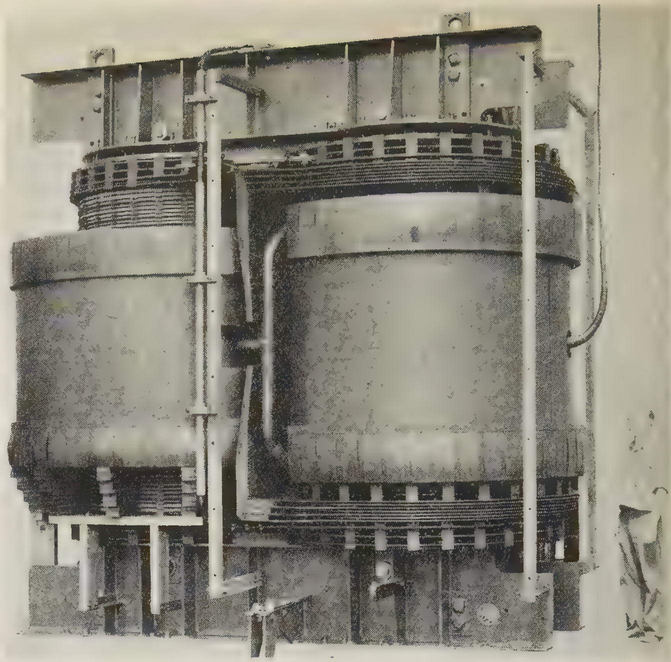


Fig. 1. Assembled core and coils of 16,320 to 287,500-volt grounded-wye 55,000-kva 60-cycle single-phase water-cooled transformer

The need for precision in these calculations cannot be reduced by mere addition of material. It can be easily shown, for example, that both the temperature and loss in windings can be increased by increasing the cross section of the winding conductors, if they are not properly proportioned. Also, in the application of a lightning impulse to a transformer, increasing the insulation between turns or between coils may increase the impulse voltage stress between them, nearly in proportion to the increase in the strength of the insulation. Furthermore, it has been demonstrated that the use of stronger dielectric materials can cause apparatus to fail at a lower 60 cycle voltage as the result of less uniform voltage stress distribution.

Methods have been developed for calculation, with the necessary precision, of local stresses produced in transformer structures by short-circuit forces, rush of magnetizing currents, transient voltages, and heating.

The reactance between a pair of windings, or, in case of multiwinding transformers, of any combination of windings, can be calculated with such accuracy that successful multiple operation with transformers of radically different types and proportions can be assured. For instance, a few years ago a 4-winding concentric-type transformer was built for multiple operation on all 4 windings with a transformer of the same voltage rating, but of interleaved design. Since several windings had taps, and since the reactances of the original transformer on different tap connections were quite different, it was necessary to match 114 reactances. This was accomplished successfully using available formulas.

The temperatures of different turns of the same

Fig. 2. Typical high voltage power transformer of 1910–14 period

winding may vary widely, depending on the amount of turn insulation, insulating tape, eddy currents, and dimensions of coils and oil ducts. To assure proper temperatures for all turns, the effect of all these factors must be determined for every part of every winding under all desired load conditions.

Surface as well as internal temperatures of every part of the core and of every turn of the windings may be computed accurately by methods now available.

Inasmuch as in multiwinding transformers the magnitude and the distribution of eddy loss in the windings depend not only on the magnitude of the loads but also on their power factors, as they affect the leakage flux distribution, eddy loss formulas must take these facts into account.

Structural Harmony

The basic problem in the design of any structure is finding a construction where requirements imposed on the shape, size, and disposition of its members by different forces and operating characteristics are in harmony with one another. This has been the principal factor in the evolution of the basic features of the present circular concentric coil transformer construction, which first came into important use about 20 years ago.

It has been found that with a concentric arrangement of windings an extraordinary harmony exists between the requirements imposed on the structure by physical laws of transient voltages, short-circuit forces, heating, losses, and reactances. The importance of this fact increases with increase in transformer size and rated voltage as may be seen from the following 7 examples.

Requirement of short-circuit forces versus cooling method. Magnetic forces resulting from short circuit can be divided into 2 classes, those which can be avoided by a suitable choice of construction and others which are present in all constructions. An unavoidable force is the force F_1 tending to separate primary and secondary. With rated voltage maintained at the terminals of a winding feeding the short circuit, the magnitude of F_1 depends only on the operating frequency f, transformer rated kilovolt-amperes, its reactance x, and the effective distance d_1 between windings. Approximately,

$$F_1 = 11.7 \frac{\text{kva}}{f} \frac{1}{x} \frac{1}{d_1} 10^3 \text{ pounds}$$

In large 60 cycle transformers, F_1 ranges between 400,000 and 1,500,000 pounds. Depending on the point of the current wave at which the short circuit takes place, this force hammers the transformer

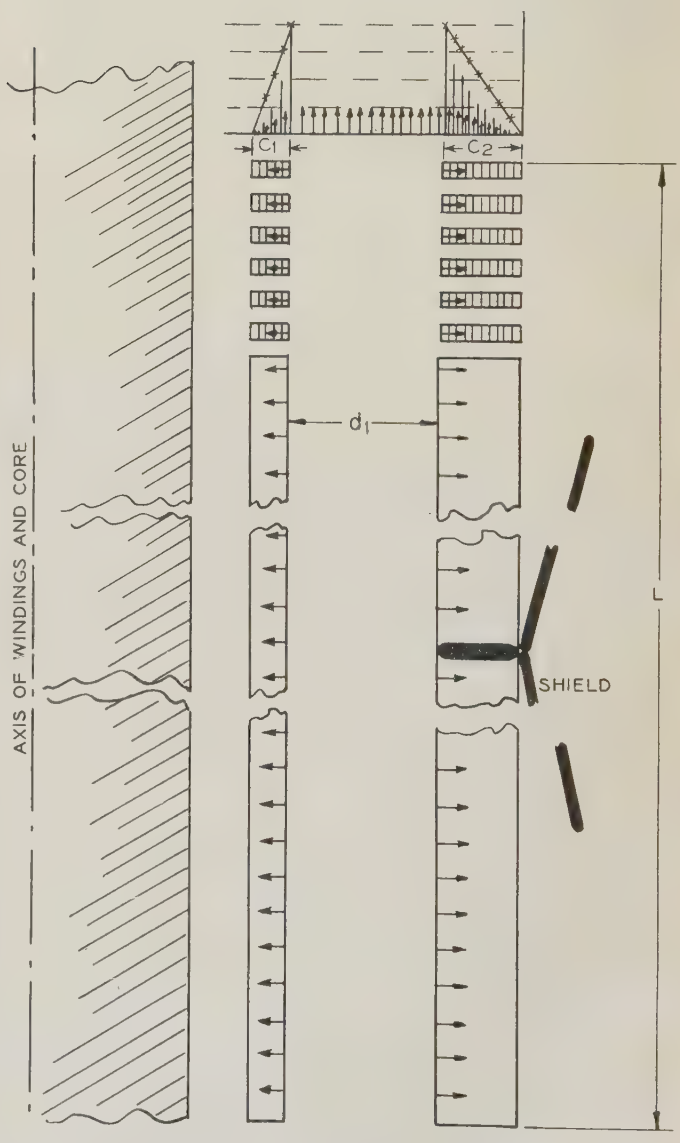
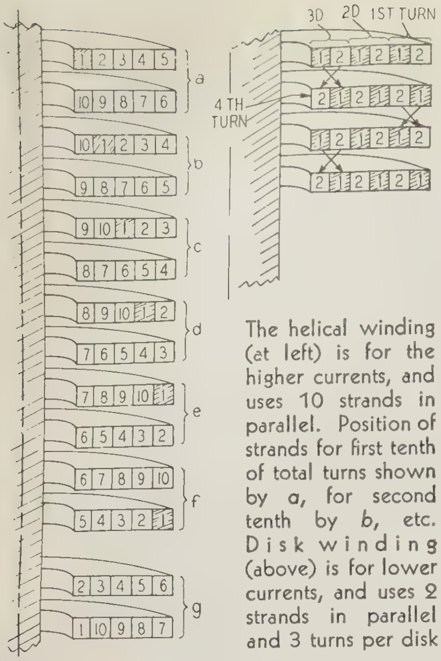


Fig. 3. Distribution of leakage flux density, eddy current loss, and short-circuit force between turns in concentric winding transformer

In the top diagram, arrows indicate the flux density, vertical lines the eddy current loss, and crosses the short-circuit forces developed in individual conductors



The helical winding (at left) is for the higher currents, and uses 10 strands in parallel. Position of strands for first tenth of total turns shown by *a*, for second tenth by *b*, etc. Disk winding (above) is for lower currents, and uses 2 strands in parallel and 3 turns per disk

Fig. 4. Perfect transpositions of conductors in concentric windings

windings either at operating frequency or at double the operating frequency.

In the case of concentric windings, F_1 tends to crush the inner winding against the core and to expand radially the outer winding. It exerts no pressure on the end clamps.

In all types of windings, short-circuit forces tend to bend conductors between spacers. The force is a maximum on the turns nearest to the opposite winding, and is a minimum on the turns farthest away. The ratio of the maximum to the minimum force is proportional to the number of conductors in a given winding in the direction of the force (figure 3).

In the concentric coil type, the conductors having the maximum force are reinforced by the other conductors in the same coil. As may be seen in figure 3, they all lie tightly together without any oil spaces between them, in the direction of the force which is in the plane of the coil. Cooling of each coil is accomplished by oil flow on all 4 sides of it, but not between its conductors. Furthermore, it is obvious that circular coils resist balanced radial forces much better than any other shape. There is no failure on record where either the inner or outer winding of a concentric type transformer was distorted by this force.

The outside winding of a concentric arrangement does not need any radial braces for resisting F_1 because the latter tends to hold it concentric with the inner winding, and the tensile strength of the conductors is ample to resist the radial expansion. The radial spacers are provided to preserve the desired minimum distance between windings and to keep the insulating cylinders in place.

To secure the desired strength for the inner coil in a concentric arrangement, the necessary number of braces, or axial spacers, is placed between the winding and the cylinder it is wound on, also between the cylinders and the core leg. Any number of such spacers can be used because their blanketing of the inner surface of the winding deprives it of only a small part of its radiating surface, which is provided principally by horizontal ducts.

Thus, requirements for short-circuit strength and the cooling methods employed are in harmony in the concentric design.

Major insulation versus avoidable short-circuit force. Generally speaking, in high voltage transformers, distances from the ends or edges of the high voltage winding to the core are much greater than they are in the low voltage winding on account of the difference in the insulation allowances.

If no other considerations were involved, dimension L of the low voltage winding, in the direction of the leakage flux, could be made correspondingly greater than that of the high voltage winding, thereby improving the space factor of the structure.

However, analysis of the distribution of short-circuit forces in transformers, in general, shows that wherever one winding extends beyond the other in the direction of the leakage flux, the force tends to tear away such an extension. On this account the dimension L of all windings should be kept the same (figure 3).

In high voltage concentric windings the above rule harmonizes with the insulation structure because for the sake of securing maximum dielectric strength the high voltage line terminal is brought to the middle of the stack with the winding progressing to the grounded neutral terminal in 2 parallel paths, one to the top and the other to the bottom end of the stack. The insulation allowances for these ends do not differ materially from those of the low voltage winding. Both windings, therefore, can be made of the same length, thereby not only preventing development of the above force but also permitting maximum use of the winding space.

Short-circuit forces versus eddy current loss. The eddy current loss in a conductor may become important not only from the standpoint of increase of total load loss, but also from the local heating that it may produce in turns nearest to the opposite winding. The eddy current loss in those turns per pound of copper is 3 times as high as the average per pound for the entire winding. Thus, if the average in a winding is 10 per cent of I^2R the eddy current loss in conductors nearest to the opposite winding is 30 per cent of their I^2R.

Eddy current loss is proportional to C^2 (C is the dimension of the conductor in the direction of short-circuit force F_1). Therefore, to reduce eddy loss, the conductor of a given cross section must have a small C. However, the beam strength of a conductor is also proportional to C^2 and therefore it reduces at the same rate as the eddy current-loss. In the concentric construction this is of no consequence as the tendency

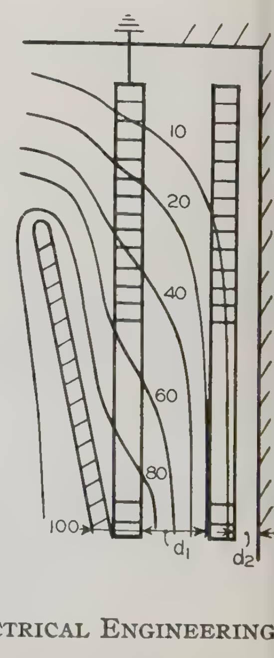


Fig. 5. Approximate distribution of electrostatic field produced by impulse or operating voltage in top half of concentric windings with shield

Bottom half is mirror image of top. Note that low voltage winding floats in the field of correspondingly low potential

to bend the first conductor is resisted by the combined strength of all conductors of a given coil, which is ample. Therefore, C can be made, even in large transformers, less than 0.1 inch, thereby giving eddy current losses of less than 3 per cent. Thus, requirements of eddy current loss and short-circuit strength do not conflict in windings arranged concentrically.

Transpositions versus short-circuit force. One of the common methods for reduction of eddy current loss consists of "splitting" the conductor in a direction perpendicular to C into n parallel strands insulated from each other and properly transposed among themselves. The eddy loss in the case of a perfect transposition is $1/n^2$ of that existing without transpositions.

In the concentric construction, because the strands or turns of each disk coil offer a combined resistance to short-circuit force, the conductor can be divided into practically any desired number of parallel strands. Therefore, the number of parallel strands is determined by the selection of their C dimension small enough to make eddy current loss as low as desired. Conductors carrying several hundreds or thousands of amperes are subdivided into 10, 20, or even more strands.

Some of the methods used for transpositions in low and high voltage windings are shown in figure 4. These methods insure uniform distribution of current among the strands.

In concentric windings, neither short-circuit forces nor any other factors conflict either with transpositions or with the choice of a sufficiently large number of strands.

Reactance versus dielectric strength. Requirements imposed on construction by these 2 principal factors of design harmonize in concentrically arranged windings.

Operating experience on power systems has established, for transformers of various ratings, certain preferred values for short-circuit reactance. The problem of a designer is to get the desired reactance with the minimum amount of material and with the minimum sacrifice of other operating characteristics.

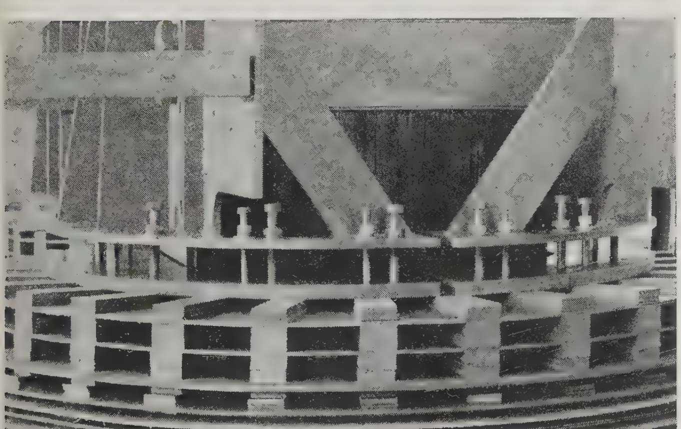


Fig. 6. Top end insulation and adjustable clamps in 55,000 kva transformer; pressure exerted by clamps on winding exceeds 100,000 pounds

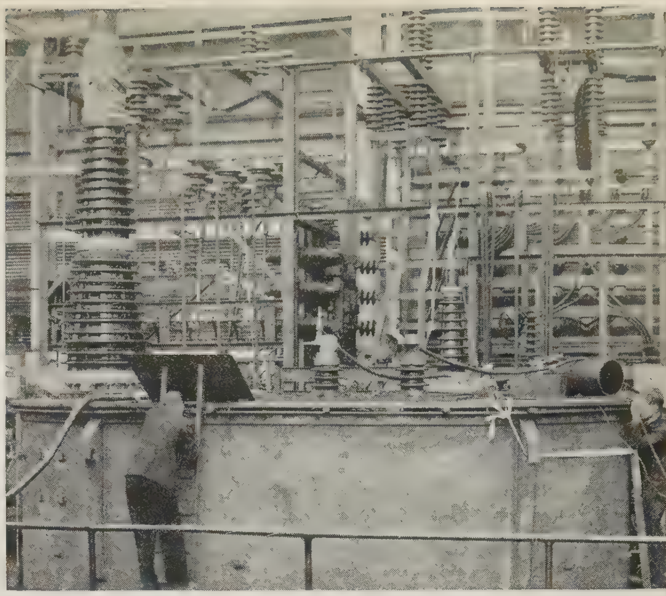


Fig. 7. Method of observation of oil surface for corona bubbles during induced voltage test of 575 kv from line terminal to ground on 55,000 kva transformers; cover is raised 1 inch

Transformer reactance, for given kilovolt-amperes and frequency, is directly proportional to

$$X = \frac{WT}{L} N_1^2$$

where WT is the effective cross section of the leakage flux path, W is its effective width, L is its effective length, and N is the number of turns in the winding considered.

The designer's problem consists of finding proportions and distribution of windings that will give minimum WT and maximum L. This permits N to be correspondingly large, which means that the core cross section can be made proportionately smaller, which in turn results in a lighter, more efficient transformer. In the concentric design, L is essentially equal to the height of the winding stack (figure 3).

The general expression for the effective width is

$$W = \left(d_1 + \frac{C_1 + C_2}{3}\right) + (K_1d_1 + K_2d_2)$$

where d_1 is the distance between windings. The quantity $C_1 + C_2$ is the total distance, in a direction perpendicular to the flux, occupied with conductors of both windings, and $(K_1d_1 + K_2d_2)$ is the total effective distance in a direction perpendicular to the leakage flux occupied with ducts between conductors. The factors K_1 and K_2 depend on the number of such conductors in each winding.

The value of W is smallest in concentric transformers because $(K_1d_1 + K_2d_2)$ is completely absent in this type.

As this factor increases with increase in transformer size and voltage, its complete absence in the concentric design is particularly important in large high voltage transformers. It is absent in the

concentric construction because in such windings the cooling and insulating ducts between coils are perpendicular to the direction of the flux and therefore do not add to the width W of the leakage path but add to its length L, thereby helping the designer to keep WT/L small.

In transformers of the concentric type with grounded neutral, L is independent of the rated voltage since the ends of high and low voltage windings can approach the top and bottom core yokes within 2 inches or so, irrespective of the value of the induced voltage and impulse tests of the high voltage winding. Therefore, full advantage can be taken of the height of the core window.

Transient voltage strength and reactance. Inductive windings develop high local voltage stresses when subject to transient voltages. The stresses produced by transient voltages are so severe that they dictate the choice of insulation allowances throughout a modern transformer structure. An effective means for radical reduction of this transient voltage concentration is the use of electrostatic shields consisting of conducting surfaces properly proportioned and insulated, placed near the winding and connected to its line terminal. To make shielding practical it is essential that the winding be substantially uniform and accessible to shielding along its length.

A helix is an ideal form of winding for such a purpose. It is uniform and its conductor has small radial dimensions. Transformer windings, when concentrically arranged, for all practical purposes possess the important characteristics of a helix; they are essentially uniform and have small radial build. Every part of a high voltage winding is accessible to shielding.

In this type of construction, shielding is in harmony with requirements of reactance, short circuit strength, eddy-current loss, and cooling, as they all are better satisfied in a tall uniform winding of small radial build, located concentrically.

Shielding of high voltage windings versus transient voltages in low voltage winding. It has been shown that the impact of an impulse on the terminals of a high voltage winding produces transient voltage stresses in the low voltage winding which can be resolved into 4 components:

1. Pure electromagnetic.
2. Pure electrostatic.
3. Essentially free oscillation of low voltage winding.
4. Essentially forced oscillation of low voltage winding produced by essentially free oscillation of the high voltage winding.

1. The electromagnetic component produces the same voltage at the low voltage terminals of all transformers having a given short-circuit inductance and turn ratio, irrespective of their construction. This voltage distributes uniformly along the low voltage winding and produces potential differences between turns and between coils, inversely proportional to their number.

2. The voltage produced by the electrostatic component at a given point of the low voltage winding is proportional to the ratio of electrostatic capacitances of that point to the high voltage winding and to the core (figure 5).

If C_2 is the capacitance of a given point of the low voltage winding to the core, C_1 its capacitance to the nearest high voltage coil, and E_{HV} the voltage of the high voltage coil at the moment of impulse impact, then the voltage of the low voltage coil

$$E_{LV} = E_{HV}\frac{C_1}{C_1 + C_2} \text{ (approximately).}$$

In the concentric construction, C_2 and C_1 can be replaced with reciprocals of d_2 and d_1, the spacings

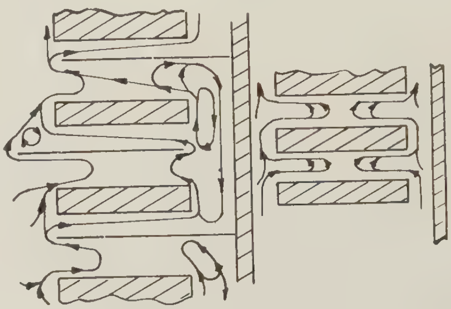


Fig. 8. Circulation of oil in windings of concentric construction as found by test

At points where oil eddies it is completely renewed in 2 or 3 minutes. Inner winding indicated by *a*, outer by *b*

between the low voltage windings and the core, and between the low voltage winding and the high voltage winding respectively, so that $E_{LV} = \dfrac{d_2}{d_1 + d_2} E_{HV}$. As d_2 is proportional to the insulation strength between the low voltage winding and the core and d_1 to that between the high voltage and low voltage windings, it follows that the transient voltage, induced electrostatically in the low voltage winding, is practically proportional to its insulation strength, and remains essentially the same no matter how high the rated voltage of the high voltage winding may be. This advantage is unique with concentrically arranged windings.

3. Generally speaking, voltages produced by the electrostatic components are likely to be the highest of all 4 components. In power transformers they last for from 1 to 3 microseconds or so. Then they begin to collapse, thereby producing free oscillation of the windings. On account of the relatively close electrostatic and electromagnetic coupling between windings, neither winding can have truly "free" oscillation, unaffected by the presence of other windings. However, in the entire phenomena of voltage transients of each winding, there is one component which is controlled principally by the constants of that winding, and, on this account it is thought of as an essentially free oscillation of the winding.

If the transient voltage applied to the high voltage winding happens to be oscillatory and of a frequency within 40 per cent of the natural frequency of the low voltage winding, extremely high stresses may be produced there. In the case of an impulse wave like the $1^1/_2$ x 40 wave applied to a high voltage winding, the stresses in the low voltage winding, due to its free oscillation, seldom exceed those electrostatically induced in the coils nearest to the high voltage line coil. Shielding of the high voltage winding reduces

the voltage gradient along the low voltage winding produced by the electrostatic component. For this reason, it also reduces the amplitude of free oscillations of the latter.

4. If the high voltage winding is not shielded, the impulse or other types of transient voltages applied to it produce in it violent oscillations which naturally force other windings to oscillate.

Conversely, if the high voltage winding, in concentric construction, is shielded, then its free oscillation is practically eliminated. In such a case the fourth component is eliminated from the low voltage winding transient. Thus internal voltages produced in the low voltage windings by the last 3 components are materially lower in shielded concentric windings than in other types.

III. Uniformity of Stress Distribution

Whenever a force applied to a structure cannot be reduced to a negligible value, such as the short-circuit force or a lightning impulse, for example, effort must be made to distribute the stresses as evenly as practicable thoughout the mass of material.

This has been accomplished in the concentric transformers described here. Stresses produced by short-circuit current, magnetizing current, transient and operating frequency voltages, and load and no load losses are distributed essentially uniformly throughout the respective elements of the structure. Two goals are attained thereby: first, the materials are stressed to lower values, resulting in proportional increases of the respective safety factors, and second, by bringing the maximum and average stresses nearer together, the significance of the A.I.E.E. tests measuring average or over-all performance is brought nearer to that intended.

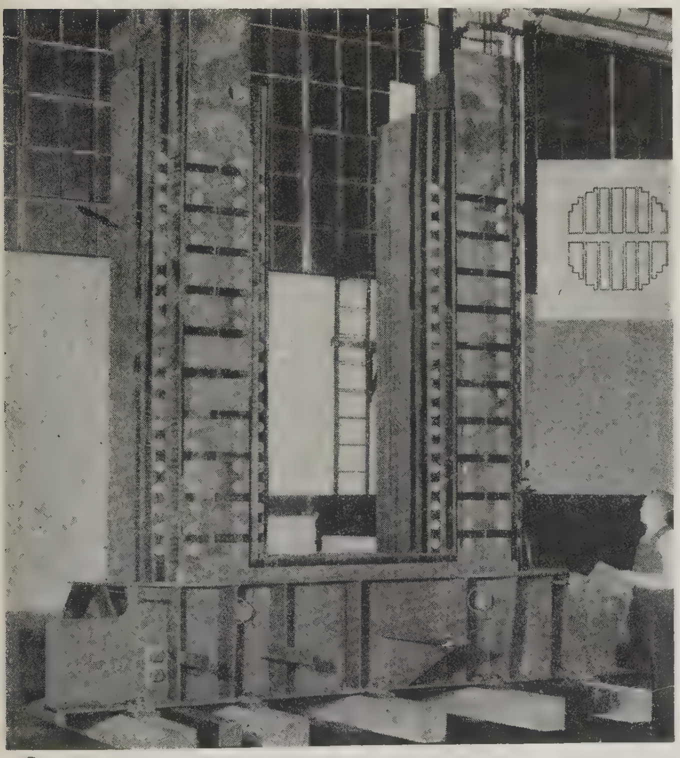
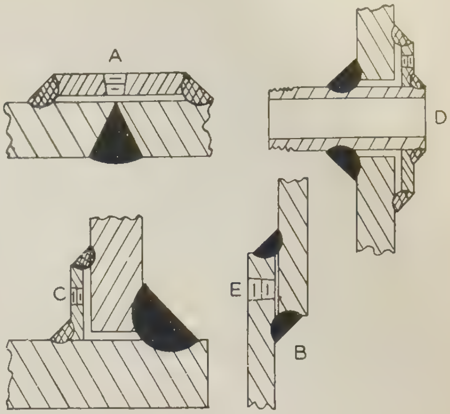


Fig. 9. Core of 55,000 kva transformer

Core legs and yokes are subdivided by oil ducts into 14 parallel cores to facilitate cooling and to reduce losses. Insert shows section of core leg

Fig. 10. Standard tank welds

Principal welds shown in black; welds and plates used for test with gas pressure of 300 to 500 pounds per square inch shown crosshatched. Welds are placed far apart to prevent accidental closing of gas passage. Gas passages are exaggerated



A—Vertical seam; B—Middle horizontal seam; C—Bottom; D—Drain pipe; E—Hole for test plug

Uniformity of short-circuit stresses. By virtue of the circular shape of coils in concentric construction, the radial stress is uniformly distributed around the circumferences of both windings. The inside winding is braced against the core leg all around, but circumferential uniformity of the force reduces pressure on these supports to a minimum. Outside windings under the stress tend to expand radially, but uniformity of the stress distribution prevents any asymmetrical deformation or displacement, while the tensile strength of the copper prevents expansion.

Local high stresses produced by axial forces in concentric designs are created (*a*) by difference in the distribution of ampere-turns along the 2 windings, as may be produced by taps, (*b*) by asymmetrical distribution of turns with respect to the middle horizontal plane, and (*c*) by a difference in lengths of the windings.

Stress concentration resulting from the first cause is reduced to negligible values by tapping out turns at 2 points in the winding, generally at $^1/_4$ and $^3/_4$ of the stack height and by proper balance of ampere-turns of the other winding at the same levels. Stress concentration due to asymmetry is eliminated completely by distributing the turns symmetrically. To avoid concentration of stress due to the third cause, windings are made of the same length.

With the present day precision in manufacturing, the heights of 2 windings and their symmetry are produced within a small fraction of an inch. However, to secure ample safety factors, clamps at the ends of the windings are designed on the assumption of the presence of force that could be created only by the dissymmetry of from 1 to 2 inches, depending, on the transformer's size. For example, due to such an assumption, the windings and clamps in the Boulder Dam transformers are designed to resist forces in excess of 100,000 pounds. Furthermore, to prevent vibration of the windings under short circuit stresses, clamps are pressed by adjusting

studs, against the windings with forces considerably in excess of the assumed short circuit forces (figure 6).

Uniformity of transient voltage stresses. Since 1929 so much has been written about stresses produced in transformers by transient voltages that it is sufficient to mention here only the basic facts. Transient voltages may cause failures of transformers not so much because their amplitudes at the transformer terminals greatly exceed the normal operating voltage, but because of their faculty of concentration in a small portion of the winding. For example, the 60 cycle rated voltages of a transformer may produce 400 volts between 2 points in the winding, while the application of an impulse wave of the same amplitude may create 20,000 volts between the same 2 points. As the amplitude of a lightning wave may be 10 or even 15 times higher than the rated 60 cycle voltage, the lightning voltage between the 2 points may be 200,000 to 300,000 volts, or 500 to 750 times normal.

The cause for this uneven voltage stress is the unfavorable distribution of electrostatic capacitance of the winding, which is common to all transformer windings. This distribution of the capacitance can be corrected, in concentrically arranged windings, by proper electrostatic shielding, eliminating serious voltage stress concentration.

The practical importance of electrostatic shielding of the windings may be measured by the fact that since the introduction of shielding in practice in 1927, over 4,000,000 kva of large power transformers have been built with shielded windings. None of these windings has failed. Shielded windings are now used in power transformers of 44 kv and higher for wye and delta operation.

Uniformity of operating frequency voltage stresses. Unless the dielectric field created by normal frequency voltage is sufficiently uniform, local breakdown of oil and subsequent damage to solid insulation may occur during A.I.E.E. insulation tests. Before such a breakdown occurs the incipient failure is evidenced by corona formation. The presence of corona during insulation tests is significant for 2 reasons: first, the insulation may be permanently impaired (see table II), and second, it demonstrates the presence of abnormally high local stresses.

The table indicates that a fresh sample will stand for one minute without failure about 95 kv. Thus, if corona is disregarded, such insulation might be used, for an operating voltage of between 95/2 = 42.5 kv and 95/3.46 = 27.4 kv without full appreciation of the potential danger.

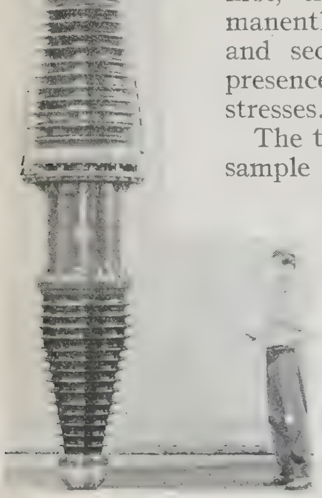


Fig. 11. A 287.5-kv 400-ampere oil-filled bushing

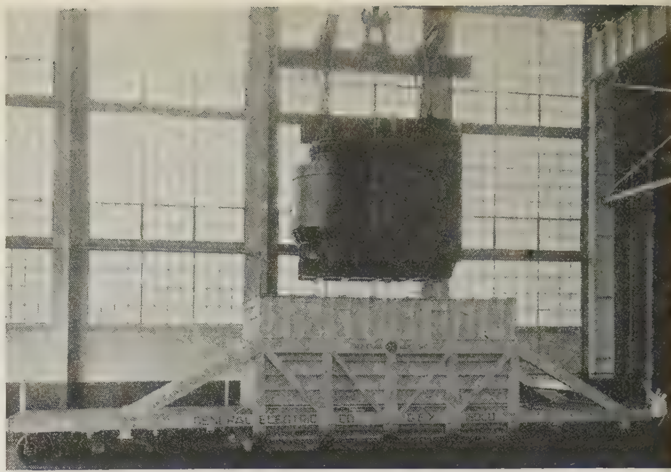


Fig. 12. Assembled core and coils of 55,000 kva transformer being loaded into special tank car, which will be filled with nitrogen gas for shipment

In the case of an operating voltage of 42.5 kv, the insulation would have been exposed during the induced voltage test to 7,200 cycles at 95 kv (2 minutes at 60 cycles) and would be subject to failure under operating stress in less than 47 days. In the case of an operating voltage of 27.4 kv and test of 95 kv, it is estimated from the table that failure would probably occur in from 3 to 5 years.

It appears that corona may stress insulation, so to say, above its "elastic limit." The damage done then is permanent and failure of such insulation is only a matter of time. As it was once aptly put, "corona is a cancer of insulation." As long as the corona point is not reached at the test voltage, the "elastic limit" of the insulation is not exceeded, and there is no residual effect from the insulation test and the life of the insulation is indefinitely long.

Effect of corona on turn insulation. Wherever corona appears next to the winding, it attacks not

Table II—Effect of 60 Cycle Corona on Puncture Strength of Pressboard Under Oil

Voltage Held, Kv	Times to Failure: Fresh Samples; Visible Corona Starts at 62.5 Kv	Samples Previously Exposed to 80 Kv for 2 Minutes; Visible Corona Starts at 62.5 Kv	Samples Previously Exposed to 80 Kv for 2 Minutes Without Corona; Visible Corona Starts Above 80 Kv
100	1 minute	Instantaneous	More than 3.5 minutes
90	2.3 minutes	0.2 minute	More than 5 minutes
80	3.5 minutes	1.4 minutes	105 minutes
70	5.0 minutes	2.2 minutes	No failure after 360 minutes
65	9.0 minutes	2.5 minutes	No test
60	130.0 minutes	12.0 minutes	Indefinitely long
55	8.14 days	1.6 days	Indefinitely long
50	64 days	14 days	Indefinitely long
45	Indefinitely long	47 days	Indefinitely long

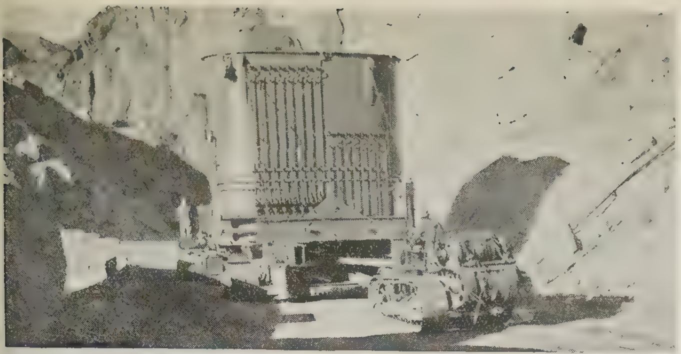


Fig. 13. Hauling 55,000 kva transformer to the edge of Boulder Canyon

only the pressboard in its neighborhood, but also the turn insulation of the conductors producing the corona. Tests show that damage done by corona often is invisible to a naked eye, and that the long-time dielectric strength of turn insulation is reduced to a small fraction of the original by corona of 1 or 2 minutes duration.

In some special cases, it is possible to produce corona in oil, for a few minutes, without doing any serious damage to the solid insulation, as for example, on the surface of heavily insulated cable in free oil. It is even possible by special means to increase the apparent strength of insulation by exposing it to corona for a short period of time.

Because in transformers generally it is impossible to determine the location and the degree of damage done by corona without disassembly of the core and coils, it is desirable not to allow formation of the corona. This is accomplished by giving the windings and other conductors the proper shapes, by proportioning the oil and solid insulation in accordance with their dielectric constants, by using a large number of narrow coils, and by placing the point of highest potential in the middle of the stack. All of these means serve to produce lower and more uniform voltage gradients throughout the structure (figure 5).

Corona may be produced by overstressing insulation between (1) high voltage winding and low voltage winding, (2) high voltage winding and core, (3) adjacent coils of the high voltage winding, and (4) high voltage leads and ground.

In concentric windings, the formation of corona in the first 2 insulation regions (major insulation) is prevented by rounding and heavily insulating the edges or ends of the stack where otherwise the voltage gradient would be the highest. In the remainder of the major insulation the gradient is low because there are no corners or edges present. The equipotential surfaces are smooth, and concentric with the windings and the core.

In the case of a transformer designed for grounded neutral (directly or through a reactor) still more uniform dielectric field is established by designing the high voltage winding with 2 multiple circuits, placing the line terminal in the middle of the stack and the neutral at the ends. This arrangement not only places the part of the winding of the highest potential at the point farthest away from the ground, but also eliminates the concentration of dielectric field at the ends of the stack since these points are essentially or actually at the same potential as the core (figure 5).

The possibility of corona from electrical stresses between coils is eliminated with the high voltage winding divided into a large number of coils (100 or so) as is inherent in the concentric type of design. Since the induced voltage between coils is inversely proportional to their number, and in the case of shielded transformers this is also true for the impulse voltage, a much higher safety factor can be secured by increasing the number of coils and proportionately reducing the space between them. This follows from the fact that the strength of coil insulation increases approximately only as the square root of the distance between the coils.

Corona on leads is prevented by making their diameter adequately large and using a liberal amount of solid insulation as can be seen in figure 1, where the lead connecting the windings on 2 legs is shown. The smaller diameter of the line cable, shown on the right side of the picture is permitted because the high voltage bushing terminates with a large heavily insulated and properly shaped metal tube through which the cable is drawn. The bottom end of the tube coincides with the entrance of the cable into the winding. Because the high voltage leads are on the outside of the winding, ample space is available and therefore their positions and diameters can always be made adequate.

To determine whether or not corona is present during insulation test, the transformer oil surface is observed during the tests, as corona tends to produce gas bubbles (figure 7). Also, electrical detectors are used. Their functioning is based on the fact that corona produces high frequency oscillation.

Uniformity of temperature. In all oil immersed transformers there is unavoidable difference in temperature between the top and bottom parts of the core and of the windings, because cooling is accomplished through vertical thermal circulation of oil.

Fig. 14. Lowering 55,000 kva transformer completely filled with oil to the power plant in Boulder Canyon

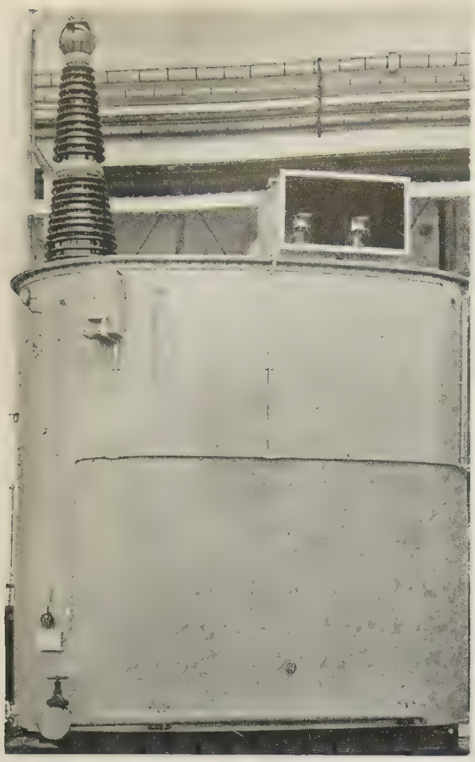


Fig. 15. Completely assembled 55,000 kva water cooled transformer

In addition to this, the excess of local temperature over the average may be caused by (*a*) heavy turn insulation, (*b*) taping of coils with insulated tape, (*c*) blanketing part of a coil with insulating channels and spacers, (*d*) high local eddy current loss, and (*e*) restriction of oil circulation in the ducts between coils by spacers or collars.

In concentric windings, individual disk coils are narrow and relatively thick; therefore, all 4 surfaces are effective in cooling. Radial spacers do not impede oil circulation. The high voltage winding, being on the outside, has ample free oil on the outside, while the low voltage winding is provided with large, straight vertical ducts. Numerous horizontal ducts, in both windings, serve to maintain even temperature (within 2 or 3 degrees centigrade) in each narrow disk coil (figure 8). Coils with heavy turn insulation or tape have proportionately larger conductor cross section. Eddy current losses are made very small by proper transposition and proportioning of conductors. No channels or any other parts are allowed to blanket coil surfaces. The result is that the "hot spot" is kept well within the allowable A.I.E.E. limit of 10 degrees centigrade and commonly is within 6 degrees centigrade of the average temperature determined by A.I.E.E. test.

It is very important to keep the internal temperature of the core within the limits safe for core sheet insulation and the surface temperature within limits safe for the oil. Internal and surface temperatures of every core can be predetermined by calculation based on available data. It is kept substantially below safe limits and is essentially uniform because the loss per unit volume is small and the core is subdivided into a number of parallel cores of small sections, each cooled by free oil circulation on all 4 sides. In the Boulder Dam transformers the cores are subdivided into 14 sections (figure 9).

Improvements in Materials

As table I indicates, core steel has been improved in the past 20 years to a remarkable degree. By establishing rigid laboratory supervision at all stages of steel manufacture, its characteristics can be controlled within narrow limits, insuring uniformity of the product. For example, as was demonstrated by the test of the Boulder Dam transformers, the maximum deviation of core loss from the average of the 7 units was found to be only 1.3 per cent, and even the deviation of exciting currents was within 5.2 per cent (table III).

Installation of means for removal of burrs from the conductors eliminates the possibility of turn insulation being mechanically punctured.

Through perfection of manufacturing processes and laboratory supervision and inspection, much greater uniformity in insulating material is secured than was possible a few years ago. Arc-resisting molded insulation, high grade pressboard, and special papers are now used in place of general purpose insulating material.

A special mica-filled insulation has been developed for core lamination insulation, which retains its dielectric strength under the high pressures existing in firmly assembled cores.

Manufacturing Processes

Although practically every old process of manufacture has been improved or superseded, mention of only a few of the most outstanding changes will be made here. They are:

(1). Complete elimination of rivets by the use of welding.

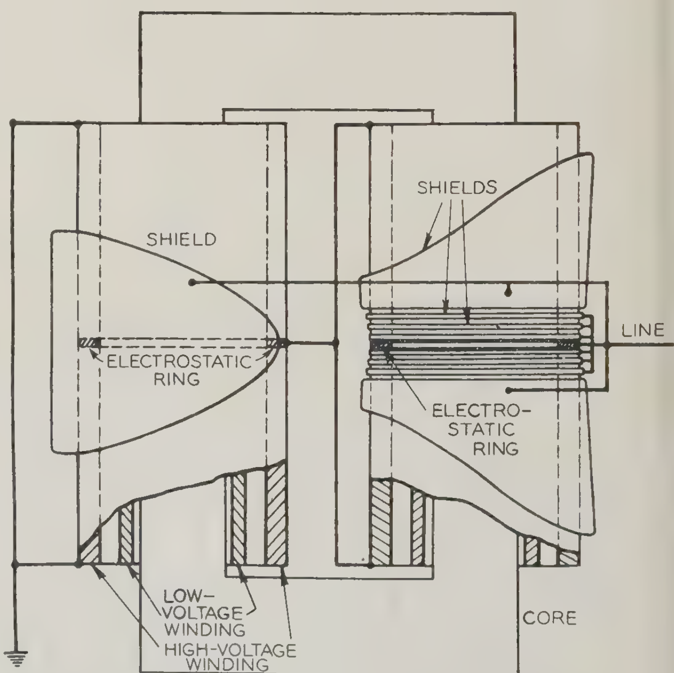


Fig. 16. Arrangement of windings and shields in 55,000 kva transformer

(2). Use of heavily coated welding electrodes. This gives stronger, denser, more ductile, homogenous, and corrosion resisting welds than those obtained with bare electrodes generally used.

(3). Standardizing on bottle tight tanks has become possible by perfection in welding and testing technique. During the last 5 years nitrogen gas under pressure of from 300 to 500 pounds per square inch has been used for the testing of welds. The gas pressure used depends on the weld (figure 10).

(4). Elimination of soldered conductor joints by the use of electric brazing and swaging.

(5). All-welded radiators have been developed with easily accessible welds of minimum length for a given radiating surface.

(6). Jointless or continuous disk coil windings for both high and low voltages.

(7). Perfect transpositions of multistrand conductors within windings (figure 4).

(8). Manufacturing of continuous molded insulation cylinders and continuous molded pressboard flanged collars for transformers of all sizes.

(9). To prevent loosening of windings in service from the gradual shrinkage of insulation, all insulation is vacuum dried and cured under high pressure, exceeding that found in service.

(10). Shot blasting of complete tanks for obtaining clean surface for painting.

(11). Developments of new tank paint and means for baking complete tanks of all sizes.

(12). Development of liquid filled bushings for the voltage range down to 8,700 volts (figure 11).

Table III—Data on 55,000 Kva 16.3 to 287.5 Kv Single-Phase Transformers

	Maximum	Average for 7 Units	Ratio, Maximum to Average
Core loss at 100 per cent excitation, watts	134,500	132,743	1.013
Exciting current in per cent of rated load current at 100 per cent excitation	3.08	2.93	1.052
Copper loss at rated load current, watts	186,900	184,430	1.013
Impedance volts in per cent at rated load current	10.15	10.06	1.01
Temperature rise above ingoing water at 25 degrees centigrade (only one unit tested)			
High voltage winding	50.6		
Low voltage winding	49.1		

Test data (All guarantees were met)

Impulse test (in accordance with A.I.E.E. recommendations): 5 impulses of $1^{1}/_{2}$ x 40 microsecond wave of 1,500 kv crest and higher

A.I.E.E. induced voltage test at low frequency with neutral terminal grounded: 7,200 continuously applied cycles of 577 kv (effective)

A.I.E.E. high potential test at low frequency on high voltage winding: 3,600 continuously applied cycles of 187 kv (effective)

A.I.E.E. high potential test at low frequency on low voltage winding: 3,600 continuously applied cycles of 51 kv (effective)

Dimensions and weights

Weight of core and coils	160,000 pounds
Total weight with oil	356,000 pounds
Height from rail to top of tank	19 feet 6 inches
Height from rail to top of bushing	31 feet 8 inches

Floor space. 11 feet 2 inches by 19 feet 10 inches

Shipping method. Tanks were shipped in halves. Cores, completely assembled with coils, were shipped in nitrogen in specially designed tank cars (figure 12). Transformers reached their destination in 14 days, a large part of the journey being made at 60 miles an hour. Shock recording instruments installed in the cars indicated several impacts approaching collision severity, but no damage of any sort was done.

Installation. Transformers were made ready for service 30 hours after arrival of core and coils. The installation operation included tanking of core and coils, bolting cover in place, filling with oil, hauling transformer $^{3}/_{4}$ mile to the edge of the canyon and then lowering it approximately 600 feet to the power plant (figures 13 and 14).

Summary

In the various ways enumerated in table I, the reliability of modern power transformers has been greatly improved over that available a few years ago. At the same time the economy of operation has been improved by marked reduction of loss, weight, and size per kilovolt-ampere transformed.

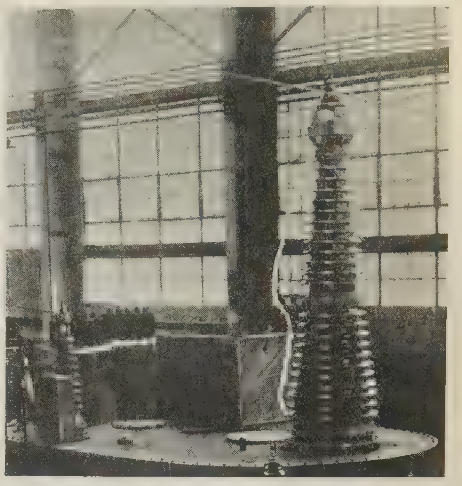


Fig. 17. Arc-over of bushing with $1^{1}/_{2}$x40 microsecond wave during tests on 55,000 kva transformer; impulse voltage approximately 1,500 kv

The principal factors responsible for these improvements are:

(*a*). Improvements in mathematical, experimental, and measuring methods and facilities permitted more accurate analysis and calculations of local stresses and of conditions throughout the structure.

(*b*). These studies showed that in circular coil concentric winding transformers the greatest possibility exists of obtaining a high degree of structural harmony, because the same proportions, dimensions, and distribution of structural elements serve to the best advantage the requirements of short-circuit forces, transient voltages, eddy current losses, reactance, and cooling.

(*c*). These studies also indicated that, unless special means are taken, high local stresses of various kinds may occur in a transformer, thereby materially lowering its reliability under service conditions.

(*d*). Improvements in insulating, magnetic, and structural materials.

These findings resulted in development of special design features for reduction of stresses of various natures and for their more uniform distribution. For example, the electrostatic shielding was developed to reduce transient voltage stresses for transformers of 44 kv and above. The effectiveness of shielding can be judged by the fact that, as far as it is known, not a single shielded winding has failed although more than 4,000,000 kva of transformers have been provided with such windings since their introduction to practice in 1927.

Also means have been found to reduce local stresses produced by low frequency voltages and thereby prevent the appearance of corona in oil even when transformer is under the A.I.E.E. standard insulation test. This, combined with co-ordination of transformer insulation with impulse arc-over strength of its bushing, eliminated the principal causes of aging of transformer insulation.

Performance of Distance Relays

The effect of grounded neutral Δ/Y or Y/Δ power transformers upon the performance of certain impedance and reactance relays is set forth in this paper, and formulas are developed for use in predetermining the performance of these relays under operating conditions.

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IN THE ordinary application of distance relays to the protection of transmission lines, the usual statement is that with faults involving more than one phase, irrespective of the type of fault, the relays will protect the same distance when the potential and current transformers supplying the relays are connected in delta. Although this is always the case when the range of the relays does not include power transformers, it is not always true when transformers are interposed between the relays and the fault.

Purpose and Outline of the Paper

For a clear comprehension of the problem by all, it may be well to start by recalling some of the main features of distance relays. Distance relays have a distance or ohm element and, in general, also a directional element. Both elements are energized by voltage and current. The indication of the ohm element depends on the vector ratio of the voltage and current acting upon it, being its magnitude in the impedance type and its reactive component in the reactance type relays. In the protection of 3 phase lines, 3 relays are used, energized from potential and current transformers connected according to either of 2 main types of connections or their equivalents, namely:

1. The potential and the current transformers are connected in delta (Δ).
2. The potential transformers are connected in Δ and the current transformers in Y.

For the case of 3 phase lines, the primary voltages and currents acting on the 2 elements of the 2 most commonly used distance relays, the "HZ" and the "GAX," are tabulated in table I and the connections of their ohm elements are shown schematically in figures 1 and 2. Where the relays are of the impedance type ("HZ"), their primary ohm indications are equal to, respectively, Z_{ab}, Z_{bc}, Z_{ca}. Where, however, they are of the reactance type ("GAX"), their primary ohm indications, provided the starting element has operated, are equal to the reactance components of $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$.

To obtain the maximum benefit from the use of distance relays, the distance protected should be the same with 3 phase, line-to-line, and line-to-line-to-ground faults, and should not be affected by operating conditions.

It is the purpose of this paper:

1. To show that the interposition of a grounded neutral Δ/Y or Y/Δ power transformer bank between the relays and the fault causes the distance protected to vary with the type of fault and, with the exception of 3 phase faults, to be affected by operating conditions.
2. To provide simple formulas for calculating the primary ohms indicated by the relays with 3 phase, line-to-line, line-to-line-to-ground, and line-to-ground faults located between the relays and the grounded neutral Δ/Y or Y/Δ bank, or past the bank.

General formulas are developed first (appendix I), to be used in making practical calculations in any case irrespective of the complexity of the network. To make their use simple, the general formulas are put in such a form that the grounded neutral Δ/Y or Y/Δ connection of the power bank may be disregarded altogether in using them.

The general formulas are next applied to the case of a stub-end line (appendix II, figures 4, 5) and the primary ohm indications expressed in terms of the impedances to positive, negative, and zero sequence currents of the circuits involved. The formulas obtained for faults between the relays and the bank correspond to the cases ordinarily considered by manufacturers in stating the distance protected by their relays. By comparing them with the formulas obtained with faults past the bank, a more definite idea than otherwise obtained from the general formulas may be formed of the effect of the interposition

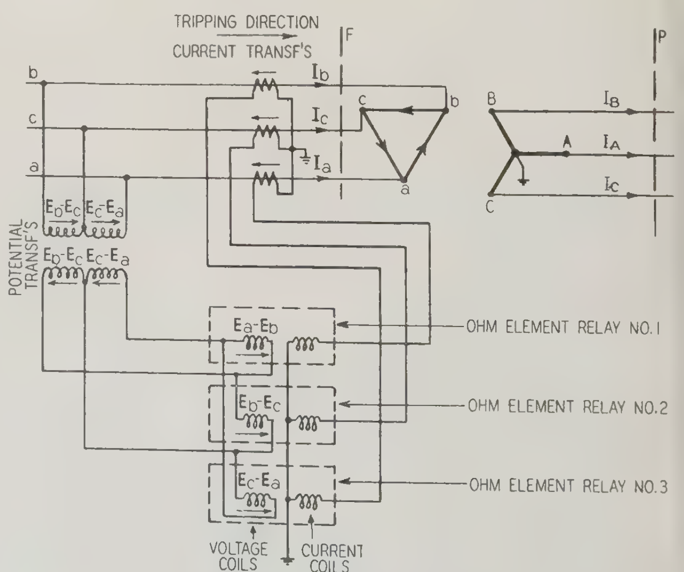

Fig. 1. A typical distance relay connection

A paper recommended for publication by the A.I.E.E. committee on protective devices, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Nov. 23, 1935; released for publication May 4, 1936.

of the latter between the relays and the fault.

A numerical example is given last (appendix IV, figure 8) further illustrating the use of the formulas and showing that, under favorable conditions, the protecting relays may reach further with line-to-line and line-to-ground faults than with 3 phase faults, but not so far with line-to-line-to-ground faults.

Though the main objective of the paper is to show how the ohm indications of the relays are affected by the interposition of a grounded neutral Δ/Y or Y/Δ power bank, no analysis would be complete that did not consider the directional elements of the relays. It was, therefore, thought proper also to outline a method for analyzing the action of the directional elements (appendix III).

As the analysis made is a straightforward application of the theory of sequence components, phases have been lettered according to the method in use among some writers on that subject. The method is different from that recommended by the A.I.E.E. and is used throughout the paper in the belief that it results in simpler formulas.

Application of Formulas

The effect of the interposition of the grounded neutral Δ/Y or Y/Δ transformer bank upon the primary ohms indicated by the relays is seen better by expressing the currents and voltages of table I in terms of sequence components. The ensuing formulas are given in table II, and their derivation is given in appendix I.

The formulas of table II are general, and apply in any case irrespective of the complexity of the network and of the type of fault. To apply them it is necessary only to compute the sequence components of the primary line-to-neutral voltages ($\boldsymbol{E}_{a1}$, $\boldsymbol{E}_{a2}$) and line currents ($\boldsymbol{I}_{a0}$, $\boldsymbol{I}_{a1}$, $\boldsymbol{I}_{a2}$) at the relay location, disregarding the effect of the connection of the power bank. The procedure to be followed in applying them, therefore, is as follows:

Connect the positive, negative, and zero sequence networks according to the type of fault, disregarding the connection of the power bank. Calculate the sequence components of the relay primary line-to-neutral voltages and line currents. Substitute in the formulas of table II, and calculate the impedances $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$. The line currents must be considered positive if flowing in the tripping direction of the relay, and negative if flowing in the opposite direction.

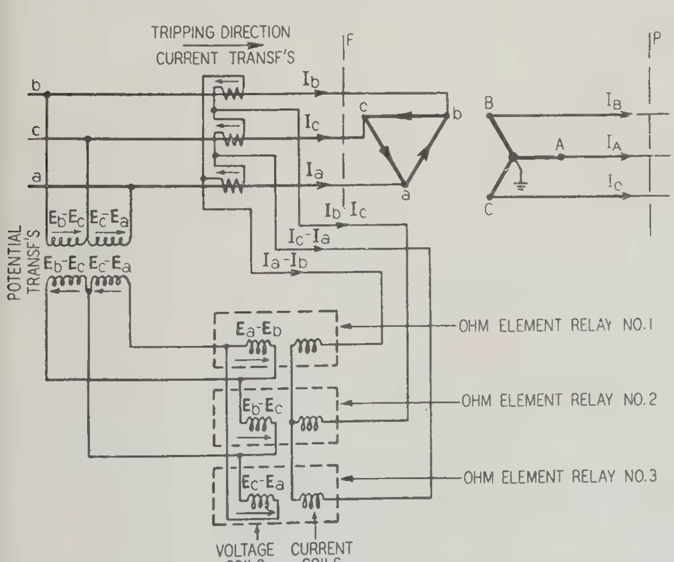

Fig. 2. A typical distance relay connection

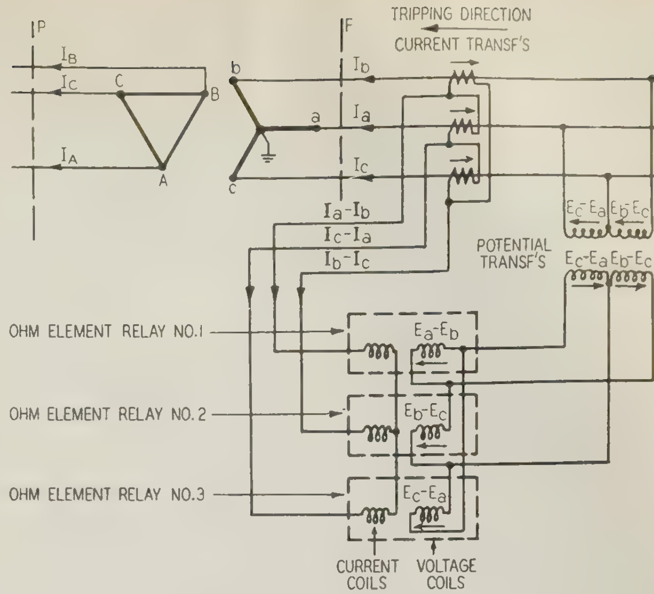

Fig. 3. A typical distance relay connection

As already noted, where the relays are of the impedance type, their primary ohm indications will be equal to Z_{ab}, Z_{bc}, Z_{ca}. However, where they are of the reactance type, the primary ohm indications, provided that the directional element has operated, will be equal to the reactance components of respectively $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$. It may be noted that this procedure offers no more difficulty than ordinarily encountered in making short circuit calculations either in long hand or on the calculating board.

In table II, 2 sets of formulas are given in the third, fourth, and sixth columns. In the first set, the actual primary voltages and currents are given, acting on the ohm elements; in the second set, the factors common to the numerator and the denominator have been eliminated. This is done for the convenience of the reader so that, if needed, the primary current or the primary voltage may be calculated directly from table II without having to recur to appendix I. The voltages and currents used in the fifth column are actual primary voltages and currents.

That the interposition of a grounded neutral Δ/Y or Y/Δ power bank between the relays and the fault affects the distance protected for faults other than 3 phase may be surmised, in a general way, from table II. From this table it may be observed that while $\boldsymbol{E}_{a1}$ and $\boldsymbol{I}_{a1}$ enter the various formulas for the 2 fault locations with the same sign, $\boldsymbol{E}_{a2}$ and $\boldsymbol{I}_{a2}$ enter them with opposite sign. During 3 phase faults $\boldsymbol{E}_{a2} = \boldsymbol{I}_{a2} = \boldsymbol{I}_{a0} = 0$, and therefore the primary impedances indicated by the relays will be equal to the ratio of the positive sequence components of the relay primary voltages and currents irrespective of whether or not a grounded neutral Δ/Y or Y/Δ power bank is interposed between the

Table I—Primary Voltages and Currents "GAX" and "HZ" Distance Relays

Relay Number (Figures 1, 2, 3)	Potential and Current Transformers in Δ: Ohm Element*	Directional Element "GAX" Relay: Voltage	Current	"HZ" Relay: Voltage	Current	Potential Transformers in Δ; Current Transformers in Y: Ohm Element*	Directional Element "GAX" Relay: Voltage	Current	"HZ" Relay: Voltage	Current
1	$\frac{E_a - E_b}{I_a - I_b} = Z_{ab}$	$E_a - E_b$	$I_a - I_b$	$E_a - E_c$	$I_a - I_b$	$\frac{E_a - E_b}{I_a} = Z_{ab}$	$E_a - E_b$	I_a	$E_a - E_c$	I_a
2	$\frac{E_b - E_c}{I_b - I_c} = Z_{bc}$	$E_b - E_c$	$I_b - I_c$	$E_b - E_a$	$I_b - I_c$	$\frac{E_b - E_c}{I_b} = Z_{bc}$	$E_b - E_c$	I_b	$E_b - E_a$	I_b
3	$\frac{E_c - E_a}{I_c - I_a} = Z_{ca}$	$E_c - E_a$	$I_c - I_a$	$E_c - E_b$	$I_c - I_a$	$\frac{E_c - E_a}{I_c} = Z_{ca}$	$E_c - E_a$	I_c	$E_c - E_b$	I_c

* "HZ" relays indicate, respectively, the magnitudes of Z_{ab}, Z_{bc}, Z_{ca}.
"GAX" relays indicate, respectively, the reactance components of Z_{ab}, Z_{bc}, Z_{ca}, provided that the corresponding directional element has operated.

relays and the fault. With faults other than 3 phase, however, E_{a2} and I_{a2} are different from zero, affecting the relay primary impedances in a manner that depends upon whether or not a grounded neutral Δ/Y or Y/Δ power bank is interposed between the relays and the fault.

With the potential transformers connected in Δ and the current transformers connected in Y, the formulas for the 2 fault locations differ not only in the sign of E_{a2} and I_{a2}, but, in addition, by the zero sequence component I_{a0} that appears in the formulas for faults between the relay and the bank, but does not appear in the formulas for faults past the bank.

While the formulas of table II are exact and, as already noted, may be used in any case irrespective of the complexity of the network, they can give only a general idea—a qualitative idea, so to say—of the effect of the interposition of a grounded neutral Δ/Y or Y/Δ power bank.

A more definite measure of the extent of this effect may be obtained by applying the formulas of table II to the case of a stub-end line as shown in figures 4 and 5. This has been done in appendix II, and the resulting formulas, expressed in terms of the impedances to currents of positive, negative, and zero sequence of the circuits involved have been tabulated in table III.

From table III, the effect of the interposition of a grounded neutral Δ/Y or Y/Δ power bank is more apparent than it is from table II.

With the formulas of table III the effect of arc resistance may easily be taken into consideration by:

1. Adding the arc resistance to Z_f for 3 phase faults and line-to-ground-line-to-ground faults.
2. Adding $1/2$ the arc resistance to Z_f for line-to-line faults.
3. Adding the arc resistance to Z_f and Z_{f0} for line-to-ground faults.

Consider first the case when the potential and current transformers are connected in Δ. A glance at the formulas in the fourth column shows that with 3 phase, line-to-line and line-to-line-to-ground faults between the relays and the power bank, there is at least one relay the primary ohm indication of which is equal* to Z_f (the impedance to positive sequence current of the circuit interposed between the relays and the faults). This corresponds to the case usually referred to when it is stated that with the potential and current transformers connected in Δ, the relays protect the same distance with 3 phase, line-to-line, and line-to-line-to-ground faults.

Consider now the formulas in the fifth column, relative to faults past the transformer bank. With 3 phase faults, the primary ohm indication of the 3 relays still is equal to the impedance Z_f. However, with line-to-line faults, none of the 3 relays will indicate a primary impedance equal to the impedance to positive sequence current Z_f of the circuit between the relays and the fault. Relay 2 will not operate at all and the indications of relays 1 and 3 will be affected by the impedance to negative sequence current Z_2 of the circuit between the source and the relays; that is, by operating conditions. Even if Z_2 should be negligible (this is not always the case) the primary ohm indications of relays 1 and 3 would be equal, respectively, to $1.155Z_f e^{+j30}$ and $1.155Z_f e^{-j30}$, and thus the relays, where of the impedance type, would indicate a primary impedance equal to $1.155Z_f$; that is, the distance protected would be less with line-to-line than with 3 phase faults.

Where the relays are of the reactance type, the primary indications of relays 1 and 3, inclusive of the effect of Z_2, assuming that the directional elements have operated, would be:

$$\text{Relay 1} = X_f + 0.577R_f + \frac{R_2}{\sqrt{3}}$$

$$\text{Relay 3} = X_f - 0.577R_f - \frac{R_2}{\sqrt{3}}$$

That is, both indications would be affected by the resistances R_f and R_2. Relay 1 would protect less with line-to-line than with 3 phase faults. Conversely, relay 3 would protect more with line-to-line than with 3 phase faults.

The effect of the arc resistance, if any, would be accounted for by adding $1/2$ its value to R_f. It would tend to make relay 3 protect still further with line-to-line than with 3 phase faults, provided, however, that it does not cause the angle of Z_{ca} to become too small or negative, thus preventing the directional element from operating. If this should happen, relay 1 would operate, if the relay current

* The expression "equal to Z_f" must be interpreted as meaning that the primary ohm indication of the relay is equal to the magnitude of Z_f where the relay is of the impedance type, and to the reactance component of Z_f where the relay is of the reactance type, as indicated in the footnote of table III.

is sufficiently large, because the voltage across its directional element, under ordinary conditions, could never lead the relay current by less than 30 degrees. In such a case, the distance protected would be less with line-to-line than with 3 phase faults. This condition is more apt to occur when X_f is small, so that the arc resistance is relatively large.

With the potential and current transformers connected in Δ and a line-to-line-to-ground fault past the transformer bank, none of the 3 relays will indicate the primary impedance $\boldsymbol{Z}_f$. The indications of all 3 relays are affected by $\boldsymbol{Z}_{f0}$, and those of 2 of them are affected also by $\boldsymbol{Z}_2$; that is, by operating conditions. They would be affected also by arc resistance. As seen from the fifth column of table III, no attempt has been made to express in terms of resistances and reactances the primary impedances obtained with this type of fault, because the resulting formulas would be too complicated.

Still, with the potential and current transformers connected in Δ, and with a line-to-ground fault past the transformer bank, the relays may afford some protection because so far as they are affected this fault is a line-to-line fault. The primary impedances indicated by the 3 relays are affected by $\boldsymbol{Z}_{f0}$ and those of 2 of them also by $\boldsymbol{Z}_2$, that is, by operating conditions. They would be affected also by arc resistance, as this should be added to both R_f and R_{f0}. The reactance term of relay 1 ($\boldsymbol{Z}_{ab}$) is:

$$X_f - 1.73(R_f + R_2) + 0.5X_{f0} - 0.866R_{f0}$$

The X_f, R_f, X_{f0}, R_{f0} appearing in this expression include, respectively, the resistances and reactances to currents of positive and zero sequence of the line between the power bank and the fault. The relative magnitude of the latter may be such as to cause 1.73 R_f + 0.866 R_{f0} to increase faster than $X_f + 0.5X_{f0}$ as the fault moves away from the bank, thus causing the reactance term of $\boldsymbol{Z}_{ca}$ to decrease. This condition takes place in the particular instance considered in appendix IV (line-to-ground faults on the cable of figure 8).

Consider now the formulas listed in the last 2 columns of table III, for the case when the potential transformers are connected in Δ and the current transformers in Y.

With 3 phase faults, all 3 relays indicate a primary impedance $\sqrt{3}e^{j30}\boldsymbol{Z}_f$ irrespective of whether or not a grounded neutral Δ/Y or Y/Δ power bank is interposed between the relays and the fault. The primary indication thus is $\sqrt{3}Z_f$ where the relays are of the impedance type, and $(1.5X_f + 0.866R_f)$ where they are of the reactance type. Both types, therefore, are affected by line resistance and by arc resistance.

With a line-to-line fault between the relays and the power bank, relay 1 does not operate. Relay 2 ($\boldsymbol{Z}_{bc}$) indicates a primary impedance $2Z_f$ where the relays are of the impedance type, and $2X_f$ where of the reactance type, provided that the starting element has operated. The primary impedance indicated by relay 3 is affected by operating conditions as reflected in the value of $\boldsymbol{Z}_2$, and by line and arc resistance. Where the relays are of the reactance type, relay 3 indicates $\{X_f + 1.73\,(R_f + R_2)\}$, provided that the starting unit has operated, thus protecting more with line-to-line than with 3 phase faults if R_f and R_2 be relatively negligible.

Where the relays are of the impedance type, relay 3 indicates a primary impedance equal to $2Z_f$, if $\boldsymbol{Z}_2$ be negligible, and

$$\sqrt{\{R_f - 1.73\,(X_f + X_2)\}^2 + \{X_f + 1.73\,(R_f + R_2)\}^2},$$

if $\boldsymbol{Z}_2$ be not negligible.

With a line-to-line-to-ground fault between the relays and the bank, relay 1 does not operate. The primary indications of relays 2 and 3 are affected by operating conditions as reflected in the values of $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_2$, by the impedance $\boldsymbol{Z}_{f0}$ to zero sequence current of the circuit between the relays and the fault, and by arc resistance. However, the effect of these various factors is too complicated to be analyzed in a general manner. The same may be said for line-to-line-to-ground faults past the transformer bank. In the case of a line-to-line fault past the transformer bank, the primary indications of all 3 relays are affected by operating conditions in arc resistance in a manner too apparent from the formulas of table III to need any further comment.

Table II—Primary Ohm Indications of Distance Relays in Terms of Sequence Components—General Case*

Relay		Potential and Current Transformers Connected in Δ		Potential Transformers Connected in Δ; Current Transformers Connected in Y	
Number (Figures 1, 2, 3)	Primary Impedance	No Grounded Neutral Δ/Y or Y/Δ Power Transf. Bank, Interposed Between the Relays and the Fault	Grounded Neutral Δ/Y or Y/Δ Power Transf. Bank Interposed Between the Relays and the Fault	No Grounded Neutral Δ/Y or Y/Δ Power Transf. Bank, Interposed Between the Relays and the Fault	Ground Neutral Δ/Y or Y/Δ Power Transf. Bank Interposed Between the Relays and the Fault
1	$\boldsymbol{Z}_{ab} =$	$\frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1}-\alpha\boldsymbol{E}_{a2})}{\sqrt{3}e^{j30}(\boldsymbol{I}_{a1}-\alpha\boldsymbol{I}_{a2})}=\frac{\boldsymbol{E}_{a1}-\alpha\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1}-\alpha\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}\alpha(\boldsymbol{E}_{a1}+\alpha\boldsymbol{E}_{a2})}{\sqrt{3}\alpha(\boldsymbol{I}_{a1}+\alpha\boldsymbol{I}_{a2})}=\frac{\boldsymbol{E}_{a1}+\alpha\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1}+\alpha\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1}-\alpha\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0}+\boldsymbol{I}_{a1}+\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}e^{j120}(\boldsymbol{E}_{a1}+\alpha\boldsymbol{E}_{a2})}{j(\boldsymbol{I}_{a1}-\boldsymbol{I}_{a2})}=\frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1}+\alpha\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a1}-\boldsymbol{I}_{a2}}$
2	$\boldsymbol{Z}_{bc} =$	$\frac{-\sqrt{3}j(\boldsymbol{E}_{a1}-\boldsymbol{E}_{a2})}{-\sqrt{3}j(\boldsymbol{I}_{a1}-\boldsymbol{I}_{a2})}=\frac{\boldsymbol{E}_{a1}-\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1}-\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}(\boldsymbol{E}_{a1}+\boldsymbol{E}_{a2})}{\sqrt{3}(\boldsymbol{I}_{a1}+\boldsymbol{I}_{a2})}=\frac{\boldsymbol{E}_{a1}+\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1}+\boldsymbol{I}_{a2}}$	$\frac{-\sqrt{3}j(\boldsymbol{E}_{a1}-\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0}+\alpha^2\boldsymbol{I}_{a1}+\alpha\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}(\boldsymbol{E}_{a1}+\boldsymbol{E}_{a2})}{j(\alpha^2\boldsymbol{I}_{a1}-\alpha\boldsymbol{I}_{a2})}=\frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1}+\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a1}-\alpha^2\boldsymbol{I}_{a2}}$
3	$\boldsymbol{Z}_{ca} =$	$\frac{-\sqrt{3}e^{-j30}(\boldsymbol{E}_{a1}-\alpha^2\boldsymbol{E}_{a2})}{-\sqrt{3}e^{-j30}(\boldsymbol{I}_{a1}-\alpha^2\boldsymbol{I}_{a2})}=\frac{\boldsymbol{E}_{a1}-\alpha^2\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1}-\alpha^2\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}\alpha^2(\boldsymbol{E}_{a1}+\alpha^2\boldsymbol{E}_{a2})}{\sqrt{3}\alpha^2(\boldsymbol{I}_{a1}+\alpha^2\boldsymbol{I}_{a2})}=\frac{\boldsymbol{E}_{a1}+\alpha^2\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1}+\alpha^2\boldsymbol{I}_{a2}}$	$\frac{-\sqrt{3}e^{-j30}(\boldsymbol{E}_{a1}-\alpha^2\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0}+\alpha\boldsymbol{I}_{a1}+\alpha^2\boldsymbol{I}_{a2}}$	$\frac{\sqrt{3}e^{-j120}(\boldsymbol{E}_{a1}+\alpha^2\boldsymbol{E}_{a2})}{j(\alpha\boldsymbol{I}_{a1}-\alpha^2\boldsymbol{I}_{a2})}=\frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1}+\alpha^2\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a1}-\alpha\boldsymbol{I}_{a2}}$

* The sequence components appearing in this table must be calculated disregarding the effect of the power bank connections; that is, they are the sequence components as ordinarily obtained from the calculating board set-up.

Summary

In concluding, the most important findings of the paper may be summarized as follows:

1. Even if the potential and current transformers are connected in delta, the interposition of a grounded neutral Δ/Y or Y/Δ power bank between the relay location and the fault causes the distance protected by impedance and reactance relays to vary (a) with the type of fault, and (b) in the case of unbalanced faults, with operating conditions.

2. In the case of a line-to-line-to-ground fault, the interposition of a grounded neutral Δ/Y or Y/Δ bank causes the distance protected to be affected by the impedance to zero sequence current of the circuits between the fault and the source or sources of zero sequence currents.

3. Relays of the impedance type are affected by arc resistance irrespective of the fault type and location. With faults located between the relay location and the power bank, relays of the reactance type are not affected by arc resistance where the potential and current transformers are connected in delta and all generator voltages are equal in phase and magnitude. However, with line-to-line and line-to-line-to-ground faults past the power bank, reactance relays are also affected by arc resistance.

4. Because of the interposition of a grounded neutral Δ/Y bank between the relay location and the fault, distance relays may afford some protection with line-to-ground faults past the bank. The distance protected is affected, however, by operating conditions and by the impedance to zero sequence currents of the circuits between the fault and the source or sources or zero sequence currents.

5. Formulas are given in tables II and III for calculating the primary ohms indicated by the relays under various fault conditions and location, with the potential transformers in Δ and the current transformers in Δ or in Y, with or without arc resistance.

Appendix I—Derivation of the Formulas of Table II From Those of Table I

The formulas of table II may be derived from those of table I in the following manner.

Referring to figures 1 and 2, let $\boldsymbol{E}_a$, $\boldsymbol{E}_b$, $\boldsymbol{E}_c$ be the relay line-to-neutral primary voltages and $\boldsymbol{E}_{a0}$, $\boldsymbol{E}_{a1}$, $\boldsymbol{E}_{a2}$, their sequence components. Let also $\boldsymbol{I}_a$, $\boldsymbol{I}_b$, $\boldsymbol{I}_c$ be the primary line currents at the relay location and $\boldsymbol{I}_{a0}$, $\boldsymbol{I}_{a1}$, $\boldsymbol{I}_{a2}$ their sequence components. As is well known:

$$\begin{aligned} \boldsymbol{E}_a &= \boldsymbol{E}_{a0} + \boldsymbol{E}_{a1} + \boldsymbol{E}_{a2} \\ \boldsymbol{E}_b &= \boldsymbol{E}_{a0} + \alpha^2\boldsymbol{E}_{a1} + \alpha\boldsymbol{E}_{a2} \\ \boldsymbol{E}_c &= \boldsymbol{E}_{a0} + \alpha\boldsymbol{E}_{a1} + \alpha^2\boldsymbol{E}_{a2} \end{aligned} \tag{1}$$

$$\begin{aligned} \boldsymbol{I}_a &= \boldsymbol{I}_{a0} + \boldsymbol{I}_{a1} + \boldsymbol{I}_{a2} \\ \boldsymbol{I}_b &= \boldsymbol{I}_{a0} + \alpha^2\boldsymbol{I}_{a1} + \alpha\boldsymbol{I}_{a2} \\ \boldsymbol{I}_c &= \boldsymbol{I}_{a0} + \alpha\boldsymbol{I}_{a1} + \alpha\boldsymbol{I}_{a2} \end{aligned} \tag{2}$$

From which the following voltages and currents appearing in table I may be obtained:

$$\begin{aligned} \boldsymbol{E}_a - \boldsymbol{E}_b &= \boldsymbol{E}_{a1}(1-\alpha^2) + \boldsymbol{E}_{a2}(1-\alpha) = \sqrt{3}(e^{j30}\boldsymbol{E}_{a1} + e^{-j30}\boldsymbol{E}_{a2}) \\ \boldsymbol{E}_b - \boldsymbol{E}_c &= \boldsymbol{E}_{a1}(\alpha^2-\alpha) + \boldsymbol{E}_{a2}(\alpha-\alpha^2) = -j\sqrt{3}(\boldsymbol{E}_{a1} - \boldsymbol{E}_{a2}) \\ \boldsymbol{E}_c - \boldsymbol{E}_a &= \boldsymbol{E}_{a1}(\alpha-1) + \boldsymbol{E}_{a2}(\alpha^2-1) = -\sqrt{3}(e^{-j30}\boldsymbol{E}_{a1} + e^{j30}\boldsymbol{E}_{a2}) \end{aligned} \tag{3}$$

$$\begin{aligned} \boldsymbol{I}_a - \boldsymbol{I}_b &= \boldsymbol{I}_{a1}(1-\alpha^2) + \boldsymbol{I}_{a2}(1-\alpha) \\ \boldsymbol{I}_b - \boldsymbol{I}_c &= \boldsymbol{I}_{a1}(\alpha^2-\alpha) + \boldsymbol{I}_{a2}(\alpha-\alpha^2) \\ \boldsymbol{I}_c - \boldsymbol{I}_a &= \boldsymbol{I}_{a1}(\alpha-1) + \boldsymbol{I}_{a2}(\alpha^2-1) \end{aligned} \tag{4}$$

These permit the expression, in terms of sequence components, of the value taken by the impedances $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$ of table I when the potential and current transformers are connected in Δ. That is:

$$\begin{aligned} \boldsymbol{Z}_{ab} &= \frac{\boldsymbol{E}_a - \boldsymbol{E}_b}{\boldsymbol{I}_a - \boldsymbol{I}_b} = \frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1} - \alpha\boldsymbol{E}_{a2})}{\sqrt{3}^{j30}(\boldsymbol{I}_{a1} - \alpha\boldsymbol{I}_{a2})} = \frac{\boldsymbol{E}_{a1} - \alpha\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1} - \alpha\boldsymbol{I}_{a2}} \\ \boldsymbol{Z}_{bc} &= \frac{\boldsymbol{E}_b - \boldsymbol{E}_c}{\boldsymbol{I}_b - \boldsymbol{I}_c} = \frac{-\sqrt{3}j(\boldsymbol{E}_{a2} - \boldsymbol{E}_{a2})}{-\sqrt{3}j(\boldsymbol{I}_{a1} - \boldsymbol{I}_{a2})} = \frac{\boldsymbol{E}_{a1} - \boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1} - \boldsymbol{I}_{a2}} \\ \boldsymbol{Z}_{ca} &= \frac{\boldsymbol{E}_c - \boldsymbol{E}_a}{\boldsymbol{I}_c - \boldsymbol{I}_a} = \frac{-\sqrt{3}e^{-j30}(\boldsymbol{E}_{a1} - \alpha^2\boldsymbol{E}_{a2})}{-\sqrt{3}e^{-j30}(\boldsymbol{I}_{a1} - \alpha^2\boldsymbol{I}_{a2})} = \frac{\boldsymbol{E}_{a1} - \alpha^2\boldsymbol{E}_{a2}}{\boldsymbol{I}_{a1} - \alpha^2\boldsymbol{I}_{a2}} \end{aligned} \tag{5}$$

In the same manner, by means of formulas 2 and 3, the values taken by the impedances $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$ when the potential transformers are connected in Δ and the current transformers in Y, may be expressed in terms of sequence components as follows:

$$\begin{aligned} \boldsymbol{Z}_{ab} &= \frac{\boldsymbol{E}_a - \boldsymbol{E}_b}{\boldsymbol{I}_a} = \frac{\sqrt{3}(e^{j30}\boldsymbol{E}_{a1} + e^{-j30}\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0} + \boldsymbol{I}_{a1} + \boldsymbol{I}_{a2}} = \frac{\sqrt{3}e^{j30}(\boldsymbol{E}_{a1} - \alpha\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0} + \boldsymbol{I}_{a1} + \boldsymbol{I}_{a2}} \\ \boldsymbol{Z}_{bc} &= \frac{\boldsymbol{E}_b - \boldsymbol{E}_c}{\boldsymbol{I}_b} = \frac{-j\sqrt{3}(\boldsymbol{E}_{a1} - \boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0} + \alpha^2\boldsymbol{I}_{a1} + \alpha\boldsymbol{I}_{a2}} \\ \boldsymbol{Z}_{ca} &= \frac{\boldsymbol{E}_c - \boldsymbol{E}_a}{\boldsymbol{I}_c} = \frac{-\sqrt{3}(e^{-j30}\boldsymbol{E}_{a1} + e^{j30}\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0} + \alpha\boldsymbol{I}_{a1} + \alpha^2\boldsymbol{I}_{a2}} \\ &= \frac{-\sqrt{3}e^{-j30}(\boldsymbol{E}_{a1} - \alpha^2\boldsymbol{E}_{a2})}{\boldsymbol{I}_{a0} + \alpha\boldsymbol{I}_{a1} + \alpha^2\boldsymbol{I}_{a2}} \end{aligned} \tag{6}$$

The preceding formulas are general and apply in any case irrespective of the complexity of the network, with only the limitation that the sequence components appearing in them are as viewed from the relay irrespective of the fault location.

Now in the ordinary practice, it is customary to make short circuit calculations, whether in long hand or on the calculating board, by properly interconnecting the sequence networks according to the type of fault, disregarding first the connections of transformers and generators and accounting for them later if necessary. The voltages and currents obtained from these calculations are as viewed from the fault.

It follows that in applying formulas 5 and 6 to faults located between the relays and the power transformer bank, such as at F of figures 1 and 2, the sequence components of the voltages and currents to be used are those obtained from the short circuit calculations made in the ordinary routine way, because currents and voltages are viewed in the same manner from the relay location and from the fault.

For faults past the transformer bank, such as at P of figures 1 and 2, however, this no longer is true, because the sequence components of the currents and voltages, on account of the grounded neutral Δ/Y transformer connection, are viewed differently from the fault and from the relay location. Therefore, the sequence components as obtained from the short circuit calculations cannot be substituted directly in equations 5 and 6, they must be transformed first to account for the grounded neutral Δ/Y connection of the power bank. The necessary relations may be found as follows:

With a fault at P (figures 1 and 2), let $\boldsymbol{I}_A$, $\boldsymbol{I}_B$, $\boldsymbol{I}_C$ be the currents flowing in the Y windings of the transformer and $\boldsymbol{I}_{A0}$, $\boldsymbol{I}_{A1}$, $\boldsymbol{I}_{A2}$ their sequence components. Because of the grounded neutral Δ/Y power transformer, the zero sequence component $\boldsymbol{I}_{a0}$ of the corresponding line current ($\boldsymbol{I}_a$, $\boldsymbol{I}_b$, $\boldsymbol{I}_c$) at the relay location, is zero. The other 2 components $\boldsymbol{I}_{a1}$, $\boldsymbol{I}_{a2}$, assuming all currents to be calculated on the same voltage basis, are tied to $\boldsymbol{I}_{A1}$, $\boldsymbol{I}_{A2}$ by the relations:

$$\begin{aligned} \boldsymbol{I}_{a1} &= j\boldsymbol{I}_{A1} \\ \boldsymbol{I}_{a2} &= -j\boldsymbol{I}_{A2} \end{aligned} \tag{7}$$

Under the same conditions, let $\boldsymbol{E}_{A1}$, $\boldsymbol{E}_{A2}$ be the positive and the negative sequence components of the line-to-neutral voltages at the relay location, as obtained from the calculations made disregarding the effect of the grounded neutral Δ/Y transformation. The zero sequence component $\boldsymbol{E}_{a0}$, of the 3 voltages $\boldsymbol{E}_a$, $\boldsymbol{E}_b$, $\boldsymbol{E}_c$ actually existing at the relay location will be zero, and the other 2 components $\boldsymbol{E}_{a1}$, $\boldsymbol{E}_{a2}$ will be tied to $\boldsymbol{E}_{A1}$, $\boldsymbol{E}_{A2}$ by the relations:

$$\begin{aligned} \boldsymbol{E}_{a1} &= j\boldsymbol{E}_{A1} \\ \boldsymbol{E}_{a2} &= -j\boldsymbol{E}_{A2} \end{aligned} \tag{8}$$

Substitution of equations 7 and 8 in equations 5 and 6 gives:

$$\begin{aligned} \boldsymbol{Z}_{ab} &= \frac{\boldsymbol{E}_{A1} + \alpha\boldsymbol{E}_{A2}}{\boldsymbol{I}_{A1} + \alpha\boldsymbol{I}_{A2}} \\ \boldsymbol{Z}_{bc} &= \frac{\boldsymbol{E}_{A1} + \boldsymbol{E}_{A2}}{\boldsymbol{I}_{A1} + \boldsymbol{I}_{A2}} \\ \boldsymbol{Z}_{ca} &= \frac{\boldsymbol{E}_{A1} + \alpha^2\boldsymbol{E}_{A2}}{\boldsymbol{I}_{A1} + \alpha^2\boldsymbol{I}_{A2}} \end{aligned} \tag{9}$$

[Tabl]e III—Distance Relay Primary Impedance Indications in Terms of Impedances to Positive, Negative, and Zero Sequence Currents of the Circuits Involved*—Stub-End Line (Figs. 4 and 5)

[Typ]e of [Fa]ult	Relay Number	Relay Primary Impedance	Potential and Current Transformers in Δ: No Grounded Neutral Δ/Y or Y/Δ Power Transformer Bank Interposed Between the Relays and the Fault (Fig. 4)	Potential and Current Transformers in Δ: Grounded Neutral Δ/Y or Y/Δ Power Transformer Bank Interposed Between the Relay and the Fault (Fig. 5)	Potential Transformers in Δ, Current Transformers in Y: No Grounded Neutral Δ/Y or Y/Δ Power Transformer Bank Interposed Between the Relays and the Fault (Fig. 4)	Potential Transformers in Δ, Current Transformers in Y: Grounded Neutral Δ/Y or Y/Δ Power Transformer Bank Interposed Between the Relay and the Fault (Fig. 5)
[Three-ph]ase [at] fig. 4 [or] fig. 5	1	$Z_{ab} =$	$Z_f = R_f + jX_f$	$Z_f = R_f + jX_f$	$\sqrt{3}e^{j30}Z_f = (1.5R_f - 0.866X_f) + j(1.5X_f + 0.866R_f)$	$\sqrt{3}e^{j30}Z_f = (1.5R_f - 0.866X_f) + j(1.5X_f + 0.866R_f)$
	2	$Z_{bc} =$				
	3	$Z_{ca} =$				
[Line t]o Line [b t]o c [(Fig]. 4) [B t]o C [(Fig]. 5)	1	$Z_{ab} =$	$-j\sqrt{3}Z_2 + 2e^{-j60}Z_f = \{R_f + 1.73(X_f + X_2)\} + j\{X_f - 1.73(R_f + R_2)\}$	$j\dfrac{Z_2}{\sqrt{3}} + 1.155\,e^{j30}Z_f = \{R_f - 0.577(X_f + X_2)\} + j\{X_f + 0.577(R_f + R_2)\}$	∞	$\sqrt{3}\left(\dfrac{jZ_2}{2} + e^{j30}Z_f\right) = \{1.5R_f - 0.866(X_2 + X_f)\} + j\{1.5X_f + 0.866(R_f + R_2)\}$
	2	$Z_{bc} =$	$Z_f = R_f + jX_f$	∞	$2Z_f = 2R_f + j2X_f$	$j2\sqrt{3}(Z_2 + Z_f) = -2\sqrt{3}\{(X_2 + X_f) - j(R_2 + R_f)\}$
	3	$Z_{ca} =$	$j\sqrt{3}Z_2 + 2e^{j60}Z_f = \{R_f - 1.73(X_f + X_2)\} + j\{X_f + 1.73(R_f + R_2)\}$	$-j\dfrac{Z_2}{\sqrt{3}} + 1.155e^{-j30}Z_f = \{(R_f + 0.577(X_f + X_2)\} + j\{X_f - 0.577(R_f + R_2)\}$	$j\sqrt{3}Z_2 + 2e^{j60}Z_f = \{R_f - 1.73(X_f + X_2)\} + j\{X_f + 1.73(R + R_{f2})\}$	$\sqrt{3}(-jZ_2 + 2e^{-j30}Z_f) = \{3R_f + 1.73(X_f + X_2)\} + j\{3X_f - 1.73(R_f + R_2)\}$
[Line t]o Line [to Gr]ound [b and] c to [Gro]und [(Fig]. 4) [B and] C to [Gro]und [(Fig]. 5)	1	$Z_{ab} =$	$Z_f + \dfrac{1 - \alpha}{\dfrac{1}{Z_0 + Z_{f0}} - \dfrac{\alpha_2}{Z_2 + Z_f}}$	$Z_f - \dfrac{\alpha^2}{\dfrac{1}{Z_{f0}} + \dfrac{1 - \alpha}{Z_2 + Z_f}}$	∞	$\sqrt{3}e^{j30}Z_f + \dfrac{j\sqrt{3}Z_2}{\dfrac{Z_2 + Z_f}{Z_{f0}} + 2}$
	2	$Z_{bc} =$	$Z_f = R_f + jX_f$	$Z_f + 2Z_{f0} = (R_f + 2R_{f0}) + j(X_f + 2X_{f0})$	$Z_f\left\{2 + \dfrac{\sqrt{3}j}{\dfrac{Z_0 + Z_{f0}}{Z_2 + Z_f} - \alpha}\right\}$	$\sqrt{3}e^{j30}\left\{Z_f\left(1 + \dfrac{\sqrt{3}e^{j30}}{\dfrac{Z_2 + Z_f}{Z_{f0}} - \alpha}\right) + \dfrac{2Z_2}{\dfrac{Z_2 + Z_f}{Z_{f0}} - \alpha}\right\}$
	3	$Z_{ca} =$	$Z_f + \dfrac{1 - \alpha^2}{\dfrac{1}{Z_0 + Z_{f0}} - \dfrac{\alpha}{Z_2 + Z_f}}$	$Z_f - \dfrac{\alpha}{\dfrac{1}{Z_{f0}} + \dfrac{1 - \alpha^2}{Z_2 + Z_f}}$	$Z_f e^{j60}\left(2 - \dfrac{\sqrt{3}j}{\dfrac{Z_0 + Z_{f0}}{Z_2 + Z_f} - \alpha^2}\right) + \dfrac{j\sqrt{3}Z_2}{1 - \alpha^2\left(\dfrac{Z_2 + Z_f}{Z_0 + Z_{f0}}\right)}$	$\sqrt{3}e^{j30}Z_f + \dfrac{3Z_f + \sqrt{3}e^{-j30}Z_2}{\dfrac{Z_2 + Z_f}{Z_{f0}} - \alpha^2}$
[Line] to [Gro]und [a to Gr]ound [(Fig.] 4) [A to Gr]ound [(Fig.] 5)	1	$Z_{ab} =$	$\dfrac{jZ_2 + (Z_0 + Z_{f0} + 2Z_f)e^{j30}}{\sqrt{3}} = \{0.5(R_0 + R_{f0}) + R_f - 0.29(X_0 + X_{f0}) - 0.577(X_f + X_2)\} + j\{0.5(X_0 + X_{f0}) + X_f + 0.29(R_0 + R_{f0}) + 0.577(R_f + R_2)\}$	$(Z_{f0} + 2Z_f)e^{-j60} - j\sqrt{3}Z_2 = \{R_f + 1.73(X_f + X_2) + 0.5R_{f0} + 0.866X_{f0}\} + j\{X_f - 1.73(R_f + R_2) + 0.5X_{f0} - 0.866R_{f0}\}$	$\dfrac{jZ_2 + (Z_0 + Z_{f0} + 2Z_f)e^{j30}}{\sqrt{3}} = \{0.5(R_0 + R_{f0}) + R_f - 0.29(X_0 + X_{f0}) - 0.577(X_f + X_2\} + j\{0.5(X_0 + X_{f0}) + X_f + 0.29(R_0 + R_{f0}) + 0.577(R_f + R_2)\}$	∞
	2	$Z_{bc} =$	∞	$Z_f + \dfrac{Z_{f0}}{2} = \left(R_f + \dfrac{R_{f0}}{2}\right) + j\left(X_f + \dfrac{X_{f0}}{2}\right)$	∞	$2Z_f + Z_{f0} = (2R_f + R_{f0}) + (2X_f + X_{f0})$
	3	$Z_{ca} =$	$\dfrac{-jZ_2 + (Z_0 + Z_{f0} + 2Z_f)e^{-j30}}{\sqrt{3}} = \{0.5(R_0 + R_{f0}) + R_f + 0.29(X_0 + X_{f0}) + 0.577(X_f + X_2)\} + j\{0.5(X_0 + X_{f0}) + X_f - 0.29(R_0 + R_{f0}) - 0.577(R_f + R_2)\}$	$(Z_{f0} + 2Z_f)e^{j60} + j\sqrt{3}Z_2 = \{R_f - 1.73(X_f + X_2) + 0.5R_{f0} - 0.866X_{f0}\} + j\{X_f + 1.73(R_f + R_2) + 0.5X_{f0} + 0.866R_{f0}\}$	∞	$(Z_{f0} + 2Z_f)e^{j60} + j\sqrt{3}Z_2 = \{R_f - 1.73(X_f + X_2) + 0.5R_{f0} - 0.866X_{f0}\} + j\{X_f + 1.73(R_f + R_2) + 0.5X_{f0} + 0.866R_{f0}\}$

[* Wher]e the relays are of the impedance type, the primary ohms indicated are equal, respectively, to the magnitudes of Z_{ab}, Z_{bc}, Z_{ca}.
[Where] the relays are of the reactance type, the primary ohms indicated are equal, respectively, to the reactance components of Z_{ab}, Z_{bc}, Z_{ca}.
[The p]erformance of the directional element must be analyzed before saying definitely what any relay will do under any specific condition.

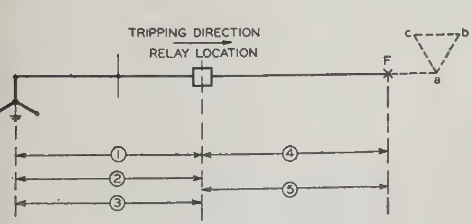

Fig. 4. Typical fault situation on stub-end line

1. $Z_1 = (R_1 + jX_1)$; positive sequence
2. $Z_2 = (R_2 + jX_2)$; negative sequence
3. $Z_0 = (R_0 + jX_0)$; zero sequence
4. $Z_f = (R_f + jX_f)$; positive and negative sequence
5. $Z_{f0} = (R_{f0} + jX_{f0})$; zero sequence

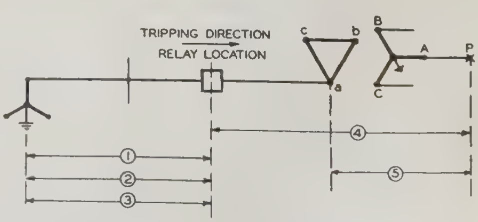

Fig. 5. Typical fault situation on stub-end line

1. $Z_1 = (R_1 + jX_1)$; positive sequence
2. $Z_2 = (R_2 + jX_2)$; negative sequence
3. $Z_0 = (R_0 + jX_0)$; zero sequence
4. $Z_f = (R_f + jX_f)$; positive and negative sequence
5. $Z_{\theta} = (R_{f0} + jX_{f0})$; zero sequence

for the case of potential and current transformers Δ-connected and

$$Z_{ab} = \frac{\sqrt{3}(e^{j30}E_{A1} - e^{-j30}E_{A2})}{I_{A1} - I_{A2}} = \frac{\sqrt{3}e^{j30}(E_{A1} + \alpha E_{A2})}{I_{A1} - I_{A2}}$$

$$Z_{bc} = \frac{-j\sqrt{3}(E_{A1} + E_{A2})}{\alpha^2 I_{A1} - \alpha I_{A2}} = \frac{\sqrt{3}e^{j30}(E_{A1} + E_{A2})}{I_{A1} - \alpha^2 I_{A2}} \quad (10)$$

$$Z_{ca} = \frac{-\sqrt{3}(e^{-j30}E_{A1} - e^{j30}E_{A2})}{\alpha I_{A1} - \alpha^2 I_{A2}} = \frac{\sqrt{3}e^{j30}(E_{A1} + \alpha^2 E_{A2})}{I_{A1} - \alpha I_{A2}}$$

for the case of potential transformers in Δ and current transformers in Y.

In calculating the primary impedances Z_{ab}, Z_{bc}, Z_{ca}, the sequence components of the voltages and currents obtained disregarding the grounded neutral Δ/Y transformation, may now be used, merely by using equations 5 or 6 for faults on the Δ side, such as at F, and equations 9 or 10 for faults on the Y side of the transformer, such as at P.

Equations 9 and 10 have been tabulated, respectively, in the fourth and sixth columns of table II.

The letter A has been replaced by a for uniformity of symbols. The sequence components of table II are as viewed from the faults; that is, are those obtained from the short circuit calculations made in the ordinary routine manner.

It can easily be shown that the equations of table II apply also to relays located on the Y side of the transformer bank (as shown in figure 3 for the case of potential and current transformers in Δ) provided that I_{a1}, I_{a2} and E_{a1}, E_{a2} are interpreted as being the positive and negative sequence components of the relay primary currents and voltages calculated disregarding the effect of the grounded neutral Δ/Y transformation.

Appendix II—Application of the Formulas of Table II to the Case of a Stub-End Line

In the case of a stub-end line, the formulas of table II may be expressed in terms of the impedances to currents of zero, positive, and negative sequence without arriving at too complicated expressions.

This is done in this appendix for the most common types of faults namely, 3 phase, line-to-line, line-to-line-to-ground, and line-to-ground. The various impedances involved are shown in figures 4 and 5, respectively, for faults (F) between the relays and a grounded neutral Δ/Y power transformer bank and for faults (P) located past the transformer bank. The results are tabulated in table III.

The starting point for the derivation of the formulas is constituted by the well-known relations:

$$E_{a1} = E - Z_1 I_{a1} \qquad (12)$$
$$E_{a2} = -Z_2 I_{a2}$$

where E, E_{a1}, E_{a2}, I_{a1}, I_{a2} are the generator internal voltage and the sequence components of the primary line-to-neutral voltages and line currents at the relay location, as viewed from the fault.

In addition to the general relations of equations 12 which are common to all types of fault, the relations, among the sequence components, characterizing each type of fault, will be used also.

A—3 Phase Faults (a to b to c, or A to B to C)

The characteristic relations for 3 phase faults are:

$$I_{a1} = \frac{E}{Z_1 + Z_f}$$

$$I_{a0} = I_{a2} = E_{a0} = E_{a2} = 0 \qquad (13)$$

From equations 13 and 12 it is obtained:

$$E_{a1} = Z_f I_{a1}$$

Direct substitution in table II gives:

$$Z_{ab} = Z_{bc} = Z_{ca} = Z_f = R_f + jX_f \qquad (14)$$

with potential and current transformers in Δ, and

$$Z_{ab} = Z_{bc} = Z_{ca} = \sqrt{3}e^{j30}Z_f = (1.5\,R_f - 0.866X_f) + j(1.5\,X_f + 0.866R_f) \qquad (15)$$

with the potential transformers in Δ and the current transformers in Y.

From equation 15 it is seen that with the potential transformers in Δ and the current transformers in Y, the primary ohms indicated by reactance relays depend also upon the resistance of the line, and therefore are affected by arc resistance.

B—Line to Line Faults (b to c or B to C)

The relations characterizing line-to-line faults are:

$$I_{a1} = -I_{a2} = \frac{E}{Z_1 + Z_2 + 2Z_f} \qquad (16)$$

$$I_{a0} = 0$$

By substituting in equations 12:

$$E_{a1} = (Z_2 + 2Z_f)I_{a1}$$

$$E_{a2} = Z_2 I_{a1}$$

From which and from equations 16:

$$E_{a1} - E_{a2} = 2Z_f I_{a1}$$

$$E_{a1} - \alpha E_{a2} = 2Z_f I_{a1} + \sqrt{3}e^{-j30}Z_2 I_{a1}$$

$$E_{a1} - \alpha^2 E_{a2} = 2Z_f I_{a1} + \sqrt{3}e^{j30}Z_2 I_{a1}$$

$$I_{a1} - I_{a2} = 2I_{a1}$$

$$I_{a1} - \alpha I_{a2} = -\alpha^2 I_{a1}$$

$$I_{a1} - \alpha^2 I_{a2} = -\alpha I_{a1}$$

Substitution in the third column of table II, gives for Z_{ab}, Z_{bc}, Z_{ca} respectively:

$$Z_{ab} = \frac{E_{a1} - \alpha E_{a2}}{I_{a1} - \alpha I_{a2}} = -j\sqrt{3}Z_2 + 2e^{-j60}Z_f$$
$$= \{R_f + 1.73(X_f + X_2)\} + j\{X_f - 1.73(R_f + R_2)\} \qquad (17)$$

$$Z_{bc} = \frac{E_{a1} - E_{a2}}{I_{a1} - I_{a2}} = Z_f = R_f + jX_f \qquad (18)$$

$$Z_{ca} = \frac{E_{a1} - \alpha^2 E_{a2}}{I_{a1} - \alpha^2 I_{a2}} = j\sqrt{3}Z_2 + 2e^{j60}Z_f$$
$$= \{R_f - 1.73(X_f + X_2)\} + j\{X_f + 1.73(R_f + R_2)\} \qquad (19)$$

These are the primary impedances for a line-to-line fault between the relays and the power bank when the potential and current transformers are connected in Δ.

With the current transformers connected in Y and a line-to-line fault at the same location:

$$e^{j30}E_{a1} + e^{-j30}E_{a2} = \{\sqrt{3}Z_2 + 2e^{j30}Z_f\}I_{a1}$$

$$e^{-j30}E_{a1} + e^{j30}E_{a2} = \{\sqrt{3}Z_2 + 2e^{-j30}Z_f\}I_{a1}$$

$$I_{a1} + I_{a2} = 0$$

$$\alpha^2 I_{a1} + \alpha I_{a2} = -j\sqrt{3}I_{a1}$$

$$\alpha I_{a1} + \alpha^2 I_{a2} = j\sqrt{3}I_{a1}$$

Substituting in the fifth column of table II:

$$Z_{ab} = \infty$$

$$Z_{bc} = \frac{-j\sqrt{3}(E_{a1} - E_{a2})}{I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}} = 2Z_f = 2R_f + 2jX_f$$

$$Z_{ca} = \frac{-\sqrt{3}(e^{-j30}E_{a1} + e^{j30}E_{a2})}{I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}} = j\sqrt{3}Z_2 + 2e^{j60}Z_f$$
$$= \{R_f - 1.73(X_f + X_2)\} + j\{X_f + 1.73(R_f + R_2)\} \qquad (20)$$

These are the primary impedances with a line-to-line fault between the relays and the transformer bank, when the potential transformers are connected in Δ and the current transformers in Y.

For line-to-line faults past the transformer bank from equations 12 and 16:

$$E_{a1} + \alpha E_{a2} = (-\alpha^2 Z_2 + 2Z_f)I_{a1}$$

$$E_{a1} + E_{a2} = 2(Z_2 + Z_f)I_{a1}$$

$$E_{a1} + \alpha^2 E_{a2} = (-\alpha Z_2 + 2Z_f)I_{a1}$$

$$I_{a1} + \alpha I_{a2} = \sqrt{3}e^{-j30}I_{a1}$$

$$I_{a1} + I_{a2} = 0$$

$$I_{a1} + \alpha^2 I_{a2} = \sqrt{3}e^{j30}I_{a1}$$

Substitution in the fourth column of table II gives for the primary impedances, with potential and current transformers in Δ:

$$Z_{ab} = \frac{E_{a1} + \alpha E_{a2}}{I_{a1} + \alpha I_{a2}} = j\frac{Z_2}{\sqrt{3}} + 1.155e^{j30}Z_f$$

$$= \{R_f - 0.577(X_f + X_2)\} + j\{X_f + 0.577(R_f + R_2)\} \qquad (21)$$

$$Z_{bc} = \infty \qquad (22)$$

$$Z_{ca} = \frac{E_{a1} + \alpha^2 E_{a2}}{I_{a1} + \alpha^2 I_{a2}} = -j\frac{Z_2}{\sqrt{3}} + 1.155e^{-j30}Z_f$$

$$= \{R_f + 0.577(X_f + X_2)\} + j\{X_f - 0.577(R_f + R_2)\} \qquad (23)$$

With faults at the same location and the current transformers connected in Y:

$$e^{j30}E_{a1} - e^{-j30}E_{a2} = (jZ_2 + 2Z_f e^{j30})I_{a1}$$

$$e^{-j30}E_{a1} - e^{j30}E_{a2} = (-jZ_2 + 2Ze_f^{-j30})I_{a1}$$

$$\alpha^2 I_{a1} - \alpha I_{a2} = -I_{a1}$$

$$\alpha I_{a1} - \alpha^2 I_{a2} = -I_{a1}$$

Substitution in the sixth column of table II gives:

$$Z_{ab} = \frac{\sqrt{3}(e^{j30}E_{a1} - e^{-j30}E_{a2})}{I_{a1} - I_{a2}} = \sqrt{3}\left(j\frac{Z_2}{2} + Z_f e^{j30}\right)$$

$$= \{1.5R_f - 0.866(X_2 + X_f)\} + j\{1.5X_f + 0.866(R_f + R_2)\} \qquad (24)$$

$$Z_{bc} = \frac{-j\sqrt{3}(E_{a1} + E_{a2})}{\alpha^2 I_{a1} - \alpha I_{a2}} = j2\sqrt{3}(Z_2 + Z_f)$$

$$= -2\sqrt{3}\{(X_2 + X_f) - j(R_2 + R_f)\} \qquad (25)$$

$$Z_{ca} = \frac{-\sqrt{3}(e^{-j30}E_{a1} - e^{j30}E_{a2})}{\alpha I_{a1} - \alpha^2 I_{a2}} = \sqrt{3}(-jZ_2 + 2e^{-j30}Z_f)$$

$$= \{3R_f + 1.73(X_f + X_2)\} + j\{3X_f - 1.73(R_f + R_2)\} \qquad (26)$$

These are the relay primary impedances with a line-to-line fault past the power bank when the potential transformers are connected in Δ and the current transformers in Y.

C—Line to Line to Ground Fault Between the Relays and the Power Bank (*b* to *c* to Ground)

The characteristic relations for this type of fault are:

$$I_{a1} = -(I_{a0} + I_{a2}) = \frac{E}{(Z_1 + Z_f) + \dfrac{(Z_0 + Z_{f0})(Z_2 + Z_f)}{Z_0 + Z_{f0} + Z_2 + Z_f}}$$

$$I_{a2} = -I_{a1}\frac{(Z_0 + Z_{f0})}{Z_0 + Z_{f0} + Z_2 + Z_f}$$

$$I_{a0} = -I_{a1}\frac{(Z_2 + Z_f)}{Z_0 + Z_{f0} + Z_2 + Z_f}$$

From these and equations 12:

$$E_{a1} = \left\{Z_f + \frac{(Z_0 + Z_{f0})(Z_2 + Z_f)}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}I_{a1}$$

$$E_{a2} = I_{a1}\frac{Z_2(Z_0 + Z_{f0})}{Z_0 + Z_{f0} + Z_2 + Z_f}$$

$$E_{a1} - E_{a2} = \left\{1 + \frac{Z_0 + Z_{f0}}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}Z_f I_{a1}$$

$$E_{a1} - \alpha E_{a2} = \left\{1 + \frac{Z_0 + Z_{f0}}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}Z_f I_{a1} + \frac{Z_2(Z_0 + Z_{f0})}{Z_0 + Z_{f0} + Z_2 + Z_f}(1 - \alpha)I_{a1}$$

$$E_{a1} - \alpha^2 E_{a2} = 1 + \frac{Z_0 + Z_{f0}}{Z_0 + Z_{f0} + Z_2 + Z_f}Z_f I_{a1} + \frac{Z_2(Z_0 + Z_{f0})(1 - \alpha^2)}{Z_0 + Z_{f0} + Z_2 + Z_f}I_{a1}$$

$$I_{a1} - \alpha I_{a2} = \left\{1 + \frac{\alpha(Z_0 + Z_{f0})}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}I_{a1}$$

$$I_{a1} - I_{a2} = \left\{1 + \frac{(Z_0 + Z_{f0})}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}I_{a1}$$

$$I_{a1} - \alpha^2 I_{a2} = \left\{1 + \frac{\alpha^2(Z_0 + Z_{f0})}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}I_{a1}$$

Substitution in the third column of table II gives:

$$Z_{ab} = \frac{E_{a1} - \alpha E_{a2}}{I_{a1} - \alpha I_{a2}} = Z_f + \frac{1 - \alpha}{\dfrac{1}{Z_0 + Z_{f0}} - \dfrac{\alpha^2}{Z_2 + Z_f}} \qquad (27)$$

$$Z_{bc} = \frac{E_{a1} - E_{a2}}{I_{a1} - I_{a2}} = Z_f = R_f + jX_f \qquad (28)$$

$$Z_{ca} = \frac{E_{a1} - \alpha^2 E_{a2}}{I_{a1} - \alpha^2 I_{a2}} = Z_f + \frac{1 - \alpha^2}{\dfrac{1}{Z_0 + Z_{f0}} - \dfrac{1}{Z_2 + Z_f}} \qquad (29)$$

These are the relay primary impedances for a line-to-line-to-ground fault between the relays and the transformer bank, when the potential and the current transformers are connected in Δ.

With the current transformers in Y and a line-to-line-to-ground fault at the same location:

$$E_{a1} = E_{a2} = \left\{1 + \frac{Z_0 + Z_{f0}}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}Z_f I_{a1}$$

$$e^{j30}E_{a1} + e^{-j30}E_{a2} = \frac{I_{a1}}{Z_0 + Z_{f0} + Z_2 + Z_f}\{2Z_f(Z_0 + Z_{f0}) + Z_f(Z_2 + Z_f)\}e^{j30} + \frac{I_{a1}\sqrt{3}}{Z_0 + Z_{f0} + Z_2 + Z_f}(Z_0 + Z_{f0})Z_2$$

$$e^{-j30}E_{a1} + e^{j30}E_{a2} = \frac{I_{a1}}{Z_0 + Z_{f0} + Z_2 + Z_f}\{2Z_f(Z_0 + Z_{f0}) + Z_f(Z_2 + Z_f)\}e^{-j30} + \frac{I_{a1}\sqrt{3}Z_2}{Z_0 + Z_{f0} + Z_2 + Z_f}(Z_0 + Z_{f0})$$

$$I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2} = (\alpha^2 - \alpha)I_{a1}\left\{\frac{(Z_0 + Z_{f0}) - \alpha(Z_2 + Z_f)}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}$$

$$I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2} = (\alpha - \alpha^2)I_{a1}\left\{\frac{(Z_0 + Z_{f0}) - \alpha^2(Z_2 + Z_f)}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}$$

Substitution in the fifth column of table II gives:

$$Z_{ab} = \infty \qquad (30)$$

$$Z_{bc} = \frac{-j(E_{a1} - E_{a2})\sqrt{3}}{I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2}} = \frac{-j\left\{1 + \dfrac{Z_0 + Z_{f0}}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}Z_f\sqrt{3}}{-j\sqrt{3}\left\{\dfrac{(Z_0 + Z_{f0}) - \alpha(Z_2 + Z_f)}{Z_0 + Z_{f0} + Z_2 + Z_f}\right\}}$$

$$Z_{bc} = \frac{(Z_0 + Z_{f0} + Z_2 + Z_f + Z_0 + Z_{f0})Z_f}{Z_0 + Z_{f0} - \alpha(Z_2 + Z_f)}$$

$$= \frac{\{2[Z_0 + Z_{f0} - \alpha(Z_2 + Z_f)] + (1 + 2\alpha)(Z_2 + Z_f)\}Z_f}{Z_0 + Z_{f0} - \alpha(Z_2 + Z_f)}$$

$$= Z_f \left\{ 2 + \frac{(1 + 2\alpha)(Z_2 + Z_f)}{Z_0 + Z_{f0} - \alpha(Z_2 + Z_f)} \right\}$$

$$= Z_f \left\{ 2 + \frac{(\alpha - \alpha^2)(Z_2 + Z_f)}{Z_0 + Z_{f0} - \alpha(Z_2 + Z_f)} \right\}$$

$$= Z_f \left\{ 2 + \frac{\sqrt{3}j}{\dfrac{Z_0 + Z_{f0}}{Z_2 + Z_f} - \alpha} \right\} \tag{31}$$

$$Z_{ca} = -\frac{\sqrt{3}(e^{-j30}E_{a1} + e^{j30}E_{a2})}{I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}}$$

$$= \frac{-\sqrt{3}Z_f\{2(Z_0 + Z_{f0}) + Z_2 + Z_f\}e^{-j30} - 3Z_2(Z_0 + Z_{f0})}{j\sqrt{3}\{Z_0 + Z_{f0} - \alpha^2(Z_2 + Z_f)\}}$$

$$= Z_f e^{j60}\left(2 - \frac{\sqrt{3}j}{\dfrac{Z_0 + Z_{f0}}{Z_2 + Z_f} - \alpha^2}\right) + \frac{\sqrt{3}jZ_2}{1 - \alpha^2\left(\dfrac{Z_2 + Z_f}{Z_0 + Z_{f0}}\right)} \tag{32}$$

D—Line to Line to Ground Faults Past the Transformer Bank (B to C)

The characteristic equations for this type of fault are:

$$I_{a1} = \frac{E}{Z_1 + Z_f + \dfrac{Z_{f0}(Z_2 + Z_f)}{Z_{f0} + Z_2 + Z_f}}$$

$$I_{a2} = -I_{a1}\frac{Z_{f0}}{Z_{f0} + Z_2 + Z_f}$$

$$I_{a0} = -I_{a1}\frac{Z_2 + Z_f}{Z_{f0} + Z_2 + Z_f}$$

$$E_{a1} = \left(Z_f + \frac{Z_{f0}(Z_2 + Z_f)}{Z_{f0} + Z_2 + Z_f}\right)I_{a1}$$

$$E_{a2} = \frac{Z_{f0}Z_2I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

from which:

$$E_{a1} + \alpha E_{a2} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_fI_{a1} - \alpha^2 Z_2Z_{f0}I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

$$E_{a1} + E_{a2} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_fI_{a1} + 2Z_2Z_{f0}I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

$$E_{a1} + \alpha^2 E_{a2} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_fI_{a1} - 2Z_{f0}Z_2I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

$$I_{a1} + \alpha I_{a2} = \frac{I_{a1}\{Z_{f0}(1 - \alpha) + Z_2 + Z_f\}}{Z_{f0} + Z_2 + Z_f}$$

$$I_{a1} + I_{a2} = I_{a1}\left\{\frac{Z_2 + Z_f}{Z_{f0} + Z_2 + Z_f}\right\}$$

$$I_{a1} + \alpha^2 I_{a2} = I_{a2}\left\{\frac{(1 - \alpha^2)Z_{f0} + Z_2 + Z_f}{Z_{f0} + Z_2 + Z_f}\right\}$$

Substitution of these values in the fourth column of table II gives for the primary impedances with the potential and current transformers connected in Δ:

$$Z_{ab} = \frac{E_{a1} + \alpha E_{a2}}{I_{a1} + \alpha I_{a2}} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_f - \alpha^2 Z_2Z_{f0}}{(1 - \alpha)Z_{f0} + Z_2 + Z_f}$$

$$Z_{ab} = Z_f - \frac{\alpha^2}{\dfrac{1}{Z_{f0}} + \dfrac{1 - \alpha}{Z_2 + Z_f}} \tag{33}$$

$$Z_{bc} = \frac{E_{a1} + E_{a2}}{I_{a1} + I_{a2}} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_f + 2Z_2Z_{f0}}{Z_2 + Z_f}$$

$$= Z_f + 2Z_{f0} = (R_f + 2R_{f0}) + j(X_f + 2X_{f0}) \tag{34}$$

$$Z_{ca} = \frac{E_{a1} + \alpha^2 E_{a2}}{I_{a1} + \alpha^2 I_{a2}} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_f - \alpha Z_{f0}Z_2}{(1 - \alpha^2)Z_{f0} + Z_2 + Z_f}$$

$$= Z_f - \frac{\alpha}{\dfrac{1}{Z_{f0}} + \dfrac{1 - \alpha^2}{Z_2 + Z_f}} \tag{35}$$

With the potential transformers connected in Δ and the current transformers in Y:

$$e^{j30}E_{a1} - e^{-j30}E_{a2} = \frac{\{Z_fe^{j30}(2Z_{f0} + Z_2 + Z_f) + jZ_2Z_{f0}\}I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

$$e^{-j39}E_{a1} - e^{j30}E_{a2} = \frac{\{Ze_f^{-j30}(2Z_{f0} + Z_2 + Z_f) - jZ_2Z_{f0}\}I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

$$E_{a1} + E_{a2} = \frac{(2Z_{f0} + Z_2 + Z_f)Z_fI_{a1} + 2Z_2Z_{f0}I_{a1}}{Z_{f0} + Z_2 + Z_f}$$

$$\alpha^2 I_{a1} - \alpha I_{a2} = I_{a1}\left\{\frac{-Z_{f0} + (Z_2 + Z_f)\alpha^2}{Z_{f0} + Z_2 + Z_f}\right\}$$

$$I_{a1} - I_{a2} = I_{a1}\left\{\frac{2Z_{f0} + Z_2 + Z_f}{Z_{f0} + Z_2 + Z_f}\right\}$$

$$\alpha I_{a1} - \alpha^2 I_{a2} = I_{a1}\left\{\frac{-Z_{f0} + (Z_2 + Z_f)\alpha}{Z_{f0} + Z_2 + Z_f}\right\}$$

Substitution of these values in the last column of table II gives:

$$Z_{ab} = \frac{\sqrt{3}(e^{j30}E_{a1} - e^{-j30}E_{a2})}{I_{a1} - I_{a2}}$$

$$= \frac{\sqrt{3}Z_fe^{j30}(2Z_{f0} + Z_2 + Z_f) + jZ_2Z_{f0}\sqrt{3}}{2Z_{f0} + Z_2 + Z_f}$$

$$= \sqrt{3}e^{j30}Z_f + \frac{j\sqrt{3}Z_2}{\dfrac{Z_2 + Z_f}{Z_{f0}} + 2} \tag{36}$$

$$Z_{bc} = \frac{-j(E_{a1} + E_{a2})\sqrt{3}}{\alpha^2 I_{a1} - \alpha I_{a2}} = \frac{-j\sqrt{3}\{(2Z_{f0} + Z_2 + Z_f)Z_f + 2Z_2Z_{f0}\}}{-Z_{f0} + (Z_2 + Z_f)\alpha^2}$$

$$= \sqrt{3}e^{j30}\left\{Z_f\left(1 + \frac{\sqrt{3}e^{j30}}{\dfrac{Z_2 + Z_f}{Z_{f0}} - \alpha}\right) + \frac{2Z_2}{\dfrac{Z_2 + Z_f}{Z_{f0}} - \alpha}\right\} \tag{37}$$

$$Z_{ca} = \frac{-\sqrt{3}(e^{-j30}E_{a1} - e^{j30}E_{a2})}{\alpha I_{a1} - \alpha^2 I_{a2}} = \sqrt{3}Z_fe^{j30} + \frac{3Z_f + \sqrt{3}e^{-j30}Z_2}{\dfrac{Z_2 + Z_f}{Z_{f0}} - \alpha^2} \tag{38}$$

E—Line-to-Ground Faults Between the Relays and the Power Transformer Bank (a to Ground)

The characteristic equations for this type of fault are:

$$I_{a0} = I_{a1} = I_{a2} = \frac{E}{Z_0 + Z_{f0} + Z_1 + Z_2 + 2Z_f}$$

$$E_{a1} = E - Z_1I_{a1} = (Z_0 + Z_2 + Z_{f0} + 2Z_f)I_{a1}$$

$$E_{a2} = -Z_2I_{a2} = -Z_2I_{a1}$$

from which:

$$E_{a1} - \alpha E_{a2} = (Z_0 + Z_{f0} + 2Z_f)I_{a1} - \alpha^2 Z_2I_{a1}$$

$$E_{a1} - E_{a2} = (2Z_2 + Z_0 + Z_{f0} + 2Z_f)I_{a1}$$

$$E_{a1} - \alpha^2 E_{a2} = (Z_0 + Z_{f0} + 2Z_f)I_{a1} - \alpha Z_2 I_{a1}$$

$$I_{a1} - \alpha I_{a2} = (1 - \alpha)I_{a1}$$

$$I_{a1} - I_{a2} = 0$$

$$I_{a1} - \alpha^2 I_{a2} = (1 - \alpha^2)I_{a1}$$

$$Z_{ab} = \frac{E_{a1} - \alpha E_{a2}}{I_{a1} - \alpha I_{a2}} = \frac{1}{\sqrt{3}}\left\{(Z_0 + Z_{f0} + 2Z_f)e^{j30} + jZ_2\right\}$$

$$= \left\{0.5(R_0 + R_{f0}) + R_f - 0.29(X_0 + X_{f0}) - 0.577(X_f + X_2)\right\} + j\left\{0.5(X_0 + X_{f0}) + X_f + 0.29(R_0 + R_{f0}) + 0.577(R_f + R_2)\right\} \quad (39)$$

$$Z_{bc} = \frac{E_{a1} - E_{a2}}{I_{a1} - I_{a2}} = \infty \quad (40)$$

$$Z_{ca} = \frac{E_{a1} - \alpha^2 E_{a2}}{I_{a1} - \alpha^2 I_{a2}} = \frac{1}{\sqrt{3}}\left\{(Z_0 + Z_{f0} + 2Z_f)e^{-j30} - jZ_2\right\}$$

$$= \left\{0.5(R_0 + R_{f0}) + R_f + 0.29(X_0 + X_{f0}) + 0.577(X_f + X_2)\right\} + j\left\{0.5(X_0 + X_{f0}) + X_f - 0.29(R_0 + R_{f0}) - 0.577(R_f + R_2)\right\} \quad (41)$$

These are the primary impedances with a line-to-ground fault when the potential and current transformers are connected in Δ.

With the potential transformers in Δ and the current transformers in Y:

$$e^{j30}E_{a1} + e^{-j30}E_{a2} = (Z_0 + Z_{f0} + 2Z_f)e^{j30}I_{a1} + jZ_2$$

$$I_{a0} + I_{a1} + I_{a2} = 3I_{a1}$$

$$I_{a0} + \alpha^2 I_{a1} + \alpha I_{a2} = 0 = I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2}$$

$$Z_{ab} = \frac{\sqrt{3}(e^{j30}E_{a1} + e^{-j30}E_{a2})}{I_{a0} + I_{a1} + I_{a2}} = \frac{1}{\sqrt{3}}\left\{(Z_0 + Z_{f0} + 2Z_f)e^{j30} + jZ_2\right\}$$

$$= \left\{0.5(R_0 + R_{f0}) + R_f - 0.29(X_0 + X_{f0}) - 0.577(X_f + X_2)\right\} + j\left\{0.5(X_0 + X_{f0}) + X_f + 0.29(R_0 + R_{f0}) + 0.577(R_f + R_2)\right\} \quad (42)$$

$$Z_{bc} = \infty \quad (43)$$

$$Z_{ca} = \infty \quad (44)$$

F—Line-to-Ground Faults
Past the Power Transformer Bank (A to Ground)

The characteristic equations for this case are:

$$I_{a0} = I_{a1} = I_{a2} = \frac{E}{Z_{f0} + Z_1 + Z_2 + 2Z_f}$$

$$E_{a1} = E - Z_1 I_{a1} = (Z_2 + Z_{f0} + 2Z_f)I_{a1}$$

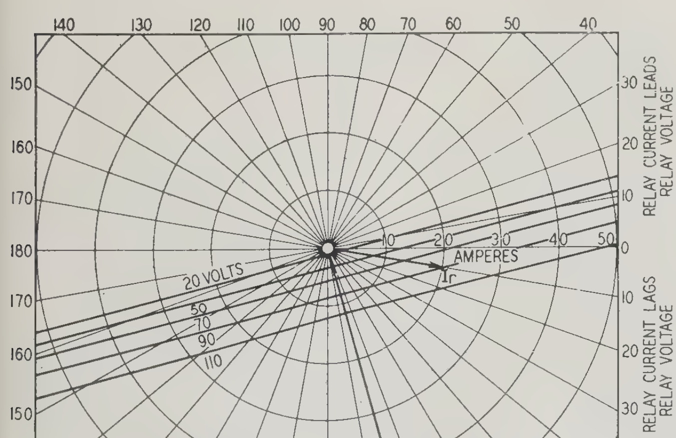

Fig. 6. Pickup characteristics of starting unit for "GAX" relay

Maximum torque when relay current lags relay voltage by 75 degrees

$$E_{a2} = -Z_2 I_2 = -Z_2 I_{a1}$$

From which:

$$E_{a1} + \alpha E_{a2} = (Z_{f0} + 2Z_f)I_{a1} + (1 - \alpha)Z_2 I_{a1}$$

$$E_{a1} + E_{a2} = (Z_{f0} + 2Z_f)I_{a1}$$

$$E_{a1} + \alpha^2 E_{a2} = (Z_{f0} + 2Z_f)I_{a1} + (1 - \alpha^2)Z_2 I_{a1}$$

$$I_{a1} + \alpha I_{a2} = -\alpha^2 I_{a1}$$

$$I_{a1} + I_{a2} = 2I_{a1}$$

$$I_{a1} + \alpha^2 I_{a2} = -\alpha I_{a1}$$

Substitution in the fourth column of table II gives:

$$Z_{ab} = \frac{E_{a1} + \alpha E_{a2}}{I_{a1} + \alpha I_{a2}} = (Z_{f0} + 2Z_f)e^{-j60} - j\sqrt{3}Z_2$$

$$= \left\{R_f + 1.73(X_f + X_2) + 0.5R_{f0} + 0.866X_{f0}\right\} + j\left\{X_f - 1.73(R_f + R_2) + 0.5X_{f0} - 0.866R_{f0}\right\} \quad (45)$$

$$Z_{bc} = \frac{E_{a1} + E_{a2}}{I_{a1} + I_{a2}} = Z_f + \frac{Z_{f0}}{2} = \left(R_f + \frac{R_{f0}}{2}\right) + j\left(X_f + \frac{X_{f0}}{2}\right) \quad (46)$$

$$Z_{ca} = \frac{E_{a1} + \alpha^2 E_{a2}}{I_{a1} + \alpha^2 I_{a2}} = (Z_{f0} + 2Z_f)e^{j60} + j\sqrt{3}Z_2$$

$$= \left\{R_f - 1.73(X_f + X_2) + 0.5R_{f0} - 0.866X_{f0}\right\} + j\left\{X_f + 1.73(R_f + R_2) + 0.5X_{f0} + 0.866R_{f0}\right\} \quad (47)$$

These are the primary impedances with a line-to-ground fault past the power bank when the potential and current transformers are connected in Δ.

With the potential transformer connected in Δ and the current transformers connected in Y:

$$e^{j30}E_{a1} - e^{-j30}E_{a2} = \left\{(Z_{f0} + 2Z_f)e^{j30} + \sqrt{3}Z_2\right\}I_{a1}$$

$$E_{a1} + E_{a2} = (Z_{f0} + 2Z_f)I_{a1}$$

$$e^{-j30}E_{a1} - e^{j30}E_{a2} = \left\{e^{-j30}(Z_{f0} + 2Z_f) + \sqrt{3}Z_2\right\}I_{a1}$$

$$I_{a1} - I_{a2} = 0$$

$$\alpha^2 I_{a1} - \alpha I_{a2} = -\sqrt{3}jI_{a1}$$

$$\alpha I_{a1} - \alpha^2 I_{a2} = \sqrt{3}jI_{a1}$$

$$Z_{ab} = \frac{\sqrt{3}(e^{j30}E_{a1} - e^{-j30}E_{a2})}{I_{a1} - I_{a2}} = \infty \quad (48)$$

$$Z_{bc} = \frac{-j(E_{a1} + E_{a2})\sqrt{3}}{\alpha^2 I_{a1} - \alpha I_{a2}} = Z_{f0} + 2Z_f = (R_{f0} + 2R_f) + j(X_{f0} + 2X_f) \quad (49)$$

$$Z_{ca} = \frac{-\sqrt{3}(e^{-j30}E_{a1} - e^{j30}E_{a2})}{\alpha I_{a1} - \alpha^2 I_{a2}} = (Z_{f0} + 2Z_f)e^{6j0} + j\sqrt{3}Z_2$$

$$= \left\{R_f - 1.73(X_f + X_2) + 0.5R_{f0} - 0.866X_{f0}\right\} + j\left\{X_f + 1.73(R_f + R_2) + 0.5X_{f0} + 0.866R_{f0}\right\} \quad (50)$$

Appendix III—Performance of the Directional Elements

In the 2 preceding appendixes formulas are given for calculating the relay primary ohms under various fault conditions. No mention is made there of the directional elements. However, no analysis of distance relays would be complete that did not take into consideration the performance of the directional elements.

The primary voltages and currents acting on the directional elements of the 2 types of distance relays most commonly used in the United States, the "HZ" and the "GAX," are given in table I.

The directional element of the "HZ" relays has true wattmeter characteristics and requires very little energy to operate. For this reason, in order to check its performance under any specific condition, it will suffice to determine the angle between its primary voltage and its primary current; when this angle is less than ±90 degrees the directional element will close its contacts, otherwise it will not. It may be noted that the angle in question is equal to

Table IV—Ratios for Calculating the Phase Angle Difference Between the Primary Voltages and Currents Acting on the Directional Elements of the Impedance Relays*

Relay		Potential and Current Transformers Connected in Δ			Potential Transformers Connected in Δ; Current Transformers Connected Y		
Number (Figures 1,2,3)	Ohm Element		No Grounded Neutral Δ/Y or Y/Δ Power Bank Interposed Between the Relays and the Fault	Grounded Neutral Δ/Y or Y/Δ Power Bank Interposed Between the Relays and the Fault		No Grounded Neutral Δ/Y or Y/Δ Power Bank Interposed Between the Relays and the Fault	Grounded Neutral Δ/Y or Y/Δ Power Bank Interposed Between the Relays and the Fault
1	Z_{ab}	$\frac{E_a - E_c}{I_a - I_b} =$	$\frac{E_{a1}e^{-j60} + E_{a2}}{I_{a1} + I_{a2}e^{-j60}}$	$\frac{E_{a1}e^{-j60} - E_{a2}}{I_{a1} - I_{a2}e^{-j60}}$	$\frac{E_a - E_c}{I_a}$	$\frac{\sqrt{3}(E_{a1}e^{-j30} + E_{a2}e^{j30})}{I_{a0} + I_{a1} + I_{a2}}$	$\frac{\sqrt{3}(E_{a1}e^{-j30} - E_{a2}e^{j30})}{I_{a1} - I_{a2}}$
2	Z_{bc}	$\frac{E_b - E_a}{I_b - I_c} =$	$\frac{E_{a1}e^{-j60} - E_{a2}e^{j60}}{I_{a1} - I_{a2}}$	$\frac{E_{a1}e^{-j60} + E_{a2}e^{-j60}}{I_{a1} + I_{a2}}$	$\frac{E_b - E_a}{I_b}$	$\frac{\sqrt{3}(E_{a1}e^{-j30} - jE_{a2})}{\alpha I_{a0} + I_{a1} + \alpha^2 I_{a2}}$	$\frac{\sqrt{3}(E_{a1}e^{-j30} + jE_{a2})}{I_{a1} - \alpha^2 I_{a2}}$
3	Z_{ca}	$\frac{E_c - E_b}{I_c - I_a} =$	$\frac{e^{-j60}(E_{a1} - E_{a2})}{I_{a1} + I_{a2}e^{j60}}$	$\frac{e^{-j60}(E_{a1} + E_{a2})}{I_{a1} - I_{a2}e^{j60}}$	$\frac{E_c - E_b}{I_c}$	$\frac{\sqrt{3}e^{-j30}(E_{a1} - E_{a2})}{\alpha^2 I_{a0} + I_{a1} + \alpha I_{a2}}$	$\frac{\sqrt{3}e^{-j30}(E_{a1} + E_{a2})}{I_{a1} - \alpha I_{a2}}$

* Where the angle of the ratio is positive the voltage leads, otherwise it lags the current.

the angle of the vector ratio between the primary voltage across, and the primary current through, the directional element and therefore may be calculated from table IV, where the ratios of the voltages to the currents acting on the directional elements have been tabulated in terms of sequence components.

The formulas of table IV are general and may be used in any case irrespective of the complexity of the network; in applying them, the necessary sequence components must be calculated disregarding the effect of the connections of the power bank in much the same manner as explained for table II.

In the case of the "GAX" relay, the directional (starting) element does not operate unless the relay current is larger than a certain value that varies with the magnitude and phase angle of the relay voltage.

The starting characteristics of the starting unit of this relay, adjusted to obtain maximum torque when the relay current lags the relay voltage by 75 degrees, are shown in figure 6. With reference to this figure, for any relay voltage, the starting element will trip if the terminal of the vector representing the relay current falls to the right of the operating line for the particular voltage applied at the relay terminals. Thus, if the relay current $\boldsymbol{I}_r$ should be 20 amperes and should lag the relay voltage by 10 degrees, the starting unit would trip if the latter were smaller than 90 volts, and would not trip if it were larger. From figure 6 the starting characteristics of the starting unit may be obtained for maximum torque at angles other than 75 degrees. All that is necessary to do is to rotate the operating lines (which in figure 6 are perpendicular to the line drawn from the origin at an angle of 75 degrees with respect to the voltage axis) until they become perpendicular to a line drawn from the origin at an angle equal to the angle at which maximum torque is desired.

The starting characteristics of figure 6 may be expressed differently by saying that for a given relay voltage the starting unit will operate if the projection of the current vector, on the line of maximum torque, is larger than a certain value I_m equal to the current intercepted, on the line of maximum torque, by the operating line for that voltage. That is, the starting unit will operate if

$$I_r \cos(\theta - \varphi) > I_m \tag{51}$$

Where I_r is the magnitude of the relay current, θ is the angle of maximum torque (75 degrees in figure 6), and φ is the angle by which the relay current lags the relay voltage. From table I, it may easily be seen that φ coincides with the angle of the primary impedance (for the element under consideration) as calculated in appendixes I and II and tabulated in tables II and III.

The variation of I_m with the voltage is shown in figure 7. The starting unit will operate if, in correspondence of the voltage E_r existing at the relay, $I_r \cos(\theta - \varphi)$ is positive and above the curve of figure 7.

By multiplying both members of equation 51 by E_r, the relay voltage, the starting characteristics of the "GAX" relay could be expressed in terms of power, which in turn could be calculated in terms of the sequence components of the primary voltages and currents. The relations obtained would, however, be too complicated to be of any real practical value. In practice, it will probably be found more convenient to check the performance of the starting unit directly from figure 6 or figure 7 after computing the relay current and voltage, with the aid of the formulas given in appendixes I and II, and their phase angle difference, from the expressions of $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$ given in tables II and III.

Appendix IV—Numerical Example

Further to illustrate the use of the formulas given in the 3 preceding appendixes and tabulated in tables I, II, and III, they will be applied to the case of a transmission line of the system with which the author is associated. In this particular case, "GAX" reactance relays are used. Their location and the impedances of the various circuits to currents of positive, negative, and zero sequence are shown in figure 8. The generating capacity connected to the H bus varies under different operating conditions. An overhead transmission line is connected to the bus D. The impedances of this transmission line to currents of different sequences, in per cent per mile at 100,000 kva are shown in figure 8.

Under 2 different operating conditions $\boldsymbol{Z}_1$, $\boldsymbol{Z}_2$, $\boldsymbol{Z}_0$ have the following values:

	Operating Conditions A	Operating Conditions B
$\boldsymbol{Z}_1$ (per cent at 100,000 kva)....	0.17 + j8.41....	0.17 + j17.21
$\boldsymbol{Z}_2$ (per cent at 100,000 kva)....	0.17 + j8.41....	0.17 + j17.21
$\boldsymbol{Z}_0$ (per cent at 100,000 kva)....	0.17 + j7.51....	0.17 + j18.71

The current and potential transformers are connected in Δ and the settings of the relays at r in per cent at 100,000 kva are as follows:

First step..	6.2 per cent
Second step......................................	10.38 per cent
Third step..	not used

In this case, with a line-to-ground fault at one of the terminals of the power transformer, neglecting the effect of the arc impedance, noting that $\boldsymbol{Z}_f = \boldsymbol{Z}_{f0} = 0$, and assuming no supply from the further end of the overhead line, from the fourth column of table III, the primary impedances in per cent at 100,000 kva may be calculated at:

Relay	Primary Impedance	Operating Conditions A	Operating Conditions B
1........................	$\boldsymbol{Z}_{ab}$ =	$-6.94 + j3.9$ (= $7.94e^{j119\text{-}40}$)	$-15.28 + j9.5$ (= $18e^{j121\text{-}45}$)
2........................	$\boldsymbol{Z}_{bc}$ =	∞	∞
3........................	$\boldsymbol{Z}_{ca}$ =	$7.12 + j3.60$ (= $8.0e^{j26\text{-}48}$)	$15.46 + j9.21$ (= $18.9e^{j30\text{-}50}$)

With line-to-ground faults at this location, the secondary current flowing in relay 1 has the same magnitude as the current flowing in relay 3. Calculated with the aid of the formulas given in appendix II and the ratio of transformation shown in figure 8, this current is found to be 20.5 amperes under operating conditions B and 44.9

amperes under operating conditions A. The primary voltage across relay 1 ($\boldsymbol{Z}_{ab}$) leads the relay primary current by 119°40′ under operating condition A and by 121°45′ under operating conditions B. The primary voltage across relay 3 ($\boldsymbol{Z}_{ca}$) leads the relay primary current by 26°48′ under operating conditions A and by 30°50′ under operating conditions B. From the curves of figure 6 it is seen that the starting unit of both relays will operate in both cases. Under the assumed conditions, neglecting arc resistance, a line-to-ground fault at the low voltage terminals of the transformer would, therefore, cause relays 1 and 3 to operate in their first step under the operating conditions A and in their second step under the operating conditions B.

The numerical values obtained for $\boldsymbol{Z}_{ab}$ and $\boldsymbol{Z}_{ca}$ for the fault under consideration show clearly that the relay ohm indications are closely dependent upon the operating conditions as reflected in the values of $\boldsymbol{Z}_0$ and $\boldsymbol{Z}_2$. The formulas of table III show also their dependence upon arc impedance and the impedance to zero sequence currents. For these reasons, as it is well known, distance relays cannot be relied upon to protect against line-to-ground faults. In the case considered, however, they would protect for faults at the specified location, should these occur under the assumed conditions or their equivalents.

Consider now the case with faults past the transformer bank.

With 3 phase faults, the relays reach partially into the transformer bank with their first step. Their second step reaches 2.85 miles into the overhead line. In fact for a fault at this location the primary impedances $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$ of table III become:

$$\boldsymbol{Z}_{ab} = \boldsymbol{Z}_{bc} = \boldsymbol{Z}_{ca} = (1.95 + j10.37) \text{ per cent at 100,000 kva}$$

With line-to-line faults (B to C),

$$R_f = 0.97 + 0.78 + 0.069x = 1.75 + 0.069x$$
$$X_f = 7.04 + 2.13 + 0.421x = 9.17 + 0.421x$$

x is the length, in miles, of overhead line between the substation D and the fault.

Under the operating conditions A, ($\boldsymbol{Z}_2 = 0.17 + j8.41$), the primary impedances assume the following values:

$$\begin{aligned}\boldsymbol{Z}_{ab} &= \{1.75 + 0.069x - 0.577(9.17 + 0.421x + 8.41)\} + \\ &\qquad j\{9.17 + 0.421x + 0.577(1.75 + 0.069x + 0.17)\} \\ &= -(8.40 + 0.174x) + j(10.275 + 0.461x) \\ \boldsymbol{Z}_{bc} &= \infty \\ \boldsymbol{Z}_{ca} &= \{1.75 + 0.069x + 0.577(9.17 + 0.421x + 8.41)\} + \\ &\qquad j\{9.17 + 0.421x - 0.577(1.75 + 0.069x + 0.17)\} \\ &= (11.90 + 0.312x) + j(8.065 + 0.381x)\end{aligned}$$

On the basis of the reactances that would be indicated by the ohm units, from the above expressions, it is seen that under the assumed conditions, the second step of relay 3 ($\boldsymbol{Z}_{ca}$) would reach 6.05 miles into the overhead line. In fact, for $x = 6.05$, the reactance component of $\boldsymbol{Z}_{ca}$ is 10.38 at 100,000 kva.

However, before saying definitely whether or not the relay will operate, the performance of the starting unit must be checked. From the formulas given in appendix II for this type of fault, and the current transformer ratio shown in figure 8, the voltage and current acting on the relay may be calculated at 80.5 volts and 27 amperes the former leading the latter by 37 degrees. Reference to figure 6 will show that the starting element will operate. Thus it may be concluded that under the assumed operating condition the second step of the relays reaches 6.05 miles into the overhead line.

For faults on the cable under the same operating conditions:

$$R_f = 0.97 + 0.0708x \qquad X_f = 7.04 + 0.1935x$$

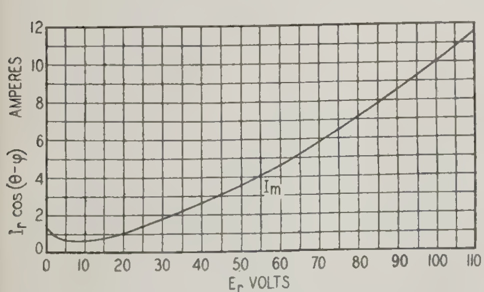

Fig. 7. Pickup characteristics of starting unit for standard type "GAX" relay

Above curve, starting unit picks up; below curve, starting unit does not pick up

where x is the distance in miles between the power bank and the fault. The primary impedances may be calculated at:

$$\begin{aligned}\boldsymbol{Z}_{ab} &= 0.97 + 0.0708x - 0.577(7.04 + 0.1935x + 8.41) + \\ &\qquad \{j\,7.04 + 0.1935x + 0.577(0.97 + 0.0708x + 0.17)\} \\ &= 0.97 + 0.0708x - 8.95 - 0.1115x + \\ &\qquad j\{7.04 + 0.1935x + 0.66 + 0.0409x\} \\ &= -(7.98 + 0.0407x) + j(7.7 + 0.2344x) \\ \boldsymbol{Z}_{bc} &= \infty \\ \boldsymbol{Z}_{ca} &= 0.97 + 0.0708x + 8.95 + 0.1115x + \\ &\qquad j(7.04 + 0.1935x - 0.66 - 0.0409x) \\ &= 9.92 + 0.1823x + j(6.38 + 0.1526x)\end{aligned}$$

From which by putting $x = 0$ it may be seen that relay 3 ($\boldsymbol{Z}_{ca}$) will trip with a line-to-line fault on the 132 kv terminals of the transformer, that is, the relays protect the transformer bank with their first step, instead of only a portion of it as under 3 phase fault conditions. For line-to-line faults on the cable ($0 < x \leq 11$) both relays 1 and 3 ($\boldsymbol{Z}_{ab}$ and $\boldsymbol{Z}_{ca}$) will trip in their second step. For faults on the overhead line past the substation D, only relay 3 ($\boldsymbol{Z}_{ca}$) will trip, reaching 6.05 to the right of D as noted on the preceding page.

Still with a line-to-line fault past the transformer bank, under operating conditions B ($\boldsymbol{Z}_2 = 0.17 + j17.21$):

Faults on the overhead line:

$$\begin{aligned}\boldsymbol{Z}_{ab} &= -(13.48 + 0.174x) + j(10.275 + 0.461x) \\ \boldsymbol{Z}_{bc} &= \infty \\ \boldsymbol{Z}_{ca} &= (11.92 + 0.312x) + j(8.065 + 0.38x) + 5.08 \\ &= (17.0 + 0.312x) + j(8.065 + 0.38x)\end{aligned}$$

For $x = 6.05$ miles

$$\begin{aligned}\boldsymbol{Z}_{ca} &= 18.89 + j10.38 = 21.6e^{j28\text{-}40} \\ R_f &= 1.75 + 0.417 = 2.167 \\ X_f &= 9.17 + 2.55 = 11.72 \\ \boldsymbol{Z}_1 &= \boldsymbol{Z}_2 = 0.17 + j17.21 \\ \boldsymbol{Z}_1 + \boldsymbol{Z}_2 + 2\boldsymbol{Z}_f &= 2(0.17 + j17.21 + 2.167 + j11.72) \\ &= 2(2.34 + j28.93) = 58.0e^{j85\text{-}23}\end{aligned}$$

From equation 16 of appendix II:

$$\boldsymbol{I}_{a1} = -\boldsymbol{I}_{a2} = \frac{4370 \times 100}{58e^{j85\text{-}23}} = 7550e^{-j85\text{-}23}$$

from which the primary current for relay 3 ($\boldsymbol{Z}_{ca}$) is calculated at:

$$\sqrt{3}\alpha^2(\boldsymbol{I}_{a1} + \alpha^2\boldsymbol{I}_{a2}) = \sqrt{3}\alpha^2\boldsymbol{I}_{a1}(1 - \alpha^2) = -3e^{j90}\boldsymbol{I}_{a1} = 22650e^{j4\text{-}37}$$

On the basis of the current transformer ratio given in figure 8, and the calculated phase angle of $\boldsymbol{Z}_{ca}$, the secondary current in relay 3 ($\boldsymbol{Z}_{ca}$) is 18.9 amperes lagging the relay voltage by 28°40′. From figure 6 it is seen that with a current of this magnitude and phase angle, the starting unit of this relay will operate irrespective of the

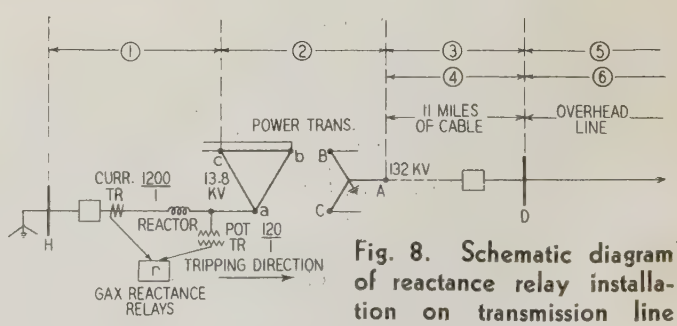

Fig. 8. Schematic diagram of reactance relay installation on transmission line

Figures for per cent impedance at 100,000 kva are as follows:

1. 0.17 + j2.11; positive, negative, and zero sequence
2. 0.97 + j7.04; positive, negative, and zero sequence
3. 0.78 + j2.13; positive and negative sequence
4. 2.42 + j1.73; zero sequence
5. 0.069 + j0.421; per cent per mile positive and negative sequence
6. 0.386 + j1.175; per cent per mile zero sequence

voltage magnitude. Thus the distance protected in this case is the same as under operating conditions A. This is because the resistance term of the impedance $\boldsymbol{Z}_2$ has been assumed the same for both operating conditions, and the change in reactance does not prevent the starting unit from operating. However, if the resistance term of $\boldsymbol{Z}_2$ should not remain constant, causing a variation of the reactance term of $\boldsymbol{Z}_{ca}$, or if the change of the reactance term of $\boldsymbol{Z}_2$ should be large enough to cause the starting unit not to operate, the distance protected would vary with the operating condition.

Under operating conditions B and line-to-line faults in the cable:

$$\boldsymbol{Z}_{ab} = -(13.06 + 0.0407x) + j(7.7 + 0.2344x)$$
$$\boldsymbol{Z}_{bc} = \infty$$
$$\boldsymbol{Z}_{ca} = 15.0 + 0.1823x + j(6.38 + 0.1526x)$$

That is, the distance protected by the first step of the reactance relays remains the same as under operating conditions A as could have been foreseen from the preceding discussion.

It has been stated that with 3 phase faults the balance point of the second step of the relays is 2.85 miles into the overhead line at the right of the substation D. With a fault between phases B and C and ground at about the same location, the relay primary impedances under the 2 specified operating conditions, neglecting the arc impedance, are shown in table V. From this table it is seen that under the assumed conditions, relays 1 and 2 cannot trip. As to relay 3, its primary ohm indication is slightly above the second step under operating condition A and more so under operating condition B. The starting unit will operate under both operating conditions, as may be easily checked with the aid of the formulas given in appendix II under item D. Thus, under operating condition A, the relay protects slightly less than or about the same distance as with 3 phase faults. Under operating conditions B, the relay protects decidedly less than with 3 phase faults. In both cases it protects less than with line-to-line faults. Any deviation in the value of the impedance to zero sequence currents $\boldsymbol{Z}_{f0}$, or any change in the operating conditions causing a change in the value of $\boldsymbol{Z}_2$, would affect the distance protected by the relays in the measure indicated by the formulas of table III, as the arc impedance would if it were different from zero.

With a line-to-ground fault on the overhead line under the operating conditions A:

$$\boldsymbol{Z}_2 = 0.17 + j8.41$$
$$\boldsymbol{Z}_f = 0.97 + j7.04 + 0.78 + j2.13 + (0.069 + j.421)x$$
$$= 1.75 + 0.069x + j(9.17 + 0.421x)$$
$$R_f = 1.75 + 0.069x$$
$$X_f = 9.17 + 0.421x$$
$$\boldsymbol{Z}_{f0} = 0.97 + j7.04 + 2.42 + j1.73 + (0.386 + j1.175)x$$
$$= 3.39 + j8.77 + 0.386x + j1.175x$$
$$R_{f0} = 3.38 + 0.386x$$
$$X_{f0} = 8.77 + 1.175x$$
$$\boldsymbol{Z}_{ab} = \{1.75 + 0.069x + 1.73\,(9.17 + 0.421x + 8.41) + 0.5\,(3.39 + 0.386x) + 0.866(8.77 + 1.175x)\} + j\{9.17 + 0.421x - 1.73(1.75 + 0.069x + 0.17) + 0.5\,(8.77 + 1.175x) - 0.866\,(3.39 + 0.386x)\}$$
$$= 41.445 + 2.012x + j(7.305 + 0.556x)$$
$$\boldsymbol{Z}_{bc} = 3.445 + 0.262x + j(13.555 + 1.009x)$$
$$\boldsymbol{Z}_{ca} = -(34.555 + 1.49x) + j(19.8 + j1.462x)$$

On the basis of the reactance indicated by the ohm units, the above expressions of $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{bc}$, $\boldsymbol{Z}_{ca}$, show that with a line-to-ground fault under the specified conditions, relay 1 ($\boldsymbol{Z}_{ab}$) would reach 5.5 miles into the overhead line. Of course, the complete analysis of the performance of each relay requires that the performance of the starting unit be analyzed. The calculated voltage and current acting on relay 1 ($\boldsymbol{Z}_{ab}$) with a line-to-ground fault at this location are, respectively, 105.3 volts and 11.3 amperes, the former leading the latter by 11°10′. With these values, and the starting unit set for maximum torque when the relay current lags the relay voltage by 75 degrees (figure 6), the "GAX" relay would not trip. However, it would trip if the relays were set for maximum torque at a smaller angle, in the neighborhood of 45 degrees lag.

Table V—B-to-C-to-Ground Fault 2.8 Miles to the Right of Substation D of Figure 8

	Operating Conditions A	Operating Conditions B
$\boldsymbol{Z}_1 = \boldsymbol{Z}_2$% at 100,000 kva	$0.17 + j8.41$	$0.17 + j17.21$
$\boldsymbol{Z}$ % at 100,000 kva	$1.94 + j10.35$	$1.94 + j10.35$
$\boldsymbol{Z}_{f0}$% at 100,000 kva	$4.47 + j12.06$	$4.47 + j12.06$
Relay 1.........$\boldsymbol{Z}_{ab}$% at 100,000 kva	$-3.77 + j13.18$	$-4.76 + j14.22$
Relay 2.........$\boldsymbol{Z}_{bc}$% at 100,000 kva	$10.88 + j34.47$	$10.88 + j34.47$
Relay 3.........$\boldsymbol{Z}_{ca}$% at 100,000 kva	$7.93 + j10.455$	$9.11 + j10.762$

Under operating condition B ($\boldsymbol{Z}_2 = 0.17 + j17.41$) and a line-to-ground fault on the overhead line it is obtained:

$$\boldsymbol{Z}_{ab} = 71.245 + 2.012x + j(7.305 + 0.556x)$$
$$\boldsymbol{Z}_{bc} = 3.445 + 0.262x + j(13.555 + 1.009x)$$
$$\boldsymbol{Z}_{ca} = -(64.355 + 1.49x) + j(19.8 + j1.462x)$$

The reactance components of $\boldsymbol{Z}_{ab}$, $\boldsymbol{Z}_{ca}$ are the same as under operating condition A, due to the fact that the resistance component of $\boldsymbol{Z}_2$ has been assumed the same for both operating conditions. However, on account of the fact that the angle by which the primary relay voltage leads the primary relay current is small (angle of $\boldsymbol{Z}_{ab}$) the starting unit of relay 1 will not operate, not even if the fault is at the substation D. It may be of interest to note that if the third step had been used, relay 2 ($\boldsymbol{Z}_{bc}$) would have reached several miles into the overhead line.

With a line-to-ground fault on the cable still under operating conditions A ($\boldsymbol{Z}_2 = 0.17 + j8.41$):

$$R_f = 0.97 + 0.0708x$$
$$X_f = 7.04 + 0.1935x$$
$$R_{f0} = 0.97 + 0.22x$$
$$X_{f0} = 7.04 + 0.157x$$
$$\boldsymbol{Z}_{ab} = (34.26 + 0.652x) + j(7.745 - 0.0415x)$$
$$\boldsymbol{Z}_{bc} = 1.455 + 0.1808x + j(10.6 + 0.272x)$$
$$\boldsymbol{Z}_{ca} = -(31.34 + 0.2892x) + j(13.375 + 0.586x)$$

With a line-to-ground fault at the 132 kv terminal of the transformer bank:

$$\boldsymbol{Z}_1 = \boldsymbol{Z}_2 = 0.17 + j8.41 \text{ per cent at 100,000 kva}$$
$$\boldsymbol{Z}_f = \boldsymbol{Z}_{f0} = 0.97 + j7.04$$
$$\boldsymbol{Z}_1 + \boldsymbol{Z}_2 + 2\boldsymbol{Z}_f + \boldsymbol{Z}_{f0} = 3.25 + j37.94 = 36.0e^{j85\text{-}5}$$
$$\boldsymbol{I}_{a0} = \boldsymbol{I}_{a1} = \boldsymbol{I}_{a2} = \frac{4{,}370 \times 100}{36}\,e^{-j85\text{-}5} = 12{,}150e^{-j85\text{-}5}$$

Relay 1 ($\boldsymbol{Z}_{ab}$) primary current:

$$\sqrt{3}\alpha(\boldsymbol{I}_{a1} + \alpha\boldsymbol{I}_{a2}) = -\sqrt{3}\boldsymbol{I}_{a1} = -21{,}000e^{-j85\text{-}5}$$

Magnitude of relay secondary current, $\dfrac{21{,}000}{1{,}200} = 17.5$ amperes

Relay primary impedance

$$\boldsymbol{Z}_{ab} = (34.26 + j7.745) \text{ per cent at 100,000 kva}$$
$$= 35.7e^{j12\text{-}35} \text{ per cent at 100,000 kva}$$

Magnitude of relay secondary voltage

$$\frac{21{,}000}{4{,}370} \times 35.7 = 171 \text{ per cent or } 1.17 \times 63.5 = 99 \text{ volts}$$

As this voltage leads the relay current by 12°35′, from figure 6, it is seen that the starting unit when set for maximum angle at 75 degrees, will not operate. It would operate if it were set for maximum angle at 45 degrees. Again it may be noted that if the third step of the relays had been used, relay 2 ($\boldsymbol{Z}_{bc}$) would have tripped on this step with line-to-ground faults on the cable.

References

1. Method of Symmetrical Co-ordinates Applied to the Solution of Polyphase Networks, C. L. Fortescue. A.I.E.E. Trans., v. 37, pt. 2, 1918, p. 1027–1115.

2. The Application of Symmetrical Co-ordinates to the Solution of Electrical Networks, G. Calabrese. *L'Elettrotecnica*, v. 15, Apr. 25, 1928, p. 327–33.

3. Symmetrical Components Applied to the Analysis of Unbalanced Electrical Circuits (a book), C. F. Wagner and R. D. Evans. McGraw-Hill Book Company, New York.

4. Westinghouse and General Electric Company publications on the subject of distance relays.

Vibration of Cables and Dampers—II

An analysis of the vibration of cables and dampers, representing the results of several years of research and laboratory and field testing, is presented in this paper. In Part I which was published in the May 1936 issue, an explanation of the causes and nature of free harmonic vibration of cable is offered, followed by an analysis of the resulting stresses. In Part II, presented herewith, various ways of reducing the maximum stresses and means of controlling the vibration are discussed, including an analysis of the action of Stockbridge dampers. Applications of the formulas to specific cases are given, and comparisons are made with experimental data from laboratory and field.

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THERE are 2 general types of methods of preventing fatigue failures in cables resulting from stresses developed in service. The total stresses in a cable result from combinations of stresses existing in the strands before stringing, those set up during stringing, and the stresses caused by vibration. The first type of prevention of fatigue failures to be discussed is the protection of the cable against the deleterious effects of vibration without trying to lessen it; the second type is the damping of the vibration to the extent that it will not be serious.

The first of these 2 general types of methods may be divided further into 2 distinct methods. One is to reduce the stiffness or fixation of the cable or its connections, thereby reducing the stresses resulting from vibration. The second is to add a protective covering to the cable in the region of high stress, with the aim of having this protective covering carry most of the oscillating stresses while the cable carries only the direct tension.

Equation 51 (equations 1 to 71, inclusive, are given in part I of this paper) gives an expression for the bending moment at the fixed end of a cable under the dead weight of the cable, and equation 58a gives an expression for bending moment at the end of the cable after the clamped end is permitted to rotate by an amount β. It may be noticed that if the end be permitted to rotate sufficiently a condition of zero moment may be obtained. For vibrating stresses, equation 51a gives the bending moment at the clamped end of a cable assuming the ends held rigidly in position, and equation 63 gives an expression for the bending moment that would occur if the end were free to oscillate by an amount β_1. Again it may be seen that if the end of the cable be permitted to rotate sufficiently without restraint it is possible for a true node to occur at the clamp and consequently the bending moment at the clamped end again would be zero.

In studying actual cases it is necessary to consider the behavior of a center span of several suspension spans. Here practical difficulties arise in attempting to bring about hinged conditions, although such conditions might appear to be advantageous. In order to obtain a true hinged end effect, adjacent spans must be synchronized exactly so that the end loop in one span is in its uppermost position while the end loop in the other is in its lowest position. If they are both up or down at the same time the end conditions are nearly fixed. Field observations on the angular oscillation of supporting clamps will give data on the amount of fixity that might be expected in practice from various types of supporting clamps. Wright and Mini[7] have made some observations along this line which serve as a start toward collecting such data.

Bellmouthed clamps may be used, which will tend to reduce the stresses at the clamped ends of cables provided the radius of the bell mouth is greater than the radius on which the cable naturally would tend to bend. The limiting stress then at a bellmouthed clamp would be that stress produced by bending the cable over a radius equal to the radius of the bell mouth. If R is the radius of the bell mouth, the maximum bending moment that can exist in a cable may be computed from

$$M = \frac{EI}{R} \tag{72}$$

The stress may be found from equation 52 and generalized as

$$S = \frac{McE_0}{3EI} \tag{52b}$$

for a 7 strand cable. Thus by properly designing bellmouthed clamps, the maximum stress variations may be reduced appreciably. While it is not practicable to eliminate all possibility of failure by this means, recent trends in design of clamps indicate attempts to minimize the residual stresses during vibration.

A paper recommended for publication by the A.I.E.E. committee on power transmission and distribution, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted Jan. 17, 1936; released for publication Apr. 21, 1936.

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7. For all numbered references, see list at end of paper.

The second means of protecting the cable against stresses is that of adding material at the points of maximum moment, the outstanding example of which is the use of armor rods. In general, the cable is strung first and the armor rods are applied after the cable is in place. In this way it is assured that the cable carries practically all the direct tension. The armor rods are wrapped tightly around the cable and extend a considerable distance on either side of the supporting clamp. The resulting effect is that the strands of the armor rods which overlay the cable will make this portion of the cable considerably stiffer with the result that any curvature caused by bending will tend to be less sharp, thereby causing less bending stress in the cable itself. Tests are now in progress to determine from actual measurements the extent to which such amelioration of the stress condition is effected by armor rods.

In a recent conference on cable vibration[8] a report on the commercial use of armor rods was given. This report indicates that armor rods provide a very substantial protection to the strands of actual conductors in service.

The second general type of prevention of fatigue failures, which is the damping of vibration to control its amplitude, is discussed in the following section of the paper.

Control of Vibration

Since the phenomenon of vibration is the result of a condition of resonance between the eddy frequencies of the air currents passing over the cable and the natural frequencies of the cable itself, a study of the nature of the building up process of vibration is essential. Ernest Bate[9] of Australia has formulated an expression for the energy transmitted to a vibrating cable by the wind. This formula may be written

$$U = 0.000022 V^2 D A L' \qquad (73)$$

where

U = energy input per loop per cycle in foot pounds
V = velocity of wind in miles per hour
D = outside diameter of the cable in inches
A = amplitude of vibration in inches
L' = loop length or distance between node points in feet

Thus it may be seen that the energy input into any given cable vibrating at a given frequency varies directly as the amplitude.

From tests made on the deflection of cables, it has been found that a hysteresis loop always occurs when a cable is deflected and the load removed. It has been found also that the width of this hysteresis loop is nearly proportional to the maximum displacement. Furthermore, the area of the hysteresis loop is a direct measure of the energy dissipated and may be represented by a constant times the product of the width of the hysteresis loop and the maximum displacement. Therefore, the energy dissipated in deflecting the cable varies as the square of the maximum displacement. If Bate's formula is correct, then it must follow that when the amplitude is very small the energy input from the wind will exceed the energy dissipated by the cable, but that as the amplitude increases the energy dissipated by the cable will reach a magnitude equal to the energy input from the wind. Thus a limiting amplitude will exist for every set of conditions. The cable will start to vibrate with a very small amplitude at first, and then the amplitude rapidly will increase to a maximum value but will not go beyond this maximum. The fact that this condition has been observed quite universally tends to show that

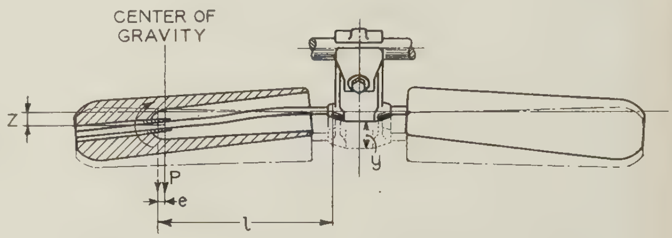


Fig. 5. Diagram of Stockbridge cable vibration damper

Bate's formula is qualitatively correct as far as amplitude is concerned. It follows then that for vibrating cables there is a natural control of the amplitude and consequently of the vibratory stresses. Information available at present is not sufficient to estimate the maximum amplitudes of vibration for commercial sizes of cables under field conditions. If the hysteresis characteristics for cables of commercial sizes at various tensions were known, the maximum amplitudes of vibration could be predicted quite definitely.

When armor rods are used the cable is stiffened effectively in the vicinity of the clamped ends and the natural frequency of the vibration for a given loop length of the cable near the supports will tend to be increased. The added weight of the armor rods, however, will tend to decrease the natural frequency. Since the frequency of the end loop is determined by the frequency of the loops in the span, there must be a change in the loop length to offset the effects of the armor rods. The loop length may become longer or shorter depending upon whether the effect of the stiffness or the effect of the weight is the greater. The armor rods also cause an increased dissipation of energy in the vicinity of the clamps resulting from the disturbing forces necessary for the cable to flex the armor rods. It has been found from laboratory and field tests that armor rods will decrease the amplitude of vibration as much as 20 per cent. This reduction of amplitude aids in reducing stresses.

If dead weights, such as balls of lead, be attached rigidly to the cable at points arranged so as to fall within the normal loop lengths of vibration, that is, at prime factors of the loop lengths, the normal vibration of the cable again is disturbed and a greater amount of energy will be dissipated for a given amplitude with dead weights than without dead weights. Thus a material reduction in amplitude of vibration would be brought about by the addition of properly sized and spaced weights. No tests have

been reported to date to show, quantitatively, the effect of dead weights.

Another way of reducing the amount of vibration in a cable is to make the cable of irregular cross section so that the eddy frequency set up by the wind in one portion of the cable will be different from the eddy frequencies set up in adjacent portions of the cable. If this were done, exact resonance would not occur and if a partly resonant vibration should be started there would be a tendency for the impulses from the wind eddies to oppose each other. As a net result, the energy put into a cable of irregular section would be less than the corresponding energy put into a smooth cable. As a result, the amplitude at which the dissipated energy would equal the input energy would not be as great as for a smooth cable. This tendency has been observed in the field, but in the author's opinion it is not sufficient to eliminate the fatigue hazard. The possibility of high concentrations of stress at supports, resulting from irregular sections, should not be overlooked.

In order to decrease the amplitude of vibration to such a point that the resulting stresses would not be serious, a great variety of damping devices has been proposed. In general, these various types of dampers may be classified as those intended to dissipate energy and those to serve merely as vibration absorbers. The use of vibration absorbers strictly as absorbers with no dissipation of energy may be dismissed at once with a consideration of the fact that if the cable vibrates at all a definite amount of energy is being delivered to it. In order to reduce the amplitude of vibration, the total energy dissipated must be greater than that dissipated by the cable itself. If a vibration absorber be placed on a line its amplitude therefore must increase continually (that is, the amplitude of the absorber itself) relative to the cable. Consequently, the relative amplitude of throw of the absorber will increase to such an extent that it either will fail or else will begin to dissipate energy.

If the damper dissipates energy at relatively low amplitudes of vibration, it is conceivable that a sufficient amount of energy may be dissipated by the damper to prevent the amplitude of vibration of the cable from becoming serious. In general, friction dampers employ 2 principles: damping by mechanical friction, such as rubbing or striking together of parts, or forcing air or liquids through small openings; and the damping resulting from hysteresis loss within the stressed parts of the damper itself.

The most notable example of a damper that dissipates energy by both friction between moving parts and internal friction in stressed metal is the Stockbridge type of damper. This type of damper consists of 2 weights resiliently suspended from the conductor cable. These weights are attached rigidly to the ends of a double cantilever of steel wire cable which in turn is attached rigidly to the conductor cable. In field and laboratory tests this form of damper has been found to give excellent results when proportioned properly for the conductor on which it is used. In view of this fact it is desirable to study the behavior of the Stockbridge type of dampers with a view to determining the proper size and design of damper for any given cable.

Analysis of Stockbridge Dampers

The analysis of a Stockbridge or any other damper requires first the establishment of conditions assumed to obtain throughout the analysis. It is desirable to assume a set of conditions that will represent most nearly the actual conditions in service. Considerable background of both field and laboratory tests has been utilized in selecting the assumptions made.

It has been observed that a cable usually tends to vibrate at a constant frequency; but when the wind changes sufficiently in either velocity or direction, a change in state of vibration occurs. The vibration may be irregular for a short time and then gradually settle down to its new state. This transition period may extend over several minutes or may be scarcely discernible. In any event it is of very short duration compared with the total time of vibration.

From elasticity:

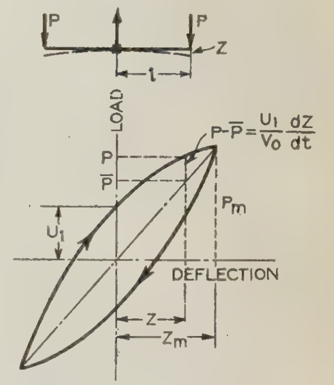
$$Z = \frac{Pl^3}{3EI} \text{ if end be not restrained}$$

$$Z_m = \frac{P_m l^3}{3EI}$$

Assume $\frac{U_1}{P_m}$ = constant = Υ

$$P_m = \frac{U_1}{\Upsilon}$$

$$Z_m = \frac{U_1 l^3}{3\Upsilon EI}; \qquad \frac{U_1}{Z_m} = \frac{3\Upsilon EI}{l^3}$$



In the steady state of vibration

$$Z = Z_m \sin wt$$

from which

$$\frac{dz}{dt} = Z_m w \cos wt$$

$$V_0 = \text{maximum value of } \frac{dz}{dt} = Z_m w$$

Then

$$\frac{U_1}{V_0} = \frac{U_1}{Z_m w} = \frac{3\Upsilon EI}{wl^3}$$

Therefore,

$$P - \bar{P} = \frac{U_1}{V_0}\frac{dz}{dt} = \frac{3\Upsilon EI}{wl^3}\frac{dz}{dt}$$

or

$$\bar{P} = P - \frac{3\Upsilon EI}{wl^3}\frac{dz}{dt}$$

Fig. 6. Damping from load

Records of vibration of damped cables show that the amplitudes of vibration of the cable remain nearly constant over comparatively long periods of time.

It may be concluded from these conditions that the damper may be considered as being operated at constant frequency and constant amplitude. The periods of transition from one frequency to the other may be neglected because of their short duration and because they change gradually from one mode of vibration to another.

Traveling waves in the cable during the transition period no doubt cause increased stresses in the damper, but the accurate evaluation of their effects is not feasible. While they will not be included in

this analysis, it may be well to point out that observations of actual transmission lines indicate that the amplitudes of the traveling waves are much smaller than those of the steady vibration. Experience has shown that if the uniform vibration is damped adequately, the traveling waves and beats also are damped so that very little overstressing may be expected. Since allowable stresses generally are selected somewhat below the maximum stress that the material will withstand safely, in order to provide against unknown conditions of temporary high stress, the occasional slight additional stressing from these neglected factors will be cared for adequately.

Therefore, the damper will be assumed to be operated at a constant frequency until dynamic equilibrium is reached. It is this state of equilibrium that is analyzed in this paper. In order to simplify the analysis, it is divided into 2 parts: The first part covers the case of an ideally elastic damper cable with no hysteresis; the second extends the analysis to the practical case of a damper cable having a definite hysteresis loop but whose average load-deflection ratio is identical with that of the first ideal cable.

The object of this analysis is to determine the following elements in the vibration of the damper for any given frequency and amplitude of vibration of the center clip:

1. Natural frequencies of damper.
2. Linear displacement of damper weights relative to center clip.
3. Phase angle between center clip motion and motion of weights.
4. Angular displacement of damper weights.
5. Force and couple exerted on damper cable by damper weight.
6. Energy dissipated by hysteresis in damper cable.
7. Stresses in damper cable caused by force and couple of the foregoing item 5.

With these factors known, it is possible to predict the behavior of the damped conductor cable and damper for any given set of conditions. The design may be altered to give the optimum damping of the conductor cable without subjecting the damper parts to damaging stresses.

Vibration Without Damping

It is assumed in this part of the analysis that the damper cable is ideally elastic and uniform, and that it is clamped rigidly in a horizontal position at the center clip. Referring to figure 5, which shows the damper in both its normal and deflected positions, one may see that in order to obtain the displacement shown the forces acting on the damper cable would necessarily be a direct force, P, and a couple, C, applied at the point of attachment of the damper weight to the cable. In order to distinguish between the actual force and couple and the idealized force and couple, the idealized force and couple are designated as $\overline{P}$ and $\overline{C}$.

The static deflection under load of a perfectly elastic damper cable with forces $\overline{P}$ and $\overline{C}$ acting at the section where the weights are attached to the cable may be found as the ordinary static deflection of a cantilever which is

$$Z = \frac{\overline{P}l^3}{3EI} - \frac{\overline{C}l^2}{2EI} \tag{74}$$

where

Z = deflection of damper cable relative to the center clip, in inches (positive as shown in figure 5)
$\overline{P}$ = force acting on idealized damper cable at weight, in pounds
$\overline{C}$ = moment of equivalent couple acting on idealized damper cable at weight, in pound inches
l = clear or effective length of damper cable between center clip and point of attachment of weight, in inches
EI = flexural rigidity of damper cable, in pound inches^2

The angular rotation at the end of the damper cable, and consequently of the damper weight, would be

$$\theta = \frac{\overline{P}l^2}{2EI} - \frac{\overline{C}l}{EI} \tag{75}$$

where θ is the angular displacement of the damper weight in radians, and the other terms are as previously defined.

From D'Alembert's principle, the dynamic forces that the damper weight will exert upon the damper cable may be treated the same as static forces equal and opposite to the product of the mass of the moving weight and the acceleration of its center of gravity. Thus,

$$\overline{P} = ma \tag{76}$$

where

m = mass of one damper weight, pounds divided by acceleration due to gravity in inches per second per second
a = acceleration of the center of gravity of the damper weight, in inches per second per second

The acceleration a may be found as the second derivative of the absolute displacement of the damper weights with respect to time. From figure 5, the absolute displacement of the center of gravity of the weight would be the displacement, y, of the center clip minus the relative displacement of the damper weight with respect to the center clip. This relative displacement is the result of the deflection of the damper cable, Z, minus the displacement of the center of gravity by virtue of the angular rotation θ. The acceleration then may be found as the second derivative of these displacements with respect to time or

$$a = \frac{d^2y}{dt^2} - \frac{d^2Z}{dt^2} + e\frac{d^2\theta}{dt^2} \tag{77}$$

where

y = displacement of the center clip at time t, in inches (positive as shown in figure 5)
t = time measured from the instant that the center clip passes upward through its neutral position, in seconds
e = distance between the center of gravity of the weight and the point of attachment, in inches (positive as shown in figure 5)

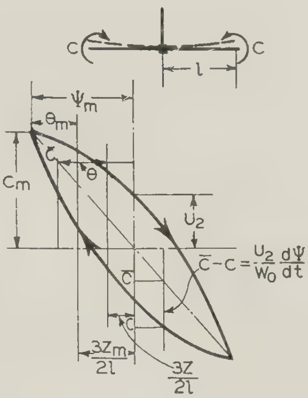
The moment acting at the point of attachment of the damper weight to the damper cable is made up of 2 parts: The first is the direct moment caused by the force P acting at the center of gravity of the

weight times the eccentricity e; the second is the solid moment of inertia of the damper weight times its angular acceleration. This moment can be expressed mathematically as

$$\bar{C} = \bar{P}e + \Pi \frac{d^2\theta}{dt^2} \tag{78}$$

where Π is the solid moment of inertia of the damper weight and is equal to its mass m multiplied by the square of its radius of gyration in inches; the other

Angle change at end of cantilever deflected an amount Z, is $\theta_1 = 3Z/2l$ if $C = 0$. The angle change resulting from a couple $\bar{C}$ is $\Phi = -\bar{C}l/EI$. The negative sign results from the fact that in the sign convention previously established a positive couple C produces a negative angle change. Assume U_2/C_m = constant = δ, as for load.



$$C_m = \frac{U_2}{\delta}$$

$$\Phi_m = -\frac{C_m l}{EI} = -\frac{U_2 l}{\delta EI}$$

$$\frac{U_2}{\Phi_m} = -\frac{\delta EI}{l}$$

In the state of steady vibration

$$\Phi = \Phi_m \sin wt$$

$$\frac{d\Phi}{dt} = \Phi_m w \cos wt$$

$$W_0 = \text{maximum value of } \frac{d\Phi}{dt}$$

$$\bar{C} - C = \frac{U_2}{W_0}\frac{d\Phi}{dt} = \frac{\delta EI}{lw}\frac{d\Phi}{dt}$$

But

$$\Phi = \theta - \frac{3Z}{2l}$$

so

$$\frac{d\Phi}{dt} = \frac{d\theta}{dt} - \frac{3}{2l}\frac{dz}{dt}$$

Therefore,

$$\bar{C} = C - \frac{\delta EI}{lw}\left(\frac{d\theta}{dt} - \frac{3}{2l}\frac{dz}{dt}\right)$$

Fig. 7. Damping from couples at free ends

terms are as previously defined.

Substituting the values of $\bar{P}$, a, and $\bar{C}$ from equations 76, 77, and 78 into equation 74,

$$EIZ = ml^2\left(\frac{l}{3} - \frac{e}{2}\right)\left[\frac{d^2y}{dt^2} - \frac{d^2Z}{dt^2} + e\frac{d^2\theta}{dt^2}\right] - \frac{\Pi l^2}{2}\frac{d^2\theta}{dt^2} \tag{79}$$

Let K be the radius of gyration of the damper weight, in inches. By definition

$$\Pi = mK^2 \tag{80}$$

Substituting this value into equation 79,

$$EIZ + ml^2\left(\frac{l}{3} - \frac{e}{2}\right)\frac{d^2Z}{dt^2} = ml^2\left(\frac{l}{3} - \frac{e}{2}\right)\frac{d^2y}{dt^2} + ml^2\left(\frac{le}{3} - \frac{e^2}{2} - \frac{K^2}{2}\right)\frac{d^2\theta}{dt^2} \tag{81}$$

It may be noted that this equation contains only Z and θ as dependent variables; y is considered a known function.

Substituting the values from equations 76, 77, and 78 into equation 75,

$$EI\theta - ml\left(\frac{le}{2} - e^2 - K^2\right)\frac{d^2\theta}{dt^2} = ml\left(\frac{l}{2} - e\right)\left(\frac{d^2y}{dt^2} - \frac{d^2Z}{dt^2}\right) \tag{83}$$

If θ be eliminated between equations 81 and 83 an equation in Z, y, and t will result, in which y will be determined by the character of the forced vibration and therefore may be considered as a known function of t. The resulting equation is

$$\frac{(EI)^2Z}{ml^2} + \frac{EI}{l}\left(\frac{l^2}{3} - le + e^2 + K^2\right)\frac{d^2Z}{dt^2} + \frac{ml^2K^2}{12}\frac{d^4Z}{dt^4} = EI\left(\frac{l}{3} - \frac{e}{2}\right)\frac{d^2y}{dt^2} + \frac{ml^2K^2}{12}\frac{d^4y}{dt^4} \tag{86}$$

A solution of this equation for any function, y, will give the deflections of the damper cable as a function of the time t. Thus the relative displacement of the end of the damper cable is known, and from this the total displacement can be found.

Equation 86 may be written in operator form as follows:

$$(aD^4 + bD^2 + c)Z = f(t) \tag{87}$$

in which

D = an operator equivalent to $\frac{d}{dt}$

$a = \frac{ml^2K^2}{12}$ $\quad b = \frac{EI}{l}\left(\frac{l^2}{3} - le + e^2 + K^2\right)$ $\quad c = \frac{(EI)^2}{ml^2}$

$f(t)$ = a function of t resulting from the differentiation of y

If the center clip be moved up and down in simple harmonic motion, which is shown to be practically the case in field installations, the motion of the center clip may be represented by

$$y = A_0 \sin wt \tag{88}$$

where

A_0 = half the total travel of the center clip, in inches
w = 2π times the frequency of vibration of the center clip in cycles per second

From this it follows that

$$\frac{d^2y}{dt^2} = -A_0w^2 \sin wt$$

and

$$\frac{d^4y}{dt^4} = A_0w^4 \sin wt$$

Substituting these values in the right-hand side of equation 86,

$$f(t) = A_0\left[\frac{ml^2K^2}{12}w^4 - EI\left(\frac{l}{3} - \frac{e}{2}\right)w^2\right]\sin wt \tag{89}$$

which may be written

$$f(t) = Q \sin wt$$

where

$$Q = A_0\left[\frac{ml^2K^2}{12}w^4 - EI\left(\frac{l}{3} - \frac{e}{2}\right)w^2\right]$$

It may be noted that Q vanishes when $w = 0$, $A_0 = 0$, or when

$$w = \sqrt{\frac{12EI\left(\frac{l}{3} - \frac{e}{2}\right)}{ml^2K^2}} \tag{90}$$

which means that in the complete solution of the differential equation the particular integral vanishes for these cases. The first 2 conditions are obviously the conditions of a fixed center clip while the third is the frequency corresponding to pure rotational vibration of the damper weight about the point of attachment.

The complementary function for equation 86 is of the form

$$Z = A_1 \sin p_1t + A_2 \cos p_1t + A_3 \sin p_2t + A_4 \cos p_2t \tag{91}$$

in which

$$p_1 = \sqrt{\frac{b - \sqrt{b^2 - 4ac}}{2a}} \tag{92}$$

$$p_2 = \sqrt{\frac{b + \sqrt{b^2 - 4ac}}{2a}} \tag{93}$$

A_1, A_2, A_3, and A_4 are arbitrary constants

In connection with the complementary function it may be noted that $p_1/2\pi$ and $p_2/2\pi$ are the natural frequencies in cycles per second at which the weights will vibrate.

The particular integral for equation 86 is of the form

$$Z = A \sin wt + B \cos wt \tag{94}$$

The values of A and B may be found by substituting the value of Z from the particular integral into the operator form (equation 87) of the original differential equation 86. The resulting equation is

$$aAw^4 \sin wt + aBw^4 \cos wt - bAw^2 \sin wt - bBw^2 \cos wt + cA \sin wt + cB \cos wt = Q \sin wt \tag{95}$$

From equation 95 it follows that

$$A(aw^4 - bw^2 + c) = Q \tag{96}$$

$$B(aw^4 - bw^2 + c) = 0 \tag{97}$$

Therefore,

$$A = \frac{Q}{(aw^4 - bw^2 + c)} \tag{98}$$

$B = 0$ provided $(aw^4 - bw^2 + c) \neq 0$.

The denominator of equation 98 may be factored into

$$a\left(w^2 - \frac{b - \sqrt{b^2 - 4ac}}{2a}\right)\left(w^2 - \frac{b + \sqrt{b^2 - 2ac}}{2a}\right) = a(w^2 - p_1^2)(w^2 - p_2^2)$$

where p_1 and p_2 have the values given in equations 92 and 93, so that

$$A = \frac{Q}{a(w^2 - p_1^2)(w^2 - p_2^2)} \tag{98a}$$

The particular integral then is

$$Z = \frac{Q \sin wt}{a(w^2 - p_1^2)(w^2 - p_2^2)} \tag{94a}$$

Thus it follows that if w be equal to either p_1 or p_2, the coefficient A, and consequently the amplitude, becomes indefinitely large.

While the analysis thus far has assumed an ideal case, the values of p_1 and p_2 as well as other factors will be useful in the analysis where damping is considered.

Vibration With Damping

If the load-deflection curve for the damper cable describes a loop when the deflection is carried through a complete cycle, some energy is dissipated and therefore damping results. Damping may be brought into the analysis by considering the deviation of the load-deflection curve from the ideal straight line assumed in the theory of elasticity. The method used in the following analysis is indicated in figures 6 and 7.

The elastic behavior is indicated by equations 74 and 75. The eccentricity, e, is taken as zero because in practice it is kept as small as possible and it may be considered separately in a quantitative way after the analysis has been completed. The values of $\overline{P}$ and $\overline{C}$ from figures 6 and 7 are

$$\overline{P} = P - \frac{3\Upsilon EI}{wl^3}\frac{dZ}{dt} \tag{99}$$

$$\overline{C} = C - \frac{\delta EI}{lw}\left(\frac{d\theta}{dt} - \frac{3}{2l}\frac{dZ}{dt}\right) \tag{100}$$

Substituting these values in equations 74 and 75 and rearranging,

$$EIZ + \frac{\Upsilon EI}{w}\frac{dZ}{dt} + \frac{3}{4}\frac{\delta EI}{w}\frac{dZ}{dt} - \frac{\delta EIl}{2w}\frac{d\theta}{dt} = \frac{Pl^3}{3} - \frac{Cl^2}{2} \tag{101}$$

$$EI\theta + \frac{3}{2}\frac{\Upsilon EI}{wl}\frac{dZ}{dt} + \frac{3}{2}\frac{\delta EI}{wl}\frac{dZ}{dt} - \frac{\delta EI}{w}\frac{d\theta}{dt} = \frac{Pl^2}{2} - Cl \tag{102}$$

Equations 76, 77, and 78 then may be written as

$$P = ma \tag{76a}$$

$$a = \frac{d^2y}{dt^2} - \frac{d^2Z}{dt^2} \tag{77a}$$

$$C = \Pi\frac{d^2\theta}{dt^2} \tag{78a}$$

These values may be substituted into equations 101 and 102. The resulting equations may be written as

$$EIZ + \frac{EI}{w}\left(\Upsilon + \frac{3}{4}\delta\right)\frac{dZ}{dt} + \frac{ml^3}{3}\left(\frac{d^2Z}{dt^2} - \frac{d^2y}{dt^2}\right) = + \frac{\delta EIl}{2w}\frac{d\theta}{dt} - \frac{\Pi l^2}{2}\frac{d^2\theta}{dt^2} \tag{103}$$

$$\frac{3EI}{2wl}(\Upsilon + \delta)\frac{dZ}{dt} + \frac{ml^2}{2}\left(\frac{d^2Z}{dt^2} - \frac{d^2y}{dt^2}\right) = -EI\theta + \frac{\delta EI}{w}\frac{d\theta}{dt} - \Pi l\frac{d^2\theta}{dt^2} \tag{104}$$

From these simultaneous equations, θ may be eliminated and an equation in Z alone obtained

(assuming y is known). Multiplying equation 104 by $l/2$ and subtracting it from equation 103,

$$EIZ + \frac{EI\Upsilon}{4w}\frac{dZ}{dt} + \frac{ml^3}{12}\left(\frac{d^2Z}{dt^2} - \frac{d^2y}{dt^2}\right) = \frac{EIl\theta}{2} \tag{105}$$

The value of θ from equation 105 together with the relation $\Pi = mK^2$ from equation 80 substituted into equation 103 gives

$$EIZ + \frac{EI}{w}\left(\Upsilon - \frac{\delta}{4}\right)\frac{dZ}{dt} + \left[ml\left(\frac{l^2}{3} + K^2\right) - \frac{\Upsilon\delta EI}{4w^2}\right]\frac{d^2Z}{dt^2} + \frac{ml}{w}\left[\frac{-\delta l^2}{12} + \frac{K^2\Upsilon}{4}\right]\frac{d^3Z}{dt^3} + \frac{m^2K^2l^4}{12EI}\frac{d^4Z}{dt^4} = \frac{ml^3}{3}\frac{d^2y}{dt^2} - \frac{\delta ml^3}{12w}\frac{d^3y}{dt^3} + \frac{m^2K^2l^4}{12EI}\frac{d^4y}{dt^4} \tag{106}$$

Equation 106 is the complete differential equation for the linear displacement of the mass center of the weights. The angular displacement may be found by substituting the solution for equation 106 into equation 105.

It may be noted that if equation 106 be multiplied by $(EI)/(ml^2)$, the coefficients of all terms not containing damping factors become identical with the corresponding coefficients in equation 86, which is the differential equation for the linear displacement of the mass center of the weights when no damping is included.

Equation 106 may be rewritten as

$$\frac{(EI)^2}{ml^2}Z + \frac{(EI)^2}{wml^2}\left(\Upsilon - \frac{\delta}{4}\right)\frac{dZ}{dt} + \left[\frac{EI}{l}\left(\frac{l^2}{\ } + K^2\right) - \frac{\Upsilon\delta(EI)^2}{4w^2ml^2}\right]\frac{d^2Z}{dt^2} - \frac{EI}{wl}\left[\frac{-\delta l^2}{12} + \frac{\Upsilon K^2}{4}\right]\frac{d^3Z}{dt^3} + \frac{mK^2l^2}{12}\frac{d^4Z}{dt^4} = \frac{EIl}{3}\frac{d^2y}{dt^2} - \frac{\delta EIl}{12w}\frac{d^3y}{dt^3} + \frac{mK^2l^2}{12}\frac{d^4y}{dt^4} \tag{106a}$$

Written in operator form this equation is

$$[aD^4 + \alpha D^3 + (b + \lambda)D^2 + \beta D + c]Z = [a_1D^4 + \alpha_1D^3 + b_1D^2]y \tag{106b}$$

in which

$$D = \frac{d}{dt}, \qquad D^2 = \frac{d^2}{dt^2}, \text{ etc.}$$

$$a = \frac{ml^2K^2}{12} \qquad \alpha = \frac{EI}{wl}\left[\frac{-l^2\delta}{12} + \frac{K^2\Upsilon}{4}\right] \qquad a_1 = a = \frac{ml^2K^2}{12}$$

$$b = \frac{EI}{l}\left(\frac{l^2}{3} + K^2\right) \qquad \beta = \frac{(EI)^2}{wml^2}\left(\Upsilon - \frac{\delta}{4}\right) \qquad \alpha_1 = \frac{-\delta EIl}{12w}$$

$$c = \frac{(EI)^2}{ml^2} \qquad \lambda = \frac{-\Upsilon\delta(EI)^2}{4w^2ml^2} \qquad b_1 = \frac{EIl}{3}$$

It may be noted that a, b, and c here are identical with the a, b, and c values given for no damping if $e = 0$, which is the case assumed.

When the value of y, defined by equation 88, is substituted into equation 106b, the right-hand side of that equation becomes

$$A_0(a_1w^4 - b_1w^2)\sin wt - A_0\alpha_1w^3\cos wt$$

This expression may be transformed into a single sine function for an angle $(wt + \phi)$ where ϕ is the phase angle necessary for this transformation. Let

$$A_0(a_1w^4 - b_1w^2) = Q_1\cos\phi$$

and

$$A_0\alpha_1w^3 = -Q_1\sin\phi$$

Substituting these values in the preceding expression, the resulting expression is

$$Q_1\cos\phi\sin wt + Q_1\sin\phi\cos wt$$

or

$$Q_1\sin(wt + \phi)$$

in which

$$Q_1 = A_0\sqrt{(a_1w^4 - b_1w^2)^2 + \alpha_1^2w^6} \tag{107}$$

$$\phi = \tan^{-1}\frac{-\alpha_1w^3}{a_1w^4 - b_1w^2} \tag{107a}$$

Again Q_1 has a characteristic minimum value when $w = \sqrt{\dfrac{4EI}{mlK^2}}$ which is the same value that makes $Q = 0$ if $e = 0$ in the first part of this section. Thus damping does not change the frequency at which Q is a minimum, but it does prevent Q from becoming zero.

The complementary function for equation 106b must contain a factor e^{-nt} such that after a sufficiently long time all vibration will cease unless some external forces act on the damper. Since this condition exists in practice, the complementary function must be transient and must disappear for all practical purposes after a time. Since only the steady state of vibration is considered in this paper, this transient vibration is dropped from the analysis. Thus the particular integral is all that need be considered.

The particular integral must be of the form

$$Z = A_1\sin(wt + \phi) + B_1\cos(wt + \phi) \tag{108}$$

where A_1 and B_1 are constants that will satisfy equation 106b. The values of A_1 and B_1 are:

$$A_1 = \frac{Q_1(aw^4 - bw^2 + c - \lambda w^2)}{(aw^4 - bw^2 + c - \lambda w^2)^2 + (\alpha w^3 - \beta w)^2} \tag{109}$$

$$B_1 = \frac{Q_1(\alpha w^3 - \beta w)}{(aw^4 - bw^2 + c - \lambda w^2)^2 + (\alpha w^3 - \beta w)^2} \tag{110}$$

This equation for Z may be transformed into a single sine function for an angle $(wt + \phi + \Psi)$, where Ψ is an additional phase angle necessary for this transformation. Now let

$$A_1 = H\cos\Psi \tag{111}$$

and

$$B_1 = H\sin\Psi \tag{112}$$

Then

$$H^2 = A_1^2 + B_1^2 \tag{113}$$

and

$$\Psi = \tan^{-1}\frac{B_1}{A_1} = \tan^{-1}\frac{(\alpha w^3 - \beta w)}{(aw^4 - bw^2 + c - \lambda w^2)} \tag{114}$$

Substituting the values for A_1 and B_1 from equations 111 and 112 in equation 108,

$$Z = H\sin(wt + \phi)\cos\Psi + H\cos(wt + \phi)\sin\Psi$$
$$= H\sin(wt + \phi + \Psi) \tag{115}$$

The value of H may be found by substituting the values of A and B from equations 109 and 110 in equation 113,

$$H = \frac{Q_1}{\sqrt{(aw^4 - bw^2 + c - \lambda w^2)^2 + (\alpha w^3 - \beta w)^2}} \tag{116}$$

The term $(aw^4 - bw^2 + c)$ may be factored as follows:

$$(aw^4 - bw^2 + c) = a\left(w^2 - \frac{b + \sqrt{b^2 - 4ac}}{2a}\right) \times \left(w^2 - \frac{b - \sqrt{b^2 - 4ac}}{2a}\right)$$

which may be written $a(w^2 - p_2^2)(w^2 - p_1^2)$, using the values of p_1 and p_2 from equations 92 and 93.

Therefore, the equation for the displacement of the mass center of the damper weights may be written as

$$Z = \frac{Q_1 \sin (wt + \phi + \Psi)}{\sqrt{[a(w^2 - p_1^2)(w^2 - p_2^2) - \lambda w^2]^2 + (\alpha w^3 - \beta w)^2}} \tag{117}$$

When the values for Q_1, a, λ, α, and β, are substituted, the equation for Z becomes

$$Z = \frac{A_0 \sin (wt + \phi + \Psi) \cdot \sqrt{\left(\frac{ml^2K^2}{12} w^4 - \frac{EIl}{3} w^2\right)^2 + \left(\frac{\delta EIl}{12}\right)^2 w^4}}{\sqrt{\left[\frac{ml^2K^2}{12}(w^2 - p_1^2)(w^2 - p_2^2) + \frac{\Upsilon\delta(EI)^2}{4ml^2}\right]^2 + \frac{(EI)^2}{l^2}\left[\frac{w^2}{4}\left(K^2\Upsilon - \frac{l^2\delta}{3}\right) - \frac{EI}{ml}\left(\Upsilon - \frac{\delta}{4}\right)\right]^2}} \tag{118}$$

Equation 118 may be written as

$$Z = H_1 A_0 \sin (wt + \phi + \Psi) \tag{118a}$$

where $H_1 A_0 = H$, so that H_1 is a numerical constant determined by the quotient of the 2 radicals in equation 118.

The angular displacement of the damper weight, θ, may be found by substituting the value of Z from equation 118*a* into equation 105. This gives

$$\frac{EIl}{2}\theta = A_0\left(H_1\left[EI - \frac{ml^3}{12}w^2\right]\sin (wt + \phi + \Psi) + \frac{H_1 EI\Upsilon}{4}\cos (wt + \phi + \Psi) + \frac{ml^3w^2}{12}\sin wt\right) \tag{119}$$

The forces acting on the damper weights and consequently on the damper cable may be found from equations 76, 77, and 78. From equations 76 and 77 it follows that

$$\bar{P} = m\left(\frac{d^2y}{dt^2} - \frac{d^2Z}{dt^2}\right) \tag{120}$$

since $e = 0$.

From the substitution of the values for y and Z from equations 88 and 118*a*, it follows that

$$\bar{P} = mA_0w^2[H_1 \sin (wt + \phi + \Psi) - \sin wt] \tag{121}$$

Figure 6 shows that the maximum value of $\bar{P}$ will be the maximum value of the actual load, P, which may be found from the following transformation of equation 121:

$$\begin{aligned}\bar{P} &= mA_0w^2[H_1 \sin wt \cos (\phi + \Psi) + H_1 \cos wt \sin (\phi + \Psi) - \sin wt] \\ &= mA_0w^2[(H_1 \cos (\phi + \Psi) - 1)\sin wt + H_1 \sin (\phi + \Psi)\cos wt] \\ &= A_0 D \sin (wt + \bar{X})\end{aligned} \tag{122}$$

where

$$D \cos \bar{X} = mw^2(H_1 \cos (\phi + \Psi) - 1)$$
$$D \sin \bar{X} = mw^2 H_1 \sin (\phi + \Psi)$$

From this,

$$D = mw^2\sqrt{H_1^2 + 1 - 2H_1 \cos (\phi + \Psi)}$$

and

$$\tan \bar{X} = \frac{H_1 \sin (\phi + \Psi)}{H_1 \cos (\phi + \Psi) - 1}$$

Therefore,

$$\bar{P} = A_0 mw^2\sqrt{H_1^2 + 1 - 2H_1 \cos (\phi + \Psi)} \cdot \sin (wt + \bar{X}) \tag{122a}$$

The value of $\bar{P}$ will be maximum when $\sin (wt + \bar{X}) = 1$, from which it follows that

$$\bar{P}_{max} = P_{max} = A_0 mw^2\sqrt{H_1^2 + 1 - 2H_1 \cos (\phi + \Psi)} \tag{123}$$

From equation 78 it follows that

$$\bar{C} = II\frac{d^2\theta}{dt^2} = mK^2\frac{d^2\theta}{dt^2} \tag{124}$$

Substituting the value for θ from equation 119 into equation 124,

$$\bar{C} = -\frac{2A_0mK^2w^2}{EIl}\left[H_1\left(EI - \frac{ml^3w^2}{12}\right)\sin (wt + \phi + \Psi) + \frac{H_1EI\Upsilon}{4}\cos (wt + \phi + \Psi) + \frac{ml^3w^2}{12}\sin wt\right] \tag{125}$$

The maximum value of $\bar{C}$ and consequently of C may be found in the same manner as the maximum value of $\bar{P}$. Equation 125 may be transformed as in previous cases to

$$\bar{C} = -\frac{2A_0mK^2w^2}{EIl}(M \sin wt + N \cos wt) \tag{126}$$

in which

$$M = \left(H_1EI - \frac{H_1ml^3w^2}{12}\right)\cos (\phi + \Psi) - \frac{H_1EI\Upsilon}{4}\sin (\phi + \Psi) + \frac{ml^3w^2}{12} \tag{127}$$

$$N = \left(H_1EI - \frac{H_1ml^3w^2}{12}\right)\sin (\phi + \Psi) + \frac{H_1EI\Upsilon}{4}\cos (\phi + \Psi) \tag{128}$$

Then

$$\bar{C}_{max} = C_{max} = -\frac{2A_0mK^2w^2}{EIl}\sqrt{M^2 + N^2} \tag{129}$$

The couple $\bar{C}$ acting with the maximum force P_{max} may be found by evaluating wt in terms of the known phase angle $\bar{X}$ at the instant of maximum force. At this instant $\sin (wt + \bar{X}) = 1$, from which it follows that at P_{max} the simplest value for wt is $\frac{\pi}{2} - \bar{X}$. Substituting this value into equation 125

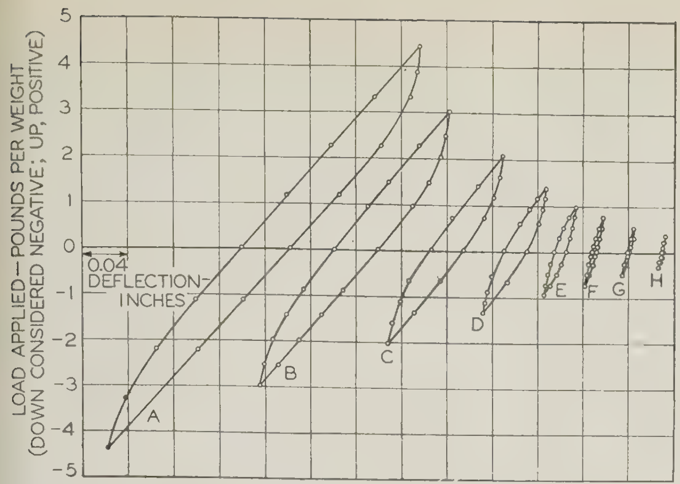


Fig. 8. Load-deflection curves obtained in flexural rigidity tests on a standard 10 pound Stockbridge damper

Loop	Area (Pound-Inches)	Vertical Width (Pounds)
A	0.3052	1.21 (max. 1.26)
B	0.1674	1.17
C	0.0835	1.01
D	0.0372	0.84
E	0.0138	0.69
F	0.00401	0.34
G	0.00081	0.108
H	0.00037	0.084

Values for loops C to H, inclusive were determined from larger scale plotting

Loads are those applied at the nominal center of gravity of each weight. Deflections are those at point of application of load. Damper cable was 3/8 inch in diameter (7 0.1214 inch strands)

the couple acting with the maximum force is found to be

$$\overline{C}_P = \frac{2A_0mK^2w^2}{EIl}\left\{H_1\left[EI - \frac{ml^3w^2}{2}\right]\sin\left(\frac{\pi}{2} - \overline{X} + \phi + \Psi\right) + \frac{H_1EI\Upsilon}{4}\cos\left(\frac{\pi}{2} - \overline{X} + \phi + \Psi\right) + \frac{ml^3w^2}{12}\sin\left(\frac{\pi}{2} - \overline{X}\right)\right\} \tag{130}$$

The maximum stresses in the damper cable at the center clip may be computed from the bending moment produced by the load P_{max}, the couple C_P, and the dead weight of the damper weight, in the same way as are stresses at the clamped end of a transmission line cable. This moment is

$$M_c = mgl + P_{max}l - \overline{C}_P$$

where mg is the weight of the damper weight in pounds, and the other terms are as previously defined.

The stresses in the top or bottom of a 7 strand damper cable at the clip may be found from equation 52*b*.

The energy dissipated per cycle for each damper weight is represented by the area of the hysteresis loops shown in figures 6 and 7.

The total energy dissipated may be taken as the area of the hysteresis loop resulting from the force alone plus the area of the loop resulting from the couple alone. The energy dissipated by P alone is found, from the area of the hysteresis loop, to be

$$U_P = \int_{\text{cycle}} (P - \overline{P})dZ = \int_{\text{cycle}} \frac{3\Upsilon EI}{wl^3}\frac{dZ}{dt}\,dZ \tag{131}$$

$$= \frac{3\Upsilon EI}{wl^3}\int_0^{\frac{2\pi}{w}} \frac{dZ}{dt}\cdot\frac{dZ}{dt}\,dt$$

$$= \frac{3\Upsilon EI}{wl^3}\int_0^{\frac{2\pi}{w}} \left(\frac{dZ}{dt}\right)^2 dt \tag{132}$$

Substituting the value of dZ/dt as obtained by differentiating equation 118*a*,

$$U_P = \frac{3\Upsilon EI}{wl^3}\cdot\frac{1}{w}\int_0^{2\pi} w^2H^2\cos^2(wt + \phi + \Psi)d(wt)$$

where $H = H_1A_0$ as before. Integrating between the limits for one cycle,

$$U_P = \frac{3\pi\Upsilon EIH^2}{l^3} \tag{133}$$

The energy dissipated by the couple C alone is found as the area of the corresponding hysteresis loop. This area may be expressed as

$$U_C = \int_{\text{cycle}} (C - \overline{C})\,d\Phi = \int_{\text{cycle}} \frac{\delta EI}{lw}\frac{d\Phi}{dt}\,d\Phi \tag{134}$$

$$U_C = \int_0^{\frac{2\pi}{w}} \frac{\delta EI}{lw}\frac{d\Phi}{dt}\cdot\frac{d\Phi}{dt}\,dt = \frac{\delta EI}{w^2l}\int_0^{2\pi}\left(\frac{d\Phi}{dt}\right)^2 d(wt) \tag{135}$$

After Φ is evaluated and substituted into this equation, the integration may be performed. From figure 7, it is found that

$$\Phi = \theta - \frac{3Z}{2l} \tag{136}$$

Substituting the value of θ from equation 119 and of Z from equation 118*a*,

$$\Phi = A_0H_1\left[\frac{1}{2l} - \frac{ml^2w^2}{6EI}\right]\sin(wt + \phi + \Psi) + \frac{A_0H_1\Upsilon}{2l}\cos(wt + \phi + \Psi) + A_0\frac{ml^2w^2}{6EI}\sin wt \tag{136a}$$

which may be reduced to

$$\Phi = A_0\left[H_1\left(\frac{1}{2l} - \frac{ml^2w^2}{6EI}\right)\cos(\phi + \Psi) - \frac{H_1\Upsilon}{2l}\sin(\phi + \Psi) + \frac{ml^2w^2}{6EI}\right]\sin wt + A_0\left[H_1\left(\frac{1}{2l} - \frac{ml^2w^2}{6EI}\right)\sin(\phi + \Psi) + \frac{H_1\Upsilon}{2l}\cos(\phi + \Psi)\right]\cos wt \tag{137}$$

This equation may be written as

$$\Phi = A_0[R\sin wt + S\cos wt] \tag{137a}$$

where

$$R = \left[H_1\left(\frac{1}{2l} - \frac{ml^2w^2}{6EI}\right)\cos(\phi + \Psi_1) - \frac{H_1\Upsilon}{2l}\sin(\phi + \Psi) + \frac{ml^2w^2}{6EI}\right] \tag{137b}$$

$$S = \left[H_1\left(\frac{1}{2l} - \frac{ml^2w^2}{6EI}\right)\sin(\phi + \Psi) + \frac{H_1\Upsilon}{2l}\cos(\phi + \Psi)\right] \tag{137c}$$

Equation 137*a* may be reduced still further to

$$\Phi = \Phi_m\sin(wt + \underline{X}) \tag{138}$$

where

$$\Phi_m = A_0\sqrt{R^2 + S^2} \tag{138a}$$

$$\underline{X} = \tan^{-1}\frac{S}{R} \qquad (138b)$$

When the value of Φ from equation 138 is substituted into equation 135 and integrated, it follows that

$$U_C = \frac{\pi\delta EI}{l}\Phi_m^2 \qquad (139)$$

The total energy dissipated is the sum of U_P and U_C which may be expressed as

$$U = U_P + U_C = \frac{\pi EI}{l}\left(\frac{3\Upsilon H^2}{l^2} + \delta\Phi_m^2\right) \qquad (140)$$

Formulas now have been developed for finding the energy dissipated by a Stockbridge damper when operated at any frequency and with any center clip displacement A_0 (half the amplitude).

Forces acting on the damper weights and the damper cable may be computed from equations 123 and 130. From these the stresses in the damper cable may be computed by equation 52*b*. Forces acting on the conductor cable will be only very slightly greater than these values because of the inertia of the damper cable and center clip. Therefore it is possible to get an indication of whether or not the forces necessary to operate the damper will be sufficient to cause a node point in the conductor cable.

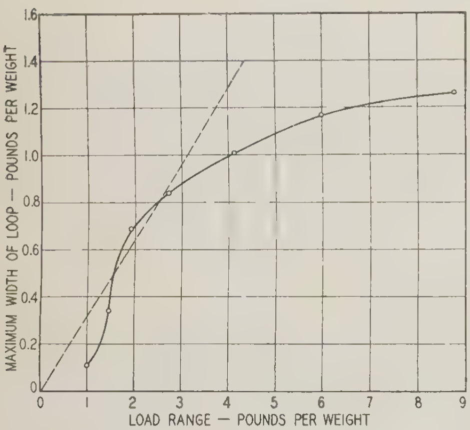


Fig. 9. Maximum width of loop versus load range for hysteresis loops shown in figure 8

The application of these principles and formulas are illustrated in the following section of this paper.

Application of Damper Analysis to a Specific Case

The particular damper considered here is a standard 10 pound damper, to which the following data apply:

Weights—each 5.0 pounds

Radius of dyration of weights—2.16 inches

Cable—$^3/_8$ inch, preformed extra high strength steel (7 0.1214 inch strands)

Effective length—5.95 inches

Point of attachment of weight to cable assumed to be at center of gravity of weight

Cyclic load-deflection tests were made on samples of the damper cable to determine its flexural rigidity and hysteresis characteristics. In these tests standard dampers were used.

The load was applied statically through a pulley system in a continuous cycle from the maximum downward load to the maximum upward load and *vice versa*. Deflections were measured with inside micrometers, the sensitivity of these measurements being well within 0.001 inch. Load-deflection diagrams showing the hysteresis loops are given in figure 8. From these data the maximum width of loop was obtained for various load ranges. Figure 9 shows the maximum width of loop plotted against the load range. It may be noted that for small load ranges a straight line relationship approximates the ratio of maximum width, $2U_1$ (see figure 6) to the load range, $2P_m$. The ratio U_1/P_m or Υ, assumed constant in the foregoing analysis, is found from the curve to be approximately 0.32.

In order to determine a value of δ (see figure 7) special tests were made. In these tests the load was applied to the damper weight at various distances from the point where the damper cable is fixed to the weight, thereby giving the equivalent of a force and a couple acting at the damper weight. The deflections were measured on the damper weight at 2 points ($2^3/_4$ inches and $5^3/_4$ inches from the center clip for the standard 10 pound damper), thereby giving the angular rotation of the end of the damper cable as well as the deflection of the fixed point. From these measurements, the values of Z_m and Φ_m were found. Then the value of δ was obtained from equation 140 using a value of total energy obtained

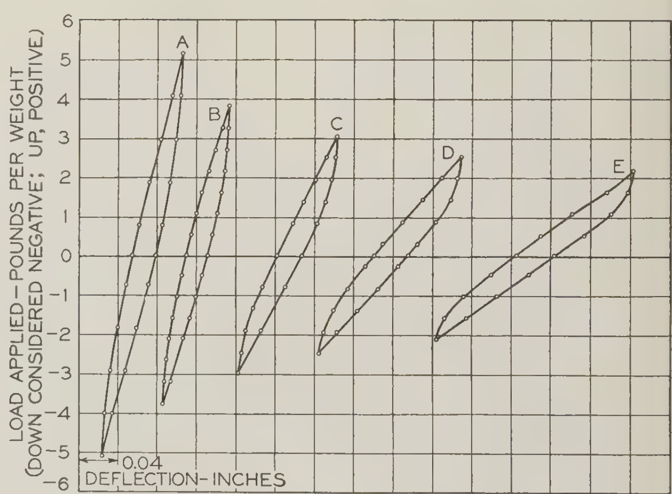


Fig. 10. Load-deflection curves obtained in flexural rigidity tests on a standard 10 pound Stockbridge damper

Loop	Area (Pound Inches)
A	0.1700
B	0.1564
C	0.1308
D	0.1172
E	0.1020

Loads are those applied to each weight. For loop *D* the load was applied at the nominal center of gravity of the weights; for loop *E*, one inch farther out from the clip; for loops *C*, *B*, and *A*, the loads were applied 1, 2, and 3 inches, respectively, nearer the clip. Deflections are at point of application of load

from the hysteresis loops of figure 10. In this way δ was found to be approximately 0.32.

In the design of the standard 10 pound damper it was assumed that the fixed point of the cable at the damper weight was at a point $^1/_3$ of the way in from the protruding end of the tapered sleeve that grips the damper cable. From the foregoing tests it was found by comparison that the effective length determined on this basis did not give consistent values between the deflection and rotation of the damper weight. A subsequent study was made to determine at which point the damper cable could be considered as fixed. It was found that the most consistent results were obtained when the fixed point was chosen as $^2/_5$ of the way in from the end of the tapered sleeve to the point of exodus from the weight. The point of fixity at the center clip originally was assumed as at the back of the protruding fillet next to the main body of the damper clip. This study showed that $^3/_5$ of the way in from the outer edge of the taper gave more consistent values. The resulting net length of the damper cable then is 5.95 inches as indicated.

This gives rise to a slight eccentricity between the fixed point at the damper weight and the mass center of the weight. Correction was made for this eccentricity in the load-deflection curves, but because of the tolerance in manufacturing it was not considered necessary to include this eccentricity in the analysis of the behavior of the damper in service.

Figure 11 shows the area of loop plotted against amplitude or range of oscillation of load points. It may be noted that a parabola fits the actual points very closely for amplitudes less than 0.05 inch. This fact confirms the findings of the analysis that the energy dissipated is proportional to the square of the amplitude of oscillation of the weights.

An average load-deflection ratio for any cycle may be obtained as the slope of the straight line joining the tips of the hysteresis loop. This has been done for several loops and the resulting slopes plotted against amplitude of deflection in figure 12. The corresponding EI values may be found by considering the load on a cantilever beam of length l. This has been done and the corresponding EI values plotted in figure 12. The necessary correction for the slight eccentricity of the loading points was accounted for in the computations.

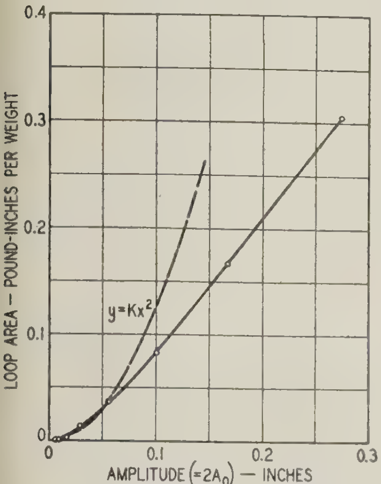


Fig. 11. Area of hysteresis loop versus amplitude or range of oscillation of load points for steel cable of Stockbridge damper

Amplitudes (or deflections) were measured at the fixed points in the weights. In the test data, the loads considered are those applied to each weight; they were applied at the nominal center of gravity of the weights. Damper cable was $^3/_8$ inch in diameter (7 0.1214 inch strands)

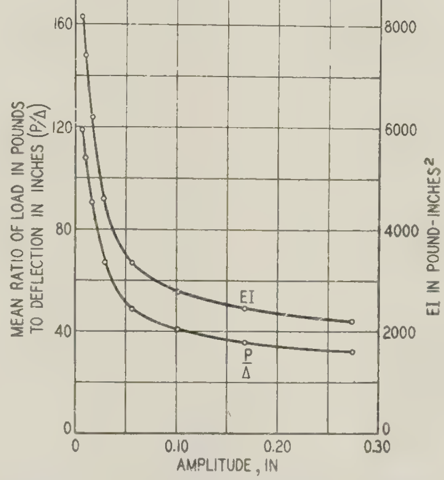


Fig. 12. Mean ratio of load to deflection versus amplitude for hysteresis loops for damper cable

Loads were applied at the nominal center of gravity of the weights. Deflections (or amplitudes) are those at the fixed points in the weights. Damper cable was $^3/_8$ inch in diameter (7 0.1214 inch strands)

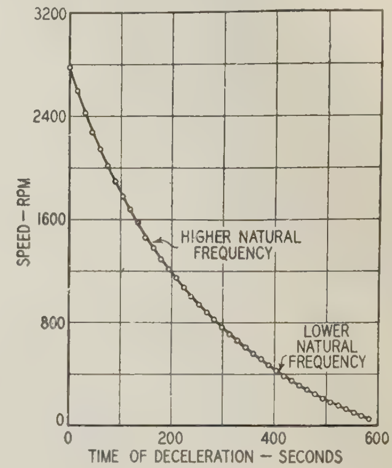


Fig. 13. A typical deceleration curve showing results of efficiency tests on Stockbridge damper

The frequencies noted are those actually observed during the test. The amplitude used was 0.05 inch

It may be seen that when the amplitude is very small, as it usually is in the field, the flexural rigidity of the cable is considerably greater than for larger amplitudes. In view of the fact that the amplitudes in the field are generally very small (less than 0.05 inch), an EI value of 3,500 pound-inches^2 corresponding to an amplitude of 0.05 inch has been chosen. This value was found to give consistent values for δ from curves shown in figure 10. For the experimental determination of the natural frequencies, however, an amplitude ($2A_0$) of the center clip of 0.05 inch was used which, at the critical speeds, produced displacements of the weights relative to the center clips as great as 0.30 inch. For such large amplitudes the value of EI is found to be less than half as large as for very small amplitudes. The value of EI corresponding to the testing conditions was chosen as 2,150 pound-inches^2.

From equations 92 and 93 the natural frequencies may be computed. For the damper under consideration these values are as follows:

For small amplitudes ($2A_0 < 0.05$ inch), $EI = 3{,}500$ pound-inches^2:
$p_1 = 53.9$ radians per second $= 515$ cylces per minute
$p_2 = 226$ radians per second $= 2{,}160$ cycles per minute

For large amplitudes ($2A_0 > 0.30$ inch), $EI = 2{,}150$ pound-inches^2:

$p_1 = 42.4$ radians per second $= 405$ cycles per minute

$p_2 = 178$ radians per second $= 1{,}700$ cycles per minute

Experimentally p_1 was found to be between 400 and 450 cycles per minute and p_2 was found to be between 1,400 and 1,500 cycles per minute.

The experimental determination of the natural frequencies consisted of attaching the center clip of a damper eccentrically to the shaft of a flywheel and accelerating the flywheel to approximately 2,800 rpm and then disconnecting the driving mechanism and permitting the flywheel to decelerate gradually until it stopped. The speeds at various time intervals were measured with a tachometer. The speeds at which the weights showed the greatest disturbance were taken as the observed natural frequencies. A typical speed-time curve is shown in figure 13.

The deceleration at the higher natural frequency was generally quite great so that the critical speed would appear to be lowered appreciably. The phenomenon of vibration during acceleration or deceleration through a critical speed has been analyzed quite fully for vibration of one degree of freedom by F. M. Lewis.[10] He shows that if the speed is decreasing the observed critical speed will be lower than that for constant speed. Thus the observed upper critical speed, p_2, during deceleration may be appreciably lower than that observed during constant speed. This fact is borne out by a comparison of the measured and computed values of the higher critical speeds.

The deceleration at the lower natural frequency was very much less than at the higher speed, so the displacement of the natural frequency would be expected to be correspondingly less. This is borne out also by a comparison between measured and computed values. The measured and computed lower natural frequencies show good agreement.

From equation 118 the maximum linear displacement of the center of gravity of the damper weight relative to the center clip may be computed. This has been done for more than 20 different speeds of operation and the results shown in curve I of figure 14. The abscissas are the ratios of the operated speed, w, to the lower natural frequency, p_1. The ordinates are the coefficients H_1 where $Z_{max} = H_1A_0$. This maximum value of Z will not always occur at the extreme position of the center clip and, in fact, at high speeds it occurs directly opposite the displacement, A_0, of the center clip. The relative positions may be indicated by the cosine of the phase angle which angle is the sum of the 2 angles ϕ and Ψ given by equations 107c and 114, respectively. The actual position of the center clip at the instant of maximum throw of the weights is given by

$$y_1 = A_0 \cos (\phi + \Psi)$$

Curve II of figure 14 shows the value of $\cos (\phi + \Psi)$ for different speeds.

Since y and Z were positive in opposite directions, the absolute displacement of the damper weight at the instant of maximum Z will be given by the difference $(Z_m - y_1)$. This value is shown as curve III in figure 14. This does not necessarily mean that when curve III crosses the zero axis the weight does not move. If the cosine of the phase angle, however, were positive unity at the same time, it would mean that the center of gravity of the damper weight was practically at rest. At the points where $w/p_1 = 2$ and $w/p_1 = 8$, these conditions nearly are fulfilled.

Curve IV of figure 14 shows the angular displacement of the damper weight relative to the position it would take if no couple were acting on it. It may be noticed that for slow speeds this displacement is practically zero. In other words, the inertia of the weight does not resist angular changes to any marked degree until the frequency of vibration approaches the higher natural frequency. It may be noticed also that at very high frequencies curve IV approaches the value of $3Z/2l$ which is the value of Φ_m for a fixed weight. This characteristic, that is, the weights remaining nearly stationary at high frequencies, has been observed in actual tests on dampers.

Curve V of figure 14 and curve III of figure 15 are identical; they show the relation between the relative displacements and energy dissipated, and between the forces acting on the damper cable and the energy dissipated. It may be noted that the energy dissipated per cycle is much greater at frequencies near the higher natural frequency than at frequencies near the lower natural frequency. This fact will be seen to have particular importance later.

Curve I of figure 15 shows the force acting on the damper cable. The variable force then acting on the conductor cable will have a maximum of twice the force shown in curve I, there being 2 weights to a damper.

Curve II of figure 15 shows the couple acting on the damper cable at the weight. Both the value for P_{max} and the value for C_{max} approach the values for a fixed damper weight which again checks with experiment.

As naturally would be expected, the energy curve (III) of figure 15 follows the curve of maximum forces. The energy per cycle dissipated by both weights of the damper is

$$U_1 = 2JA_0^2 \tag{142}$$

where A_0 is the throw of the damper weight, that is, half the amplitude, in inches.

The energy per cycle imparted to a cable by the wind may be evaluated in terms of the cable diameter, loop length, amplitude, and wind velocity from equation 73. The energy input for a span vibrating in n loops would be n times the energy per loop or

$$U_n = 0.000022\,V^2DAnL'$$

and since

$$nL' = \text{total span} = L_s',$$

$$U_n = 0.000022\,V^2DAL_s' \tag{143}$$

If an efficient damper be placed on a line, the amplitude of vibration will be quite small and therefore the energy dissipated by the cable will be extremely small. Assuming that the energy dissipated by the cable is negligible, it follows that the energy

input from the wind must equal the energy dissipated by the damper or dampers on the line.

Equating the energy dissipated by the dampers to the energy input on a span of L_s',

$$N \cdot 2 \frac{JA_0^2}{12} = 0.000022 V^2 D A L_s' \qquad (144)$$

where N is the number of dampers on the span. The factor 12 changes the inch pounds of energy dissipated to foot pounds. Now, assuming that the damper moves with the cable and that it is located near the center of the loop, it follows that $A = 2A_0$. When this value is substituted into equation 144, it follows that

$$A_0 = \frac{0.000264 V^2 D L_s'}{NJ} \qquad (145)$$

If a damper be appreciably off center of a loop, the node points will tend to shift so as to bring the center of the loop near the damper.

The value of J depends upon the frequency, and the frequency upon the wind velocity and cable diameter. The frequency of vibration expected for any given cable size and wind velocity is given in equation 1.

Take for example 2 standard 10 pound dampers, as previously described, on a transmission line having a 1,000 foot span and 397,500-circular mil steel-reinforced aluminum conductors. The diameter of the conductor is 0.81 inch. Sample computations for 3 wind velocities are given to illustrate the application of these formulas.

For wind velocity of 5 miles per hour

$$f = \frac{3.26V}{D} = \frac{3.26 \times 5}{0.81} = 20.1 \text{ cycles per second}$$

$$= 1{,}206 \text{ cycles per minute}$$

The relative frequency $\dfrac{w}{p_1} = \dfrac{f}{p_1} = \dfrac{1{,}206}{515} = 2.34$

From curve V of figure 14, the value of J for $w/p_1 = 2.34$ is found to be 80. The value of A_0 from equation 145 then is

$$A_0 = \frac{0.000264 \times 25 \times 0.81 \times 1{,}000}{2 \times 80} = 0.0334 \text{ inch}$$

For a wind velocity of 7 miles per hour,

$$f = 28.2 \text{ cycles per second (1,690 cycles per minute)}$$

$$\frac{w}{p_1} = 3.28$$

$$J = 320$$

$$A_0 = \frac{0.000264 \times 49 \times 0.81 \times 1{,}000}{2 \times 320} = 0.0164 \text{ inch}$$

For a wind velocity of 12 miles per hour,

$$f = 48.3 \text{ cycles per second (2,900 cycles per minute)}$$

$$\frac{w}{p_1} = 5.64$$

$$J = 850$$

$$A_0 = \frac{0.000264 \times 144 \times 0.81 \times 1{,}000}{2 \times 850} = 0.0181 \text{ inch}$$

Curve I in figure 16, based upon an assumed constant EI value and full movement of the damper, shows the general relation between wind velocity and amplitude for a 1,000 foot span of 397,500-circular mil steel-reinforced aluminum cable damped with one standard 10 pound Stockbridge damper at each end of the span. It may be noted that the amplitude of vibration at a wind velocity of about 5 miles per hour is about 0.067 inch. This amplitude, however, is large enough that the effective EI value of the damper cable will be reduced slightly. This reduction in EI value will reduce the critical speeds of the damper with the result that the relative speed w/p_1 will be greater. Corresponding to the greater relative speed, the value of J will be greater so that the actual amplitude will be slightly less. Using figure 12 as a guide for the reduction in EI value for various amplitudes, an estimation of the final amplitudes was made. These amplitudes are indicated by curve II of figure 16.

The significant fact that may be gleaned from these curves is that if the second peak of the energy dissipation curve (curve V of figure 14) is relatively wide compared with the trough between peaks, the damper will be very efficient over a wide range, whereas if the second peak is relatively narrow the damper will not be efficient over as wide a range of frequencies. Thus in the design of dampers, particular attention may be paid to proportioning the parts to obtain greater efficiency. From the curves of several types of designs, the effects of different proportions may be seen. Experimental data on the damping characteristics of different types and sizes of damper cables may be obtained by the same procedure used to obtain the data given herein.

From the energy curves for the damper considered, it may be concluded that if this damper were placed on a smaller cable so that the frequencies set up by the wind would be greater, the resulting efficiency would be greater. The limitation to this argument is that the forces exerted by the cable must be sufficient to operate the damper or a partial node point will be formed. If the maximum forces exerted by the damper for a given amplitude exceed the maximum force exerted by the cable, the damper will not be operated through as large an amplitude as the loops in the central portion of the span. Since the force necessary to operate that damper is directly proportional to the amplitude, the probable amplitude of the center clip of the damper may be approximated by the formula:

$$2A_0 = \frac{F_c A}{F_c + F_d} \qquad (146)$$

where

A_0 = half the amplitude of the center clip as previously defined, in inches

A = amplitude of vibration of a normal loop of the cable, in inches

F_c = maximum force exerted by such a loop of the cable, in pounds

F_d = maximum force exerted by damper (2 weights) operated at amplitude $2A_0$, in pounds

Thus if the force exerted by a single loop of the cable, F_c, be quite large with respect to that exerted by the damper, F_d, the relationships formerly con-

sidered will be valid, whereas if F_d be of the same order of magnitude as F_c a correction should be applied. This correction consists of using the value of A_0 from equation 146 in equation 144.

Before the resulting equation can be solved it will be necessary, of course, to evaluate forces F_c and F_d. The maximum force exerted by the cable may be taken as the sum of the products of each element of mass times its maximum acceleration.

The maximum acceleration of any point within a loop may be found by differentiating equation 3 twice with respect to time and setting $\sin wt = 1$. This gives the maximum acceleration of each point as

$$a_{max} = Aw^2 \sin \frac{\pi x}{L} \tag{147}$$

The maximum force, dF, on each element then would be

$$dF = a_{max}\, dm = Aw^2 \frac{W}{g} \sin \frac{\pi x}{l}\, dx,$$

where

dm = mass of an elemental length of cable, Wdx/g
W = weight of cable per unit of length, pounds per inch
w = frequency of vibration, radians per second

and the other terms as previously defined.

The total force exerted by a single loop will be

$$F_c = \int_0^L Aw^2 \frac{W}{g} \sin \frac{\pi x}{l}\, dx \tag{148}$$

which by integration gives

$$F_c = 2Aw^2 \frac{W}{g} \frac{l}{\pi} \tag{149}$$

The value of w in equation 149 is taken as radians per second, but the formula may be changed to give the force in terms of the frequency in cycles per second. This gives

$$F_c = \frac{8\pi A f^2 W l}{g} \tag{149a}$$

The frequency for a particular cable may be found from equation 1.

Applying this formula to 397,500-circular mil steel-reinforced aluminum cable under a tension of 5,000 pounds, the following values are obtained.

For a wind velocity of 5 miles per hour, the frequency is 20.1 cycles per second and the loop length (obtained from equation 2 or from nomographic charts[5]) is about 150 inches, from which

$$F_c = \frac{8\pi A \times 20.1^2 \times 0.052 \times 150}{386.4} = 205\ A \text{ pounds}$$

By referring to curve I of figure 15, it may be noticed that for this speed (that is, $w/p_1 = 2.34$) the maximum damper force is very much less than that possible from the cable. In other words, the cable will carry the damper with it.

For a wind velocity of 12 miles per hour, the frequency is 48.3 cycles per second and the loop length about 63 inches, from which

$$F_c = \frac{8\pi A \times 48.3^2 \times 0.052 \times 63}{386.4} = 496\ A \text{ pounds}$$

Again referring to figure 15 it may be noticed that for this speed (that is, $w/p_1 = 5.64$) the maximum damper force is about 775 A_0.

Referring to equation 146 it may be seen that

$$2A_0 = \frac{496\ A^2}{496\ A + 775\ A_0}$$

from which

$$A_0 = 0.33\ A$$

Using this value of A_0 in equation 144, it is found that

$$\frac{2 \times (0.33)^2 NJA^2}{12} = 0.000022\, V^2 DAL_s'$$

from which

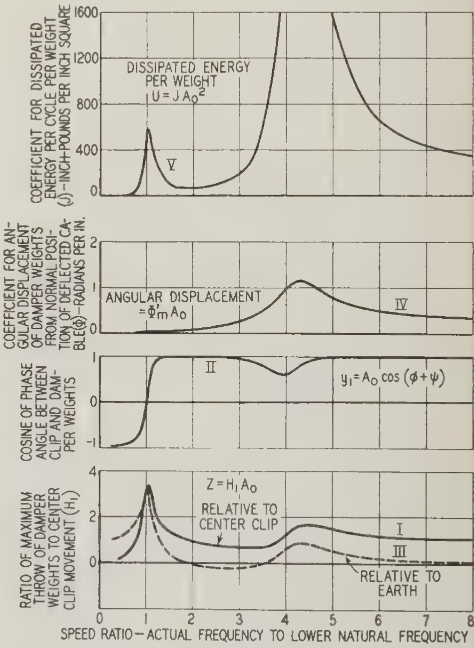
$$A = \frac{0.000264 \times 144 \times 0.81 \times 1{,}000}{2 \times 850 \times 0.218} = 0.083 \text{ inch}$$

From this,

$A_0 = 0.027$ inch, and the amplitude $2A_0 = 0.054$ inch

The values of amplitude in the end loop computed in this way are shown by curve III in figure 16. The fact that the amplitude of the cable at the damper is less than that out in the span gives rise to the condition of a partial node being formed at the damper. Expressed in other words, the actual nodes as observed in the cable would move toward the damper, making the loop length at the damper shorter than those out in the span. If heavier damper weights be used so that greater forces are required for their operation, the nodes will approach the damper until for very heavy weights a single node point is formed at the damper and a point of reflection is established.

Fig. 14. Theoretical amplitudes of damper weights and energy dissipated per weight, for various speeds of operation for standard 10 pound damper



If the wind velocity increases, and at the same time there are no gusts, the frequency of vibration increases and the loop length correspondingly de-

creases. As the loop length shortens, the conductor cable itself will begin to dissipate an appreciable amount of energy at relatively small amplitudes. Load-deflection tests on large cables indicate that when the deflection is about $^1/_{1,000}$ the span length or greater at nominal service tensions, an appreciable hysteresis loop is obtained in the load-deflection curves. This leads then to the conclusion that when the amplitude exceeds $^1/_{1,000}$ the loop length, the cable itself tends to damp the vibration. Curve IV in figure 16 illustrates the effect of the cable thus cutting in on the damping action at the higher wind velocities. This curve represents the expected amplitudes of vibration of a 397,500-circular mil steel-reinforced aluminum conductor strung at a tension of about 5,000 pounds at 60 degrees fahrenheit and having 2 standard 10 pound dampers on a 1000 foot span.

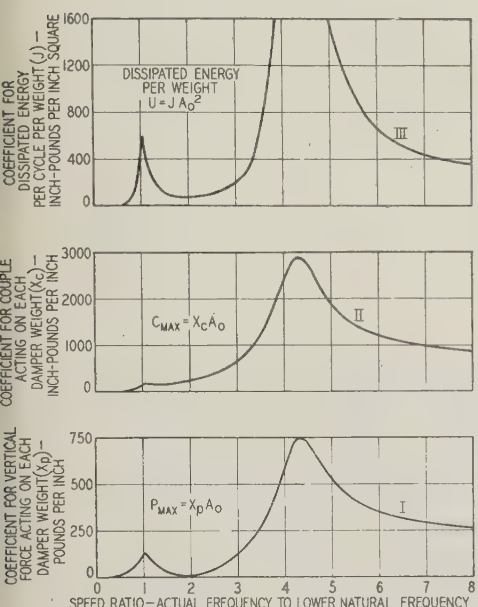


Fig. 15. Theoretical values of maximum forces and couples and energy dissipation per weight, for various speeds of operation for standard 10 pound damper

For lighter cables where the frequencies of vibration set up by normal winds are very high (say over 5,000 cyles per minute) the standard 10 pound damper no longer will be the most efficient one that can be used, even though it may serve very satisfactorily for the lower wind velocities. A lighter damper having higher natural frequencies will serve better for the higher wind velocities. If occasional low-velocity steady winds occur, it may be desirable to design a special damper having low natural frequencies to meet the particular conditions encountered. In some rare cases it may be advisable to provide against possible very low frequency vibration or against very high frequency vibration as well as to cover the usual range of normal frequencies. This can be done by means of a coordinated system of dampers comprising the usual standard dampers plus auxiliary dampers having either very low or very high natural frequencies, as may be required. Low frequency auxiliary dampers should be attached farther from the support than the standard dampers with which they are used.

Curve I—Computed residual vibration assuming that the damper is carried freely by the cable and that EI is constant

Curve II—Same as curve I with allowance for changes in EI with amplitude

Curve III—Same as curve II with allowances for tendency of damper to form partial node

Curve IV—Final expected amplitude of vibration including estimate of interstrand damping in conductor cable with high velocity wind

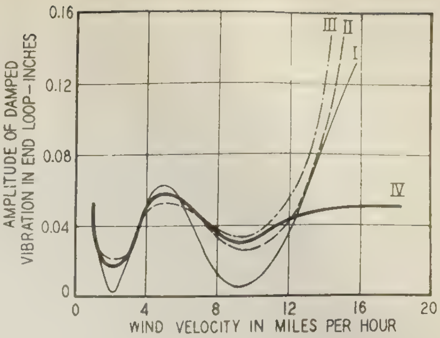


Fig. 16. Variation of amplitude of damped vibration in end loop with wind velocity

1,000 foot span of 397,500-circular mil steel-reinforced aluminum cable with 2 standard 10 pound Stockbridge dampers

High frequency auxiliary dampers, however, should be attached nearer the support than the standard dampers.

Knowing the hysteresis characteristics of the damper cables, the effects of different sizes and shapes of damper weights on the efficiency of a damper on a given size conductor may be determined theoretically. By considering several modifications in design, the most efficient type may be chosen on the basis of fundamental short-time tests and the theoretical analysis.

Stresses in Damper Cables

The stresses occurring in the damper cables may be found by applying equation 52 as soon as the bending moment is found. The bending moment at the damper weight will be given directly by the value for C, the maximum value of which is shown in curve II of figure 15. For a wind velocity of 12 miles per hour, the stresses in the damper cable may be computed as follows:

The couple acting on the damper cable at the damper weight is indicated by curve II in figure 15. For a wind velocity of 12 miles per hour ($w/p_1 = 5.64$), the value of C_{max} is 1,360 inch-pounds for $A_0 = 1$ inch (amplitude = 2 inches). From figure 16 the final amplitude is found to be about 0.05 inch ($A_0 = 0.025$ inch). This gives a maximum couple of 34 inch-pounds at the damper weight. The maximum stress then is found from equation 52 to be

$$S = \frac{34 \times 0.0607 \times 29{,}000{,}000}{3{,}500} = 17{,}200 \text{ pounds per square inch}$$

A value for significant stress at the center clip may be found by finding the couple, $\overline{C}_p$, existing at the instant of maximum direct force, $\overline{P}_{max}$. Actual computations show that this value of $\overline{C}_p$ is practically equal to C_{max}.

From figure 15 it is found that for a value of $w/p_1 = 5.64$,

$$P_{max} = 390 \text{ pounds for } A_0 = 1 \text{ in.}$$

For an amplitude of 0.05 inch (A_0 = 0.025 inch) this amounts to a force of 9.8 pounds, and for $w/p_1 = 5.64$,

$$C_{max} = 1{,}360 \text{ inch-pounds for } A_0 = 1 \text{ inch}$$

which gives

$$C_{max} = 34 \text{ inch-pounds}$$

The maximum moment at the clip then is

$$M_c = (9.8 \times 5.95) - 34 = 24.2 \text{ inch-pounds}$$

without the dead weight which for the upward stroke would be added, and for the downward stroke subtacted

The dynamic moment then is

$$M_c = 24.2 + (5 \times 5.95) = 54.0 \text{ inch-pounds}$$

and the maximum stress, according to equation 52, is

$$S = \frac{54.0 \times 0.0607 \times 29{,}000{,}000}{3{,}500} = 27{,}200 \text{ pounds per square inch}$$

For a preformed extra-high-strength steel cable these stresses are safe and represent a good factor of safety.

A study of figures 15 and 16 in the light of the foregoing computations shows that at higher frequencies or higher wind velocities the stresses in the damper cable will not be greatly different than for the case considered, and at lower frequencies or lower wind velocities stresses will be appreciably less. The indication is that the damper should give long service under normal operating conditions. Similar studies of other sizes of dampers are in progress, and the results found are being correlated with test results and actual observations on dampers in service.

Comparative results of computations for different dampers indicate their probable relative life as well as their relative efficiency, and also indicate which factors may be changed to increase the life of dampers without materially decreasing their efficiency or to increase their efficiency without reducing their life. The analysis also makes possible the design of dampers to meet any given conditions encountered in service. Final verification of the validity of conclusions reached in this way has been obtained during the past few years for several standard dampers.

References

5. Vibration of Overhead Transmission Lines, R. A. Monroe and R. L. Templin. A.I.E.E. Trans., v. 51, Dec. 1932, p. 1059–73.

6. Stress-Strain Studies of Transmission Line Conductors, G. W. Stickley. A.I.E.E. Trans., v. 51, Dec. 1932, p. 1052–8.

7. Field Tests on Conductor Vibration, E. M. Wright and J. Mini, Jr. Elec. Engg., v. 53, July 1934, p. 1123–7.

8. Are Wire Failures Progressive Within Suppressive Equipment or Underneath Reinforcing Equipment? F. W. Deck. Unpublished discussion presented at the technical conference on vibration held on June 27, 1935, during the A.I.E.E. summer convention, Ithaca, N. Y. (See Elec. Engg., v. 54, Aug. 1935, p. 906.)

9. Vibration of Transmission Line Conductors, Ernest Bate. Institution of Engrs., Australia, *Trans.*, v. 11, 1930, p. 277.

10. Vibration During Acceleration Through A Critical Speed, F. M. Lewis. A.S.M.E. *Trans.*, APM-54-24; 1932, p. 253–9.

A Faster Carrier Pilot Relay System

Carrier pilot relaying has been regarded by many as the nearest approach to a perfect protective relay system; however, it has not been adopted generally because of its high cost. With the intention of making still further improvements and reducing the cost of carrier pilot systems, a careful investigation has been made. The results of the investigation are presented in this paper, and a new carrier pilot system, designed in accordance with these findings, is described.

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DURING the last 3 years carrier pilot relaying has passed from the experimental stage to become an accepted tool in the protective practice of the central station industry. The experience of many users of pilot relaying has demonstrated conclusively its reliability and its ability to provide the best available relay protection for transmission circuits. Several technical papers attest that it gives fast and selective clearing of short circuits obtainable in no other way with equal assurance that undesirable tripping will not be caused from load swings or external faults.

Although pilot protection has received general acceptance as the nearest approach to an ideal relay system, its widespread adoption has been impeded greatly by its cost. To many engineers this form of protection therefore has seemed to be a last resort, considered only when nothing else would suffice.

In order to make carrier pilot relaying more generally useful, the subject has been given a careful survey to determine the extent to which it might be improved further, and whether the cost might be lowered sufficiently to make carrier protection well worth the extra cost over other forms of relaying. If this could be accomplished, the transmission circuits of the United States could be given far better relay protection than they have now, with such consequent improvement in service that the cost soon would be repaid.

A paper recommended for publication by the A.I.E.E. committee on protective devices, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted April 10, 1936; released for publication May 1, 1936.

Objectives

Until a new method of accomplishing something has demonstrated conclusively its practicality and has received recognition, it seems logical to construct it of the best available elements, connected in the best arrangement known. As time passes and its performance is shown to be satisfactory, a stage is reached at which it seems advisable to design devices especially adapted to the requirements of the method, and to make the changes that evolution inevitably will produce in any progressive art. A review of the status of the art showed that the fundamental features of the earlier systems of carrier relaying should be retained, and that simplification of the devices and connections was possible without sacrificing those desirable features. One important aim, therefore, was to simplify, and reduce the cost of, each terminal.

Speed is another respect in which improvement was found possible. The operating time of carrier relaying had been reduced from 10 cycles to 2 or 3 cycles; still it was not quite equal in speed to the fastest high speed relays of other types working under favorable conditions. Hence, a decrease in tripping time was sought.

In all carrier relaying systems, the carrier portion of the equipment actually is needed only for the brief period during which a short circuit exists external to the line, and it has been suggested that it might be employed for other purposes during the time of normal line conditions. If this proved to be practical, the cost could be distributed over the 2 services and thus make each a more economical investment.

Analysis of Desirable Features

In order to utilize carrier transmission only over assuredly sound circuits, all carrier relaying systems in the United States use the carrier as a blocking medium to prevent relay tripping during external faults. Although the carrier always has been used to block tripping during faults, there were 2 methods of operation during normal conditions, that is, while no faults existed. In one method the carrier equipment was idle until an external short circuit occurred, whereupon the relays turned on the carrier at the line terminal nearest the fault to block its own and the remote end. In the other method the carrier equipment was in continuous operation, always maintaining the block except when fault current flowed into the line at all terminals. A combination of these has been described, in which the carrier is not normally on the line, but the blocking relays are held in the blocking position at all times except during internal faults. Each of these methods has some good characteristics, but none excels in all respects.

With carrier used only as a blocking means to prevent tripping during external faults, the conductor is a perfect medium for a carrier channel because the fact that it is carrying short circuit current proves its continuity. The only apparatus requiring maintenance, therefore, will be in the station yard, where all equipment is readily accessible. This valuable principle, assuring a sound channel for blocking, certainly should be retained.

Another characteristic common to the different systems of carrier pilot relaying is the use of directional relays to determine the fault location by a comparison of the power direction at the ends of each line. It is highly desirable to retain the advantages incidental to the use of such simple and familiar devices.

In all systems of carrier relaying now on the market the carrier equipment performs a simple telegraphic function. In any revision of the relay scheme this desirable simplicity should be kept prominent.

In the intermittent carrier system the blocking action was initiated by the outward flow of short circuit current; therefore, this arrangement had the advantage of permitting the clearing of an internal fault, even though it was fed from only one end of the line, no source of power being available at the other end. This means that the relays were free to trip a faulted line at any time, unless there was positive indication of an external fault. The infrequent use of the carrier equipment prolonged the life of the tubes and made it possible to employ the carrier for other purposes when it was not required for relaying.

Prevention of tripping during periods of instability was accomplished readily, because loss of synchronism is identified with a sequence of apparent fault locations that never occur under other circumstances, and are therefore distinguishable from all other fault conditions.

The continuous carrier and normally blocked intermittent carrier systems possessed an advantage in speed, because the normally blocked condition of the

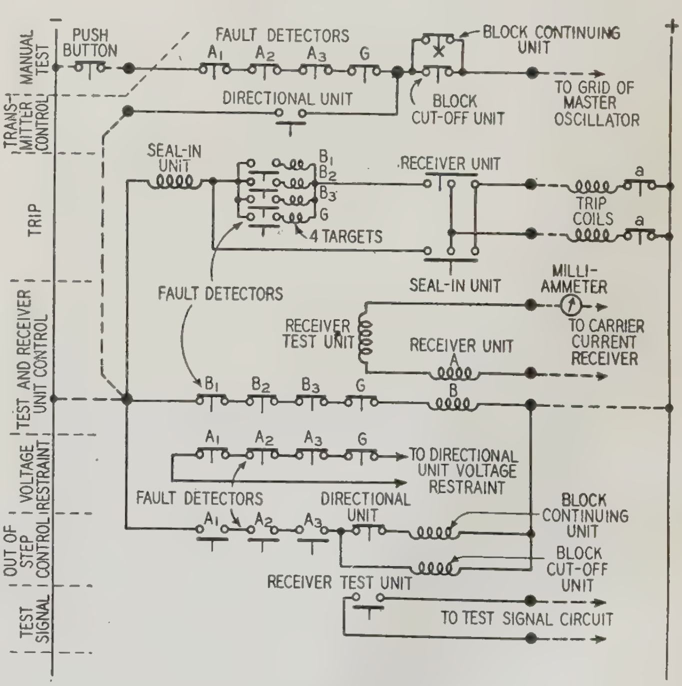

Fig. 1. Diagram of relay control circuits at one terminal of a line

Contacts are shown in normal positions

carrier receiver relay made unnecessary any intentional delay in the tripping relays to allow time for the block to be established during external faults.

Desired Characteristics

The characteristics that should be adopted from the older methods in any new one are:

1. Use of directional comparison.
2. Use of carrier to block tripping during external faults.
3. Freedom of relays to trip on any fault, unless receiving positive indication of an external fault.
4. High speed.
5. Use of a simple telegraph carrier channel.
6. Use of carrier only at times of fault.

and optionally,

7. Ability to block tripping during power swings or loss of synchronism.

Further improvements that are believed desirable for incorporation in a new method are:

1. Simplified unit construction of the relay equipment.
2. Higher speed.
3. Optional groups of relay units for differing service conditions.
4. Accessories for making possible the joint use of the carrier channel.

Description of the New Arrangement

Starting with these premises and the experience gained from the construction and operation of the many carrier relay installations now in use, a new and fast method embodying the foregoing characteristics has been devised. Like the earlier types, it is a blocking system in which directional relays permit a blocking signal to be transmitted from the end of a line nearest an external fault to its own receiver and to those at the ends remote from the fault. The carrier signal is transmitted only during faults outside of the line, except for momentary "flicks" which will be referred to later. Notwithstanding, the receiver unit is held locally in a normally blocked position. As explained later, the use of grid control in the transmitter solves the problem of contact

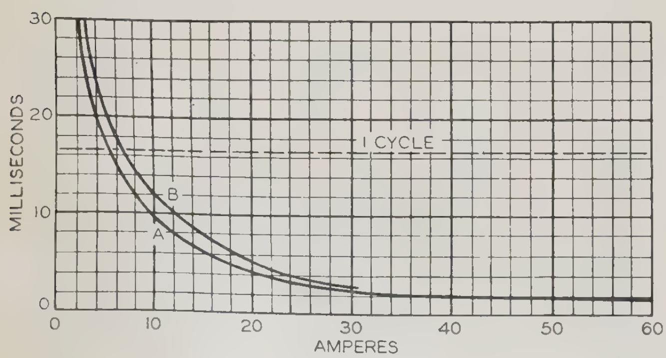

Fig. 2. Time characteristics of phase fault detectors

A—Fault impedance zero
B—Fault impedance 4 ohms
Pickup setting, both curves: 10 amperes, 100 volts; 2 amperes, zero volts. Rated voltage and no load before fault

racing at either the beginning or end of a fault in some other line, or when an instantaneous reversal results from the opening of a circuit breaker in a parallel circuit. Hence, tripping may occur whenever an internal short circuit occurs, regardless of the previous condition of the system and the positions of the relay contacts just prior to the short circuit.

Figure 1 is a diagram of the relay control circuits at one terminal of a line. From this figure it may be seen that the relay equipment consists essentially of:

Fault detectors A with one circuit closing contact and 2 circuit opening contacts, to act as starting units.

Fault detectors B with one circuit opening contact and one circuit closing contact, for trip circuit control.

A combined polyphase and ground current directional unit having one circuit closing and one circuit opening contact.

A receiver unit having contacts that are open when the unit is energized.

A series seal-in unit.

A receiver test unit.

Targets.

Out-of-step auxiliaries.

Action During Faults

To make certain of correct blocking under circumstances that cause the current to rise slowly above the fault detector setting, 2 sets of fault detectors are used, and the several contact functions are divided between them as indicated by the letters A and B. The units labeled A are set for a sensitivity somewhat greater than those marked B at the opposite end of the line. One set of circuit opening contacts on the A fault detectors applies a negative bias to the transmitter grid that renders it inoperative normally. Whenever a fault or other similar condition occurs in the vicinity of the protected section, either inside or outside of it, one or more of the fault detectors opens the bias circuit and starts the transmitter within a fraction of a cycle.

Circuit opening contacts on the B fault detectors control a local circuit to coil B of the receiver unit which is normally completed to hold its contacts open. Thus at the instant the fault detectors operate they transfer the holding action from the local holding circuit, to a carrier holding action by means of the local transmitter and receiver; therefore the contacts of the receiver unit remain open. If the occasion for the foregoing operation of the fault detectors is an external fault, no further action takes place at the line terminal where the power flows out of the line; therefore, carrier transmission continues to hold open all receiver units of the protected section until the fault terminates and the detectors reset themselves.

At a line terminal where power flows into the line, the same procedure is followed with the additional closing of the directional unit contact which reapplies the negative grid bias and stops carrier transmission at that end. Carrier received from the other end, however, prevents the completion of the trip circuit.

For internal faults the carrier first is turned on by the detectors at any end where there is fault current;

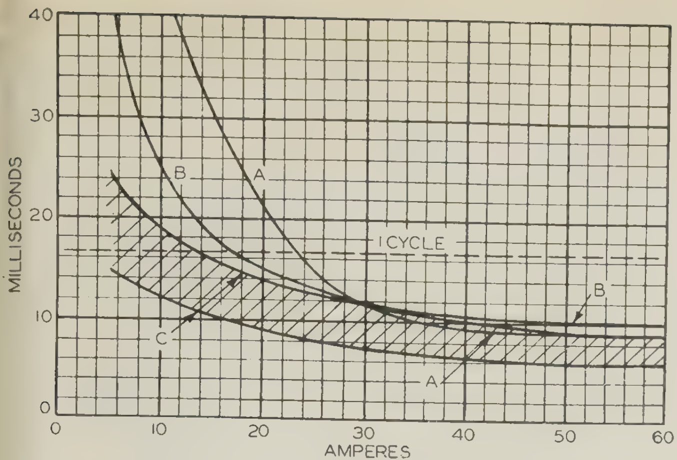

Fig. 3. Time characteristics of polyphase directional unit

A—3 phase fault, 0.25 ohm reactance
B—3 phase fault, 1.75 ohms reactance
C—Phase-to-phase fault, voltages 0–100 and 100 volts
Shaded area shows range of maximum and minimum time values expected

then it is turned off promptly by the directional units. As soon as reception of carrier ceases, the receiver units drop out and close the trip circuit. To prevent any false tripping, in case the local holding circuit is open, the circuit closing contacts of the *B* detectors are included also in the trip circuit. The tripping time is, therefore, the sum of the closing time of the directional unit and the drop-out time of the receiver unit.

It is evident that any terminal feeding fault current into a line may be cleared, even though some other terminal supplies no current because the fault detectors will not turn on the carrier transmitter. This point is important in connection with lines that may have synchronous apparatus at only one end at times and in connection with lines having more than 2 ends.

In the foregoing description it was assumed that the bias circuit contacts of the directional units were both open at the start. This is a condition that is realized by including a voltage restraint torque in these units. The sensitivity to fault conditions is increased by removing the restraint with the third set of circuit opening contacts on the fault detectors; however, it makes no real difference whether the directional units are open or closed, because one opens more rapidly than the other closes. Thus, there is no time subsequent to the start of an external fault, when one end or the other is not transmitting carrier and hence continuing the block at both ends.

Similarly, a familiar condition that has upset many good relay schemes is rendered harmless, namely, the sudden reversal occasioned by the opening of an end of 1 of 2 parallel lines. Under this condition, carrier is sent first in one direction and then must be sent in the other direction without any cessation, to avoid tripping the unfaulted line. Since an opening contact on one directional unit starts carrier at its end while a closing contact stops it at the other, there is actually a brief time during which both ends are transmitting and thus preventing undesirable tripping.

When 2 faults occur simultaneously in different line sections, both affect the directional units. The more widely separated terminals always act properly, as though the line were one big section, but the other 2 intervening terminals may be subjected to a variety of conditions wherein each of these internal faults is, to some degree, external to the other section, and the directional tripping torque therefore is counteracted to some extent. Proper functioning is assured, however, in one or the other of the following ways.

If there is little or no synchronous apparatus to feed both lines from a bus between the 2 faults, one of these torques will predominate; accordingly, 1 of the 2 intervening directional units will indicate an internal fault and the other will indicate an external fault. The first will clear its line without delay, and the second will follow immediately.

If, on the contrary, there is sufficient synchronous apparatus connected between the fault locations, it will add to the tripping torque of the directional units at both intervening terminals; consequently, the 2 defective line sections will be cleared simultaneously and almost instantaneously.

Action During Other Abnormal Conditions

The foregoing description applies to any type of short circuit that may occur. In addition to faults there are other abnormal conditions that affect the relays in much the same way as a fault. For example, loss of synchronism first has the appearance of an external 3 phase fault condition of increasing severity. As phase opposition of the synchronous apparatus at the ends of the line is approached, the indication changes to a 3 phase internal fault of brief duration, then it appears to be external in the opposite direction, with diminishing severity. This cycle is repeated for each slip cycle of the synchronous apparatus at the 2 ends of the circuit. Very severe power oscillations not accompanied by loss of synchronism follow a similar cycle, except that the apparent fault direction is the same before and after the internal indication, instead of opposite. A slow approach to the relay settings invariably will cause the *A* fault detectors to operate to start carrier and remove re-

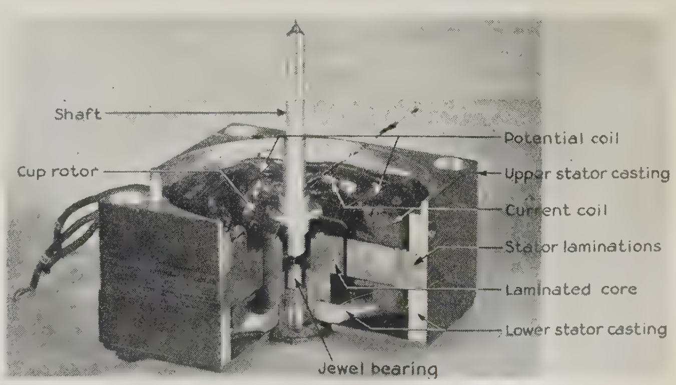

Fig. 4. The cup type of directional element

straint from the directional unit before the B fault detectors remove the local holding circuit of the receiver unit and otherwise prepare for tripping. By this means, blocking is assured for all conditions of excessive through power, whether it is caused by a short circuit or merely an abnormal load. During out-of-step conditions, blocking will be accomplished

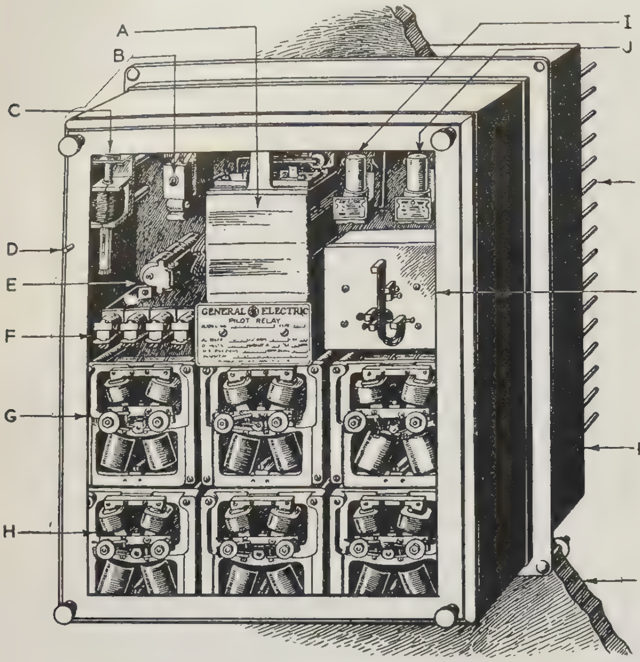

Fig. 5. Complete pilot relay equipment in one case

A—Directional unit
B—Seal-in unit
C—Ground fault detector
D—Target reset
E—Receiver test unit
F—Targets
G—*B* fault detectors
H—*A* fault detectors
I—Block cut-off units
J—Block continuing unit
K—Connection studs
L—Receiver unit
M—Compartment for auxiliaries and wiring
N—Panel

from one end or the other at all times except for a brief period during which the currents at the 2 ends are approximately 180 degrees out of phase; therefore, it is necessary to bridge only this short interval by continuing the carrier signal for about one second if an excess current in all 3 phases is indicated when the directional relay otherwise would cut off carrier. This block continuing action is terminated abruptly whenever any one of the 3 fault detectors resets itself to normal. The continuing period is controlled by an auxiliary unit that picks up almost instantly and, because of a copper jacket, drops out on time delay. The auxiliary unit is controlled by the circuit closing contacts of the A fault detectors, and by the back contacts of the directional units. The instantaneous cessation is brought about by a second auxiliary unit that is instantaneous in both directions and cuts off carrier immediately when any A fault detector resets to normal.

So long as there is an indication of a 3 phase external fault, both auxiliary units will pick up and open the grid bias. As the torque of the directional unit reverses and it moves to the trip position to reapply the grid bias, it will first de-energize the time auxiliary unit coil, but the bias will not be applied until the time unit also drops out. Before it drops out and turns off the carrier, the apparent fault location will have changed again to external, and one of the directional units will have opened its bias contact to start the other carrier transmitter. A continuous transmission of a blocking signal thereby is secured.

Should any internal short circuit, except a 3 phase one, occur during an out-of-step condition, one or more of the fault detectors will drop out for part of the slip cycle. This will let the instantaneous auxiliary unit drop out and by-pass the time unit contacts so that the directional unit can turn off the carrier immediately to permit tripping. A circumstance that introduces slight delay in the clearing of a fault is the occurrence of a 3 phase internal short circuit during loss of synchronism or immediately following an external 3 phase fault.

For circuits in which instability or power oscillations do not occur, and in which there are no other reasons for current and voltage values that approach the setting of the fault detectors, one set of phase fault detectors and the instability block auxiliaries may be omitted. This results in some simplification in equipment and has no effect upon the performance during short circuits.

Description of Various Units

Fault Detectors. The fault detecting units are conventional impedance relays in which current is balanced against voltage, but torque has been increased and inertia has been reduced, so that an exceedingly high speed is secured. With normal voltage and zero current, a 4 volt-ampere input to the potential coil gives the very strong pull of 200 grams in the plane of the contacts. The time-current characteristic is shown in figure 2. Adjustment is possible over a range of from 10 to 20 ohms.

The ground fault detector is a simple overcurrent unit of the plunger type, and of intermediate size.

Directional Unit. So far as relay design is concerned, one of the chief factors in the high speed secured is the new polyphase directional unit of the induction cup type, which was designed particularly for this application. In this cup type construction the entire periphery of the rotor is active and produces torque.

Furthermore, the diameter of the cup type rotor is small, and therefore the moment of inertia is kept low. With this double advantage of high torque and low inertia, the ratio of torque-to-inertia, which is a measure of speed, is very high.

The problem in designing a fast induction relay is to produce, at a minimum radius, the maximum force available from a given input. For this reason all the coils of the unit are so arranged around the outside that the active ends of their poles may be focused on a rotor of small diameter and at the same time the coils may be in the best position to dissipate their heat losses, as shown in figure 4.

Securing fast action is, of course, only half of the problem. Stopping the motion of the contacts without rebounding is just as important. In the poly-

phase directional unit this problem was solved by the use of a movable mass normally resting against the back of the fixed contact. This movable mass is so proportioned that it will accept the kinetic energy of the moving contact, shaft, and rotor, and retreat with that energy, leaving the original moving mass standing quietly at contact. The familiar analogy to this action is the striking of one billiard ball squarely with another. The moving ball stops abruptly, and the previously static one assumes a speed substantially equal to the speed of the first ball at the moment of impact. In the relay the absorbing mass also is in the shape of a ball. It is confined in an inclined tube with fairly close clearance. This makes for a dashpot action which, between the outward and the return movements, expends all the energy and allows the ball to ease back against the fixed contact without creating a new shock.

This new polyphase unit has 3 elements condensed into one, thus saving space and affording a more economical design. The unit tends to be as fast as if 3 elements all jointed on one shaft were used, because the ratio of torque to inertia is the same for 1 or 3 cups. The over-all torque necessarily is less with one element than would be possible with 3 of the same size; however, the efficiency of the cup enables a single element to produce a torque (and hence contact pressure) greater than that of the 3 disk type elements that have long been considered adequate.

Another innovation is the addition of a directional ground element on the same shaft with the polyphase elements. In this way the problem of giving the ground units preferential control is solved without the complication of interlocking contacts, so that regardless of the type of fault, the torque produced by the fault current will predominate at once.

A complete description of the many good points of this new unit, the concepts back of the design, and the exceptional performance, would make a story much too long to tell here.

The time characteristics are shown in figure 3. Observe particularly the speedy action shown by curve *C* for single phase line-to-line short circuits As this represents the great majority of faults between phases, this curve is the most representative one. It is a performance not adversely influenced by nearness of the relay to the fault. In fact, the action is fastest and most thoroughly reliable on a short circuit right at the station.

The advantage of this characteristic is great, for complete and proper protection is secured not only for all types and locations of single phase faults, but also for 3 phase faults that were single phase for the first cycle.

To insure correct action for the very rare case of an initial 3 phase fault, the secondary potential should be one volt or more. This is because of possible transients during the first cycle and because of the high speed of the unit.

Lightning, the most prevalent source of trouble almost invariably will create an arc in any fault it causes, and this arc persists during the short time required for relay tripping. The arc alone, because of its minimum length (insulator flashover distance) will give sufficient secondary potential on any voltage system to polarize properly the directional unit to give correct results.

Receiver Units. The receiver units, which serve to block undesired tripping, have undergone comparatively little change and are still of the same substantial, fully insulated construction as formerly, but they have been speeded up, as have the fault detectors and the directional units, so that they drop out and close contacts in $^1/_3$ cycle.

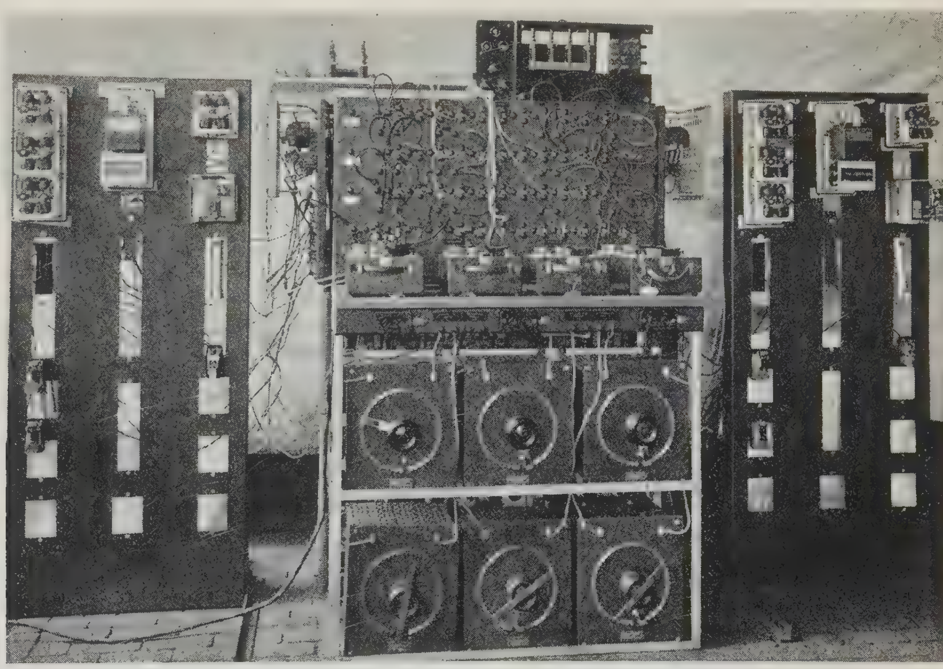

Fig. 6. Transmission line table set up for carrier pilot relaying test

Seal-In Unit. Once the tripping circuit has been made, it is sealed by the familiar auxiliary relay, until opened by the circuit breaker auxiliary switch.

Receiver Test Unit. The receiver test unit shows whether the receiver signal strength is ample for its purpose. The unit is of the telephone type and is purposely less sensitive than the main receiver unit, which is in the protective circuit. If, therefore, on periodic tests the test unit functions, it will be a clear indication of a good margin of carrier to control the more sensitive receiver unit in the trip circuit.

Targets. The relay equipment includes 4 electrically operated targets, 3 of which indicate whether the fault is phase-to-phase, and which phases are involved. The fourth target indicates if tripping was due to a ground.

Out-of-Step Auxiliaries. The block continuing and the block cut-off auxiliary units are similar to telephone relays, except that they are provided with vacuum contacts. The time delay drop-out feature for the block continuing unit is provided by a copper slug to trap the flux and cause it to die out in about 0.8 second.

Only One Case. A new feature is the assembly of all the units with their accessories in one case as illustrated in figure 5. This saves an appreciable amount of space and eliminates all back-of-board connections except the power supply leads. The only studs to be brought out of the relay are those required for the remote connections. Figure 7, which shows the backs of the panels, with a test arrangement of the previous type of equipment using individual relays, is a good example of the amount of back-of-board wiring that recently was necessary, but now is avoidable. This, in general, represents extra connections that are not required inside the case because every coil and contact is brought to readily accessible connecting blocks in the back of the case where the connections between elements are made.

It is as simple and as easy to remove any unit from within this single case as if it were a separate relay mounted in the familiar fashion. This now becomes a relay equipment that is easier to test, install, and maintain than a group of separate relays. The same relay may be used with a wire channel, if for any reason this should prove preferable.

Test Table

Although field tests have their proper place in the development of new equipment, they are not a satisfactory means for design, because of the difficulty of accurately controlling the many variables involved. The complete equipment herein described has been subjected to severe and exhaustive factory tests that were comparable with a long period of actual service.

To make possible this comprehensive testing of carrier current protection under equivalent system conditions all under control from one point, a variable artificial line has been constructed with reactance characteristics, in miniature, closely approximating the transmission circuit being studied from a protective viewpoint.

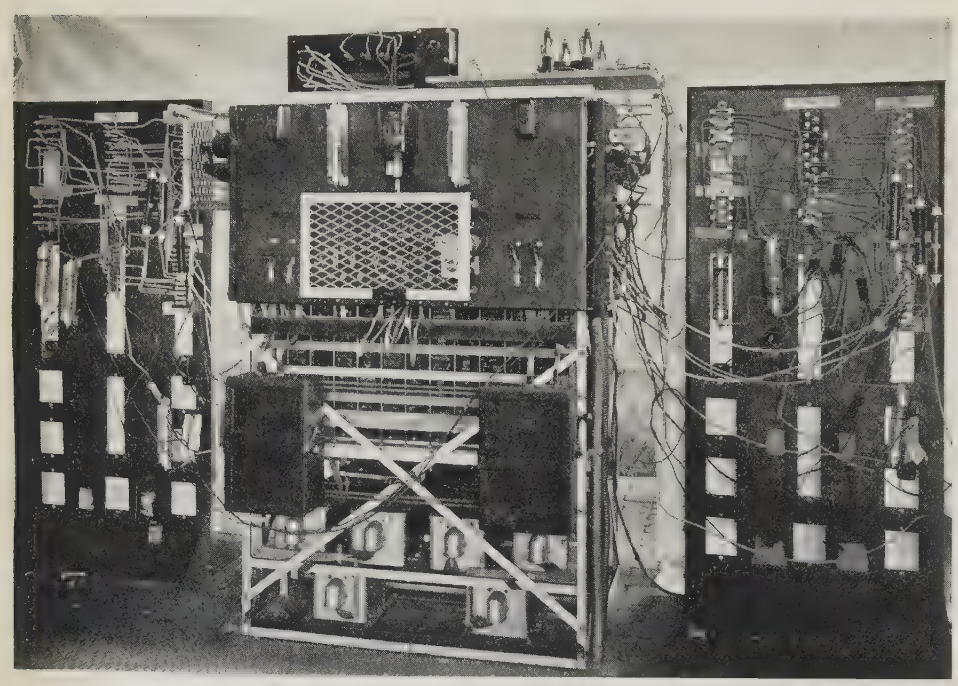

Fig. 7. Back of the carrier pilot test table

This transmission line table is equipped with 54 tapped reactors wired to 4 rows of terminals, representing 3 phases and ground, and capable of being interconnected to form a total system reactance up to 26 ohms on a 220 volt, 5 ampere basis. Of this total, any desired portion can be chosen to correspond to the line being studied, and tests may be made almost as accurately as they could be made in service, and far more completely than has been attempted on a high voltage line. No capacitors are

used in this artificial circuit, because line capacitance plays a negligible part in the performance of relays.

The table, together with 2 skeleton panels which carry the individual relays now grouped as units in one case, is shown in figure 6 and the rear view is shown in figure 7.

On each end of the line, and under control of the test relay equipment in a conventional way, are 2 cycle circuit breakers.

Short circuits can be thrown on any phase to ground or between any phases either inside or outside the line section under observation, or they may be both inside and outside, as in the case of simultaneous double grounds.

The alternators at each end of the system are large in comparison with the rating of the table; therefore, their speed is affected only slightly by changes in the power transmitted through the table. They can be made to operate with a normal interchange of power, or they can be thrown out of step and run at any desired slip frequency. This provides an opportunity to check the relay performance under all conditions of operation.

Test Performance

Many tests have been made, with correct results, and in addition, a series of demonstrations has been made before groups of engineers who have manipulated the controls with no embarrassment to the relay or its sponsors. Oscillograms in figures 8, 9, 10, and 11 show the intervals between the functioning of the various contacts and the over-all time where tripping was in order.

Manual and Automatic Test Equipment

Manual testing is performed by depressing the pushbutton in the grid bias circuit to turn on the transmitter. This operates both receiver test units to indicate correct functioning. It is customary for the operator at the remote end to answer by pushing his button and thus check the over-all operation of the carrier equipment. The frequency of test varies from once an hour to once a day on different systems. A milliammeter furnished for the receiver plate circuit may be used to determine the received signal.

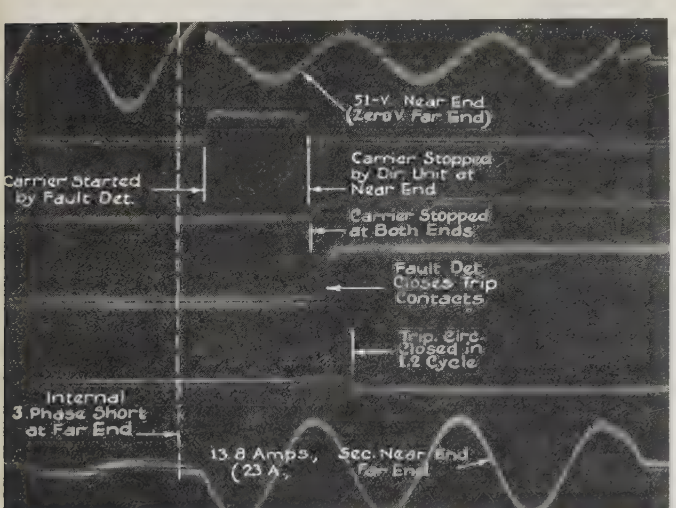

Fig. 8. Oscillogram of 3 phase internal fault at one end of a line

Although the foregoing provision usually is considered quite adequate, automatic test equipment is available for those that desire it. Carrier is transmitted periodically from the originating station and received at the responding station. The responding station then follows with an answering carrier signal that will be the occasion for an alarm if it is not received by the originating station within the time expected.

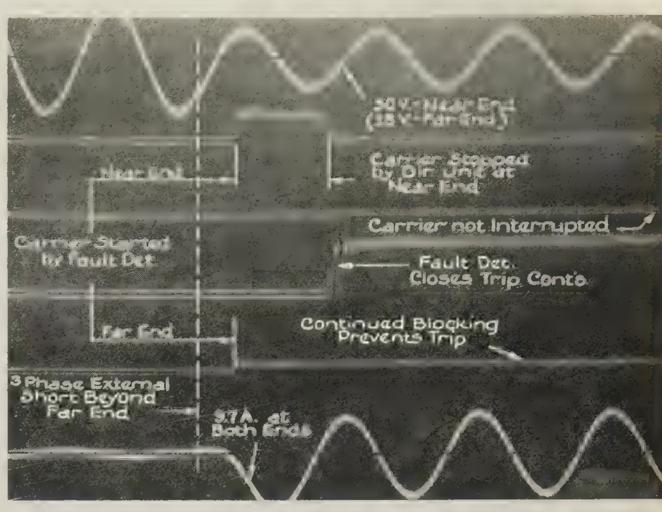

Fig. 9. Oscillogram of 3 phase external fault

The equipment is entirely safeguarded against interference with its protective functions. No test signals can be sent or returned if the tripping units begin to function.

Joint Use of Carrier Equipment

The infrequent and brief use of the carrier equipment for protective purposes leaves the carrier channel idle most of the time. The use of the channel for other purposes when not required for protection, therefore, gives an added value. Thus, carrier relaying apparatus makes available, at small additional cost, a telephone or telemeter circuit that will help share the expense of the joint channel.

The protective functions are not affected, for the fault detectors will turn the carrier on whenever they operate and leave the directional unit in full control. During these periods the other use of the channel obviously is interrupted; but since these periods constitute only an extremely small proportion of the total time, no inconvenience will result.

There are several possibilities of incorporating the relay carrier equipment into the communication system. By means of a simple modulating unit, any section of carrier relaying may be used as a station-to-station simplex telephone channel.

Where communication is desired between 2 points connected by 2 transmission line routes, duplex serv-

ice is possible. Speech in one direction passes over one route and in the other direction over the other route.

Other variants are possible, but it is believed that these 2 applications will cover many of the communication requirements.

The relaying use of the carrier equipment only at the time of faults and the "trip-free during test" feature permit any section of carrier relaying to serve as a channel for any type of telemetering system that requires making and breaking a circuit. Where the indication is to be transmitted from one end to the other of a single section of line, the contacts of the sending instrument are inserted in series with the test pushbutton. At the receiving end a long life telephone relay is added externally, and takes the place of the usual test unit. Its contacts control the indicating or recording telemeter just as though a wire channel connected the ends. If the indication is to be conveyed over 2 or more line sections in series, the receiver relay contacts of the first section are used to turn on and off the transmitter in the second section, and the process continues until the destination is reached. The channel may include also wire sections for extending the indication to points not directly reached by the carrier channel. Hence, merely by adding any type of "on-and-off" telemeter and adding an external receiver relay of a type designed to withstand a duty of several operations per second, indications may be transmitted at small additional cost.

Conclusions

New designs of relay units and improved control connections, now have given carrier pilot relaying twice the speed of previous systems giving equal protection, and simplification has reduced the cost, which should greatly broaden the field of application. The availability of the carrier channel for joint use with telephone or telemetering functions increases its economic possibilities still further. Now that a speed of one cycle has been attained for the more severe short circuits, with no offsetting disadvantages, further speed reduction will have no real significance. Thus, there is available a relaying system that gives the recognized advantages of the pilot system—fast clearing of any kind of fault; reduced duration of voltage dips; less damage; less shock to the system; less loss of load—hence, better satisfied customers.

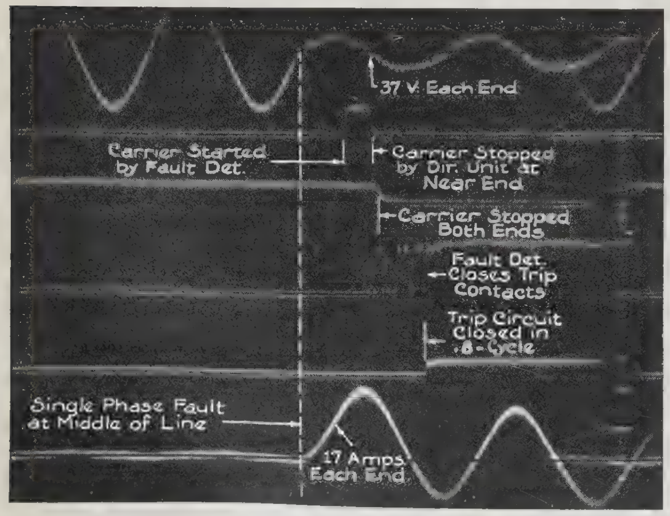

Fig. 10. Oscillogram of internal single phase line-to-line fault at middle of a line

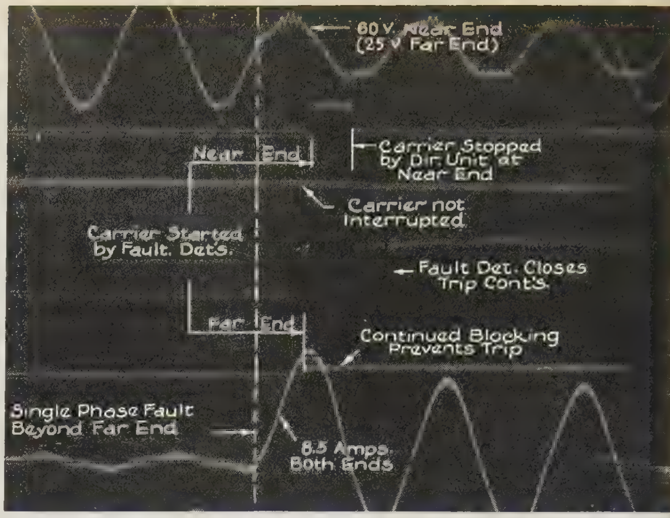

Fig. 11. Oscillogram of external single phase line-to-line fault

References

1. A Carrier-Current Pilot System of Transmission Line Protection, A. S. Fitzgerald. A.I.E.E. Trans., v. 47, Jan. 1928, p. 22–29.

2. Carrier-Current Relaying Proves Its Effectiveness, Philip Sporn and C. A. Muller. *Elec. World*, v. 100, Sept. 10, 1932, p. 332–36.

3. Pilot Protection by Power-Directional Relays Using Carrier-Current, Oliver C. Traver, John Auchincloss, and E. H. Bancker. *Gen. Elec. Rev.*, v. 35, Nov. 1932, p. 566–70.

4. Carrier-Current Relaying Simplified, Philip Sporn and C. A. Muller. *Elec. World*, v. 102, Oct. 28, 1933, p. 556–60.

5. Some Recent Relay Developments, Lloyd Hunt and A. A. Kroneberg. Elec. Engg. (A.I.E.E. Trans.), v. 53, April 1934, p. 530–35.

6. A Carrier Current Relay Installation, O. A. Browne and W. L. Vest. Elec. Engg. (A.I.E.E. Trans.), v. 54, Jan. 1935, p. 109–15.

7. Fault and Out-of-Step Protection of Lines, H. D. Braley and J. L. Harvey. Elec. Engg. (A.I.E.E. Trans.), v. 54, Feb. 1935, p. 189–200.

8. Protection Sélective Par Haute Fréquence, J. Fallou. Bulletin of Societé Française des Electriciens, Sept. 1931, p. 956–70.

9. Les Reseaux de Transmission D'Energie (a book), J. Fallou. Gauthier-Villars, Paris, 1935.

10. Schnellabshaltung beim Selektivschutz, H. Neugebauer. *E.T.Z.*, v. 55, Feb. 1934, p. 181–84.

11. Streckenschutz mit Hochfrequensverbindung, H. Neugebauer. *Siemens-Zeitschrift*, v. 10, March 1934, p. 83–88.

12. Les Principes de la Protection des Feeders et leur Application a Trois Systemes Modernes, B. H. Leeson and H. Leben. Int. Conf. on High Voltage Networks, Session 1931, paper 106.

13. La Protection des Lignes Aeriennes et des Feeders, W. W. Clothier. Int. Conf. on High Voltage Networks, Session 1931, paper 106.

14. Étude sur les Systemes de Protection Selective a Fonctionnement Rapide, Dr. Schleicher and H. Neugebauer. Int. Conf. on High Voltage Networks, Session 1933, paper 101.

15. Die Entwicklung der Schnellschaltenden Schutzsystem in Amerika, Deutschland, England, Frankreich, R. Schimpf. Bulletin des Association Suisse Electriciens No. 13, v. 125, 1934, p. 336–41.

16. One-Cycle Carrier Relaying Accomplished, Philip Sporn and C. A. Muller. *Elec. World*, v. 105, Oct. 12, 1935, p. 26–28.

A New Distance Ground Relay

The theory and design of a new distance relay suitable for ground protection are described in this paper, and the results of 2 groups of extensive field tests are presented. These results demonstrate that the new relay performs in a manner that can be predicted mathematically, and that it avoids certain difficulties involved in the application of conventional distance relays to ground protection.

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SINCE the introduction of the distance principle for phase relaying, and particularly since the introduction of high speed phase relays, there has been a need for this same type of protection in connection with ground faults.

Distance relays for phase protection have proved their value in minimizing damage to lines and apparatus and interruption of service, and in raising the stability limit by means of their high speed operation and inherent selectivity. Although from the standpoint of stability, ground faults need not be cleared as rapidly as phase faults, their preponderance and the danger of their developing into phase faults if not quickly cleared, makes desirable the application of the distance principle to the clearing of this type of fault. Several difficulties have prevented the ready application of distance relays to ground faults heretofore; now there is available a relay that overcomes these difficulties. Extensive field tests have demonstrated that the theory and design of the relay is sound basically. A summary of the theory upon which the design is based and the results of the field tests may be of interest to protection engineers.

Summary and Conclusions

1. Conventional methods of distance relaying for ground faults are subject to errors due to: (*a*) the relatively large resistances present during ground faults; (*b*) changes in system connections external to the protected section; and (*c*) voltages induced in the protected line by the zero sequence current in a parallel line.

2. A relay has been designed based on the theory developed by Lewis and Tippett. It incorporates compensating elements that cope effectively with the difficult conditions encountered on ground faults.

3. Thorough field tests under widely different conditions have demonstrated the ability of the relay to provide against ground faults the same accurate high speed distance protection that has been available for many years against phase faults.

General Considerations

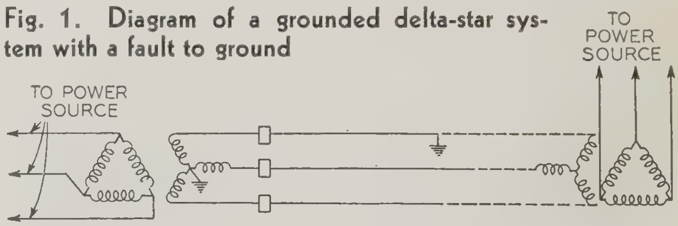
In order to measure accurately the distance from the relays to a ground fault, it is necessary to obtain the quotient of the proper quantities. Usually there must be a voltage quantity and a current quantity; furthermore, the voltage quantity must be proportional to the voltage drop caused by the current between the relay and the fault, and the current quantity must be proportional to the current producing the voltage drop. In general, the quantities that fulfill these conditions are the phase-to-ground voltage and the line current. These are phase quantities; therefore, 3 relays are required—one for each phase.

In the interest of reducing the number of relays, it has occurred to many engineers that it might be possible to employ one ground relay energized by the zero sequence voltage and current to obtain the zero sequence impedance. These quantities, however, do not give the correct impedance because the zero sequence voltage at the relay is not proportional to the impedance from the relay to the fault, but is proportional to the zero sequence impedance from the source of ground current to the relay.

Since the source of zero sequence voltage is assumed to be at the fault, the zero sequence voltage at the relay is $-I_0 Z_{0R}$, where Z_{0R} is the zero sequence impedance between the relay to the source of ground current, whereas the quantity desired is $-I_0 (Z_{0F} - Z_{0R})$, in which Z_{0F} is the impedance from the fault to the ground current source.

Previous attempts have been made to reduce the number of relays by employing various switching

Fig. 1. Diagram of a grounded delta-star system with a fault to ground



means whereby only 3 relays are required for protection against both phase-to-ground faults and phase-to-phase faults.[1] These switching schemes always require selector and transfer relays to change the relay voltage connections and relay settings, and the difficult and undesirable process of switching in the

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1. For all numbered references, see list at end of paper.

current circuit often is required. In addition to the inherent disadvantages of these switching schemes it is extremely doubtful whether any simplification in relay equipment is accomplished. There are certain fundamental requirements of an accurate distance ground relay that are extremely difficult, if not impossible, to meet with 3 relays and switching means to protect against all interphase and ground faults.

It may be concluded, therefore, that successful results can be obtained only by 3 relays utilizing line-to-ground voltage and line current, or voltages and

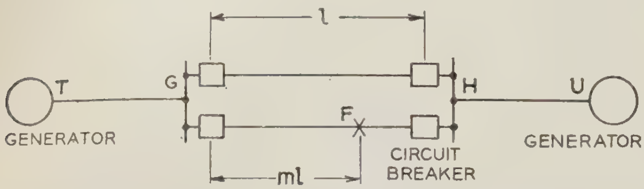

Fig. 2. Single line diagram of a system equivalent to the system of figure 1

currents derived from these quantities; however, conventional distance relays that are energized only by line-to-ground potential and line current will not give satisfactory results, for several reasons.

First, such a relay system is subject to considerable error because of the variation in the distribution of the positive, negative, and zero sequence impedances outside the protected section. For example, on a radial line fed from a grounded delta-star transformer bank, all of the positive, negative, and zero sequence currents flow through the relay (see figure 1). A distance relay with conventional connections can be adjusted to measure correctly the distance to a single phase ground fault under these conditions.

It should be noted that the zero sequence current is equal to $^1/_3$ of the line current; furthermore, the voltage applied to the relay is made up of the voltage drops caused by the positive, negative, and zero sequence currents flowing through their respective impedances. The zero sequence impedance, however, is materially greater than the positive sequence impedance and the contribution to the total line-to-ground voltage drop made by the zero sequence current therefore is considerably higher than the contribution of the other 2 sequence components.

Now suppose that the far end of the line be connected to a generator through an ungrounded transformer. It is obvious that the distribution of the sequence impedances has been altered since positive and negative currents flow from both ends of the line to the fault, but the zero sequence current still comes only from the relay end. The zero sequence current component in the relay no longer is equal to $^1/_3$ of the line current. In other words, a change of connections outside the protected line section has changed the distribution of the sequence components of the relay current, but the sequence voltages that made up the relay restraint have not been changed in the same manner, because the zero sequence current component operates on an impedance entirely different in value from the other component impedances. It is obvious, therefore, that a relay set to operate correctly for the first condition will not be correct for the second condition.

A satisfactory ground relay must be provided with a means to compensate automatically for the different distribution of positive, negative, and zero sequence currents due to system changes external to the section being protected. During ground faults there are 2 more fundamental conditions that are imposed upon a distance relay using conventional connections.

First, the fault resistance values encountered in ground faults generally are higher than those in phase-to-phase faults. This is due mostly to the tower footing resistance or to the particular manner in which the object causing the fault is connected to ground. Although distance relays of the impedance type have been proved entirely suitable for phase protection, it is obvious that the larger fault resistance of grounds would affect materially an impedance relay. A satisfactory ground relay, therefore, must operate as a reactance relay.

Second, the zero sequence mutual impedance between parallel lines materially changes the apparent distance to a ground fault at various locations, and a satisfactory relay must embody a means for correcting this error. Lewis and Tippett[2] have calculated in terms of symmetrical components the voltages and currents in all types of faults on transmission lines and also the apparent distance indications that would be given by conventional impedance and reactance relays.

An examination of the equations indicates 2 methods of eliminating the undesirable errors, a method of current compensation and a method of voltage compensation; that is, a subtraction of the undesirable voltage components from the line-to-ground voltage or the addition of the proper current components to neutralize the voltage components. The latter method has been used, since it possesses ad-

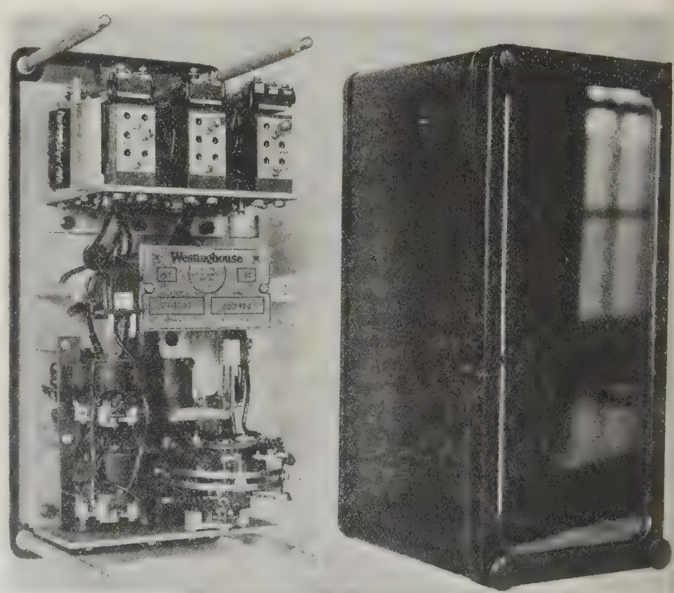

Fig. 3. The new reactance type of ground relay

vantages that are apparent from the Lewis and Tippett paper.

The general expression for the line-to-ground voltage for a line-to-ground fault is (see figure 2):

$$\mathbf{E}_{aG} = m\,\mathbf{Z}_{1GH}\left[\mathbf{I}_{aGF} + \mathbf{I}_{0GF}\frac{\mathbf{Z}_{0GH} - \mathbf{Z}_{1GH}}{\mathbf{Z}_{1GH}} + \mathbf{I}_{0GH}\frac{\mathbf{M}_{0GH}}{\mathbf{Z}_{1GH}}\right] + (\mathbf{I}_{aGF} + \mathbf{I}_{aHF})\,R_G \quad (1)$$

where

$\mathbf{E}_{aG}$ = line-to-ground voltage
$\mathbf{Z}_{1GH}$ = positive sequence impedance station G to H
$\mathbf{Z}_{0GH}$ = zero sequence impedance station G to H
$\mathbf{M}_{0GH}$ = mutual impedance between lines from G to H
m = fraction of distance from G to H at which fault is assumed to occur
$\mathbf{I}_{aGF}$ = phase A fault current
$\mathbf{I}_{0GF}$ = zero sequence fault current on faulted line
$\mathbf{I}_{0GH}$ = zero sequence fault current on parallel line
Bold face characters represent vector values.

Inspection of equation 1 will confirm what was said previously regarding the errors inherent in a relay connected conventionally. Multiplying the terms in the bracket by $m\,\mathbf{Z}_{1GH}$ results in

$$m\,\mathbf{Z}_{1GH}\mathbf{I}_{aGF} + \mathbf{I}_{0GF}\,m\,(\mathbf{Z}_{0GH} - \mathbf{Z}_{1GH}) + \mathbf{I}_{0GH}\,m\,\mathbf{M}_{0GH}$$

The first term represents the voltage drop caused by the line current in the positive sequence impedance. The second term represents the voltage drop caused by the zero sequence current, and the third the voltage drop caused by the zero sequence mutual impedance between parallel lines. A conventional relay using line current and line-to-ground voltage takes account of the first term only, since the second term is not represented in the relay current element. It may be seen that variations in the distribution of line current and residual current cause trouble only because the zero sequence impedance is different

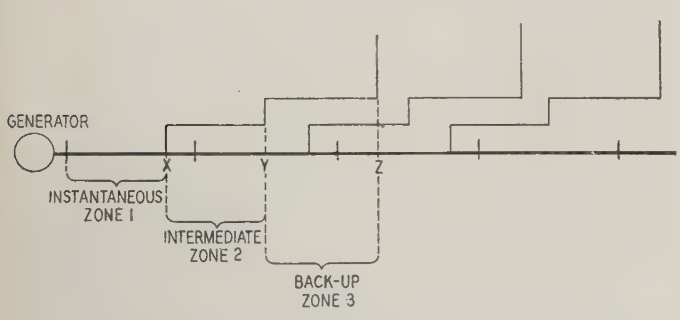

Fig. 4. Distance-time characteristic of balance-point distance ground relay

from the positive sequence impedance, for if they were the same, the second term would vanish.

It is evident that if a relay current I_r equal to the value of the expression in brackets (equation 1) is used, the indicated impedance is

$$\mathbf{Z}_R = m\,\mathbf{Z}_{1GH} + \frac{(\mathbf{I}_{aGF} + \mathbf{I}_{aHF})}{\mathbf{I}_r}\,R_G \quad (2)$$

In general, all impedances involved have nearly the same phase angle; hence, the ratio $(\mathbf{I}_{aGF} + \mathbf{I}_{aHF})/\mathbf{I}_r$ will be a pure number and the resistance term in equation 2 will be disregarded by a relay responsive only to reactance. The first term is directly proportional to the distance to the fault and is, of course, independent of conditions external to the faulted section.

Since the line current is present in the relays, they may be expected to operate on phase-to-phase and

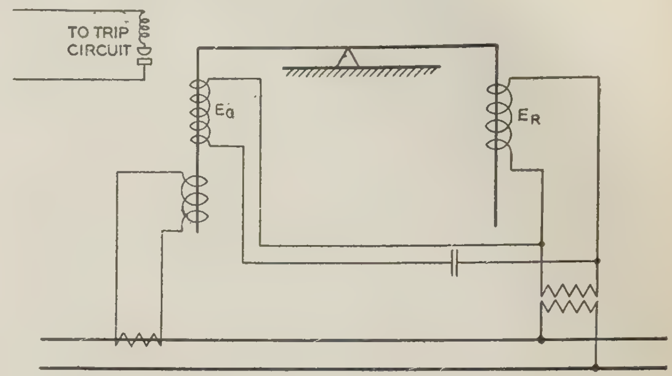

Fig. 5. Diagram of ground relay reactance element

double line-to-ground faults. Lewis and Tippett[2] have analyzed the conditions existing during a double line-to-ground fault, and have obtained the expressions

$$\mathbf{Z}_{R(B)} = m\,\mathbf{Z}_{1GH} + \frac{R_S}{2}\left[\frac{\mathbf{I}_{bGF} + \mathbf{I}_{bHF}}{\mathbf{I}_{yGF}}\right] + 3\,R_F\left[\frac{\mathbf{I}_{0GF} + \mathbf{I}_{0HF}}{\mathbf{I}_{yGF}}\right] \quad (3)$$

and

$$\mathbf{Z}_{R(C)} = m\,\mathbf{Z}_{1GH} + \frac{R_S}{2}\left[\frac{\mathbf{I}_{cGF} + \mathbf{I}_{cHF}}{\mathbf{I}_{zGF}}\right] + 3\,R_G\left[\frac{\mathbf{I}_{0GF} + \mathbf{I}_{0HF}}{\mathbf{I}_{zGF}}\right] \quad (4)$$

for the apparent impedance values as seen by phase B and phase C relays during grounds on B and C phases, respectively.

In this type of fault the line and residual currents usually are considerably out of phase; therefore, the relay currents $\mathbf{I}_{yGF}$ and $\mathbf{I}_{zGF}$, being made up of both line and residual currents, will be intermediate in phase between the line and residual currents, and hence the resistance terms will contain apparent reactance components. The individual reactance components in each expression will tend to cancel each other, however, since the relaying current has a phase position intermediate between the line current and residual current and will therefore lag one and lead the other. If the fault resistance R_S between phases and the ground fault resistance R_G are not large compared with the line reactance, and do not run to opposite extremes, it might be expected that the errors would not cause undue trouble. Because a phase-to-phase fault represents small R_S values and an infinite value of R_G, however, it appears that average service conditions easily may present a very wide variation in the 2 resistance terms. For these reasons it may be anticipated that a satisfactory relay must be arranged so as to prevent tripping on all faults other than single phase-to-ground faults.

This will result in no disadvantage, since distance relays for phase protection usually will be available to clear the double-ground faults. According to a description by A. R. van C. Warrington,[1] an attempt has been made to obtain accurate interphase and ground distance measurement by means of 3 relays and a switching arrangement. It is apparent, however, that the scheme described will not provide correct protection, since no compensation for mutual induction is employed; moreover, incorrect tripping on double ground faults may occur because of the unpredictable errors outlined in the foregoing discussion.

A New Distance Relay

Figure 3 shows a relay of the reactance type provided with the additional operating current components required to eliminate the errors discussed previously.

Structurally, this new relay is similar to the beam type[3] of impedance relay now so successfully used for phase-to-phase protection; it employs 3 separate balanced beam elements to give the time-distance characteristic shown in figure 4.

The elements providing zones 1 and 2 are mechanical duplicates of the familiar beam type impedance element, except that they have been modified to respond to reactance instead of impedance. This is accomplished by the addition of a potential coil on the current or operating side of the beam (see figure 5). The operating force is proportional to the square of the vector sum of the currents in the potential and current coils, but the restraint is proportional to the square of the voltage. From the cosine law of trigonometry the operating force is

$$F_0 = K\,E_0^2 + I^2 + 2K\,E_0 I \cos(\alpha + \theta)$$

where α is the angle by which the current in the additional operating coil E_0 leads the applied voltage, and the restraining force is

$$F_R = K_1 E_R^2$$

where E_R is the voltage applied to the restraining coil.

At the balance point the operating and restraining forces are just equal; therefore,

$$K_1 E_R^2 = K\,E_0^2 + I^2 + 2K\,E_0 I \cos(\alpha + \theta)$$

If the angle $\alpha = 90$ degrees and the pull of the E_0 coil equals that of the E_R coil, then $K_1 = K$ and the equation reduces to

$$I^2 = 2K\,E_0 I \sin\theta$$

which is the equation of the reactance relay.[4]

The third element is of the ordinary impedance type, and it functions as a back-up element and a fault detector element to prevent incorrect operation of the reactance elements during normal load conditions, and to allow proper functioning during fault conditions. The inductor-loop directional element and synchronous timer are duplicates of elements that have been used in beam type impedance relays for many years.

Zero sequence current and voltage are used on the directional element to eliminate the necessity of directional control of the reactance elements, since the directional element is de-energized during normal conditions. In addition to these elements, 3 small direct current operated switches are used to provide the contacts necessary for the third (impedance) element to control the reactance elements and to block all 3 relays on phase-to-phase faults.

Normally, the back contacts of switch 3 of figure 6 short-circuit the potential coils on the operating side of the first and second reactance elements, which is entirely feasible, since these potential coils are in series with capacitors mounted externally. The short circuit on the potential coils effectively nulli-

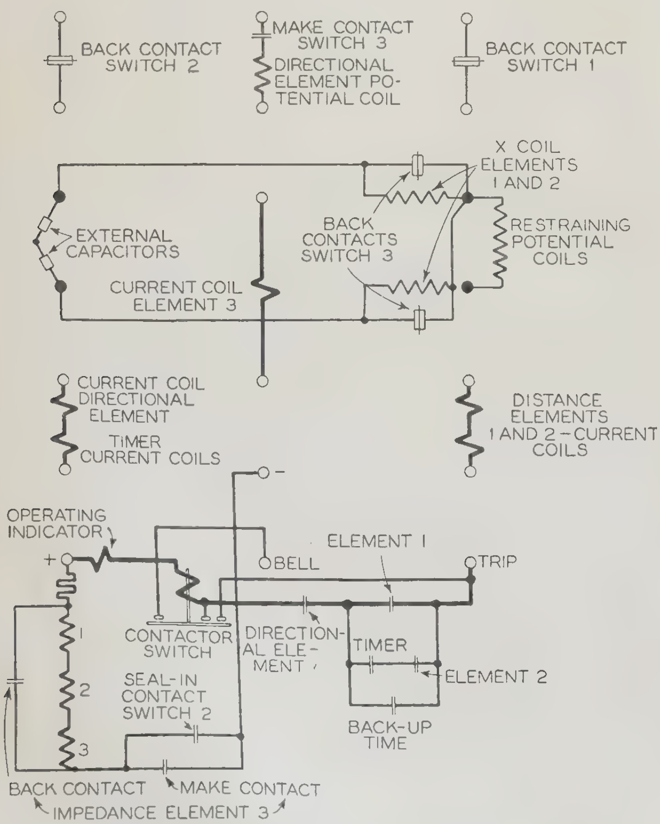

Fig. 6. Internal connections of reactance ground relay

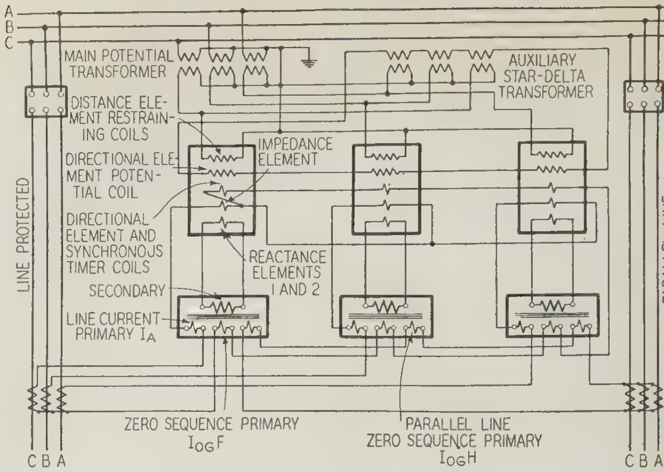

Fig. 7. External connections for 3 ground relays

fies the operating flux and prevents operation of the reactance elements under normal load conditions, in which the apparent reactance, as seen by the relay, is very small or even negative. The directional element potential coil of each relay is in series with the "make" contact of switch 3 and is wired externally in series with the back contact of switch 1 or 2 in each of the other 2 relays. Operation of the third (impedance) element energizes switches 1, 2, and 3 through its "make" contact, thus rendering the reactance elements operative and energizing the directional element potential coil. If the third element in more than one relay operates, however, the energization of the directional elements in all relays will be prevented because the back contacts on switches 1 and 2 will open. It is evident, therefore, that this arrangement allows the relays to operate only on a single phase-to-ground fault.

Figures 7 and 8 show the external connections for 3 relays. The line current of the protected line and the residual currents of the protected line and parallel line, if there are any, are fed into the primaries of the external adjusting transformer, of which the secondary feeds the current coils of the first and second reactance elements. The transformer secondary coils and the reactance elements are provided with taps for making the proper settings. The line current, after passing through the transformer primary, flows through the third (impedance) element after which a neutral is formed, and the residual current passes through the directional element and synchronous timer current coils. The voltage coil of the directional element receives zero sequence voltage, but the restraint potential coils of the distance elements receive the line-to-ground voltage.

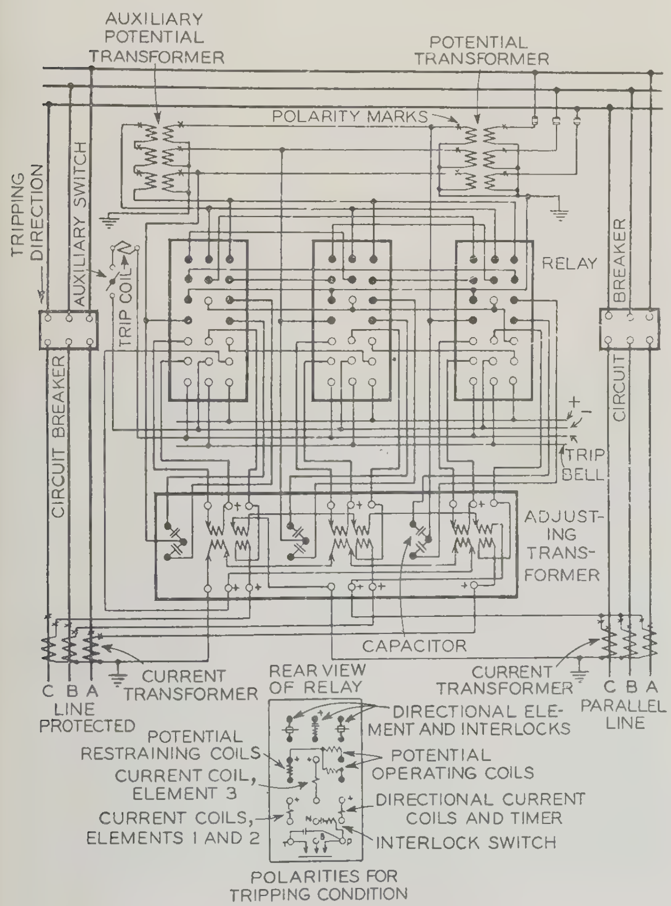

Fig. 8. Detailed diagram of external connections for 3 relays

It might appear that the proper setting of such a relay would be complicated; however, this is not the case. In fact, the setting process is only slightly more complicated than the setting of a conventional phase-to-phase distance relay. As indicated by the first term in equation 1, the balance point would be set on the basis of a 3 phase fault; that is, the $\boldsymbol{I}_{0GF}$ and $\boldsymbol{I}_{0GH}$ currents are assumed not to exist and $m\,\boldsymbol{Z}_{1GH}$ is the line-to-neutral impedance in ohms of the line to the balance point. This setting is made to agree with the formula

$$\frac{T_3}{T_2} = KZ_s$$

where

T_2 = transformer secondary taps
T_3 = current coil taps on reactance elements
Z_s = line-to-neutral impedance in ohms (based on secondary values)
K = a constant fixed by relay design

A relay, set according to this formula, will operate correctly, as far as the distance elements are concerned, on 3 phase faults not involving grounds. On ground faults, however, an additional component appears in the voltage, and in order to balance it, the proper component must be contributed by the residual current $\boldsymbol{I}_{0GF}$. By means of taps the number of turns in this winding is fixed by the equation

$$T_0 = T_A \frac{(\boldsymbol{Z}_{0GH} - \boldsymbol{Z}_{1GH})}{3\,\boldsymbol{Z}_{1GH}}$$

where

T_A = number of turns in $\boldsymbol{I}_{aGF}$ primary winding
T_0 = number of turns in $\boldsymbol{I}_{0GF}$ primary winding

The factor 3 occurs in the denominator because residual current is equal to 3 $\boldsymbol{I}_{0GF}$.

When a parallel line is present, the turns in the winding that receives the residual current from this line are adjusted according to the relation

$$T_M = T_A \frac{M_{0GH}}{3\,Z_{1GH}}$$

where T_M = turns in $\boldsymbol{I}_{0GH}$ winding.

It is clear, from this method of setting, that all compensation indicated by the general equation 1 has been provided automatically.

Field Tests

Extensive field tests have been conducted to prove the theory and operation of this new relay. These tests covered a wide range of conditions both in regard to the line characteristics and the general system characteristics.

The first group of tests was conducted on a pair of 110 kv lines of the Carolina Power and Light Company. These lines are located near Raleigh, N. C.,

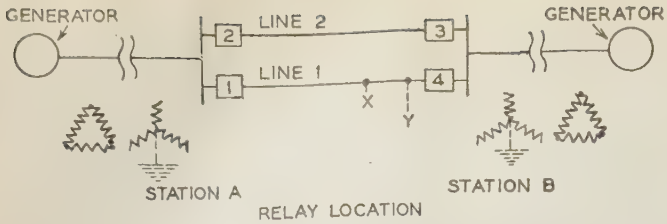

Fig. 9. Diagram of field test circuits

and are approximately 25 miles long. The results of the impedance measurements are:

$Z_1 = 13.3 + j\,22.3$
$Z_0 = 24.45 + j\,57.3$

The second group of tests was conducted on 2 22 kv lines of the Duquesne Light Company located near Pittsburgh, Pa. These lines are approximately 6 miles long. The results of the impedance measurements are:

$Z_1 = 1.26 + j\,3.5$
$Z_0 = 9.74 + j\,14.3$
$M_0 = 8.5 + j\,10.0$

Figure 9 shows the system arrangement applying to both groups of tests. The switching arrangement was flexible enough to allow a change of connections in any manner desired.

In each group of tests the procedure was to calculate the relay setting to locate the balance point of the first distance element at the fault location, which usually was at the next bus, and to check this setting by solid line-to-ground faults with the parallel line out of service and the faulted line fed from one end only. The tests indicated clearly that the method of setting, as discussed previously, gives correct results, for in each case a slight movement of the balance point setting back toward the relay prevented operation of the distance elements.

On single lines fed from one end only, the effect of fault resistance was determined. On the 110 kv lines, fixed resistance up to twice the positive sequence impedance and arcs up to 25 feet in length were inserted at the fault. On the 22 kv lines, resistance up to 10.5 ohms, or approximately 3 times the line impedance was used, but no arc faults were produced. It is interesting, however, that on the basis of 300 volts per foot and 1,000 amperes, a 10 ohm resistance is equivalent to a 33 foot arc. These values of fault resistance did not shift the balance point as much as 6 per cent, which is the smallest variation provided by the relay taps. Furthermore, in all other tests in which the primary purpose was to test other factors, similar resistances were introduced into the fault, and in no case did the distance elements change their apparent setting. This shows that the relay is independent of fault resistance on ground faults.

Several faults, both solid and with fault resistance, were produced to test the effectiveness of the residual current component in eliminating errors due to changes in system connections outside the protected section. After checking the accuracy of the setting for a fault at point y (figure 9) with breakers 3 and 4 open and the power transformer neutral grounded at the relay end, breaker 4 was closed and power was fed into station B with the transformer neutral solidly grounded. This produced no shift in the balance point for either a solid or resistance single phase fault to ground at y. On the 110 kv system it was not possible to insert resistance in the power transformer neutrals, but on the 22 kv system the resistance in the neutral at the relay end was varied from 3.75 to 15 ohms with no discernible shift in the relay balance point. These tests amply demonstrated the ability of the residual component in the relay to compensate for any shift in the line and residual current distribution due to changes in system connections at the far end of the line.

With power supply at station A, but with the neutral ungrounded, and with grounded neutral but no power supply at station B (line 2 out of service), the results were not the same for the 2 types of tested lines. On the 110 kv system, the apparent relay setting was not shifted in any way; however, on the 22 kv system the balance point was shifted back toward the relay approximately $^1/_{10}$. The relay did not trip the breaker, since no ground current could flow from the ungrounded end, and the directional element received no zero sequence current. For similar systems, the directional elements could be energized by phase currents and phase-to-phase voltages, as in other directional relays.

In both groups of tests several double line-to-ground faults were produced with various values of resistances R_S between phases and R_G to ground in order to observe the action of the distance elements. On the 110 kv line, with phases A and B faulted to ground, the phase B relay balance points shifted materially beyond the fault point, regardless of whether R_S was greater than R_G or the reverse, except on one fault. This fault to ground cleared itself, leaving a plain phase-to-phase fault on which the A phase relay operated and the B phase relay did not. On the 22 kv system the results were, in general, just the reverse; the A phase relay shifted beyond the fault and the B phase relay shifted its

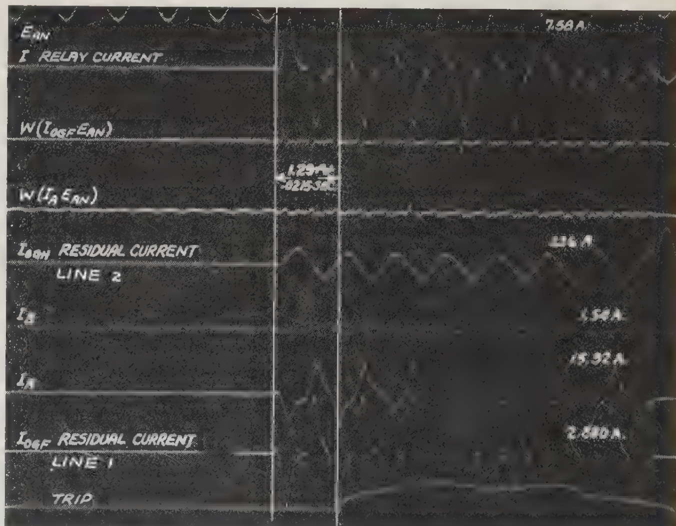

Fig. 10. Oscillogram showing typical operation of the new distance ground relay

balance point back. These tests indicate quite clearly that the relay errors on double line-to-ground faults, as expressed in equations 3 and 4, are variable and in general unpredictable, thus justifying the switching arrangement whereby tripping is prevented on double line-to-ground faults.

The effect of the mutual zero sequence impedance of the parallel line and the ability of the additional residual current I_{0GH} to compensate for it were investigated only in the 22 kv tests, since the compensating winding was not incorporated in the relays used on the 110 kv tests. For both solid and 10 ohm ground faults at y, the relay balance point did not shift as much as 6 per cent, since the setting was the same as required under the same conditions without the parallel line in service. Failure of the relays to operate when connected for the protection of line 2 against faults at x on line 1, was investigated, and a correct action was obtained.

Although the relays were not equipped for mutual effects on the 110 kv lines, it was noted that the parallel line, which was on the same towers, caused a shift in the relay balance point of approximately $^1/_3$. These tests indicate that where parallel lines are on the same towers, compensation must be made for the mutual effects, if accurate distance measurement is desired. Figure 10 shows an oscillogram of a single phase fault close to the relay location. Note that tripping occurs in 1.29 cycles. For faults close to the balance point, the time was approximately 3 cycles.

References

1. Control of Distance Relay Potential Connections, A. R. van C. Warrington. Elec. Engg. (A.I.E.E. Trans.), v. 53, Jan. 1934, p. 206–13.

2. Fundamental Basis for Distance Relaying, W. A. Lewis and L. S. Tippet. Elec. Engg., v. 50, June 1931, p. 420–2.

3. High-Speed Relays Increase System Stability, S. L. Goldsborough. *Elec. Jl.*, v. 27, July 1930, p. 400–01.

4. A High-Speed Relay for Short Lines, S. L. Goldsborough and W. A. Lewis. Elec. Engg., v. 51, Mar. 1932, p. 157–60.

Hydrogen Cooling of Rotating Machines

Hydrogen cooled synchronous condensers now in use have a total rating of several hundred thousand kilovolt-amperes. During the last 2 years the use of hydrogen cooling has been extended to frequency converters, and most recently to turbine generators. Several such machines now are being constructed. This method of cooling not only results in a reduction of machine losses and maintenance cost, but also makes feasible the construction of units with capacities greater than those of the existing air cooled units.

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ENGINEERS responsible for the design, construction, and performance of electrical machines always have considered the problem of ventilating these machines to be important. The outstanding progress that has been made toward obtaining a better solution of this problem embraces 2 principal features: first, a material reduction of the losses; second, the development of more effective methods of dissipating them. From the beginning of the industry, air has been used almost universally as the cooling medium for all classes of rotating electrical machines. Since air is the most plentiful and the cheapest of all gases and an efficient heat transfer medium, it undoubtedly will continue to be used as the coolant for the major number of classes of such machines in the future.

The development of air-ventilation of the larger and more important electrical machines can be grouped chronologically into 3 main periods. During the first and earliest period, a continuous stream of cooling air was carried to the machine through intake ducts, passed through the unit, and discharged through outlet ducts to regions beyond the source of supply. The principal disadvantage of such a system was that the air available at the site of utilization usually was contaminated with dirt and other solid impurities. In the second period, a move toward the alleviation of this difficulty was the introduction of air filtering devices in the incoming air ducts. Such devices were reasonably effective and actually they removed a large proportion of the solid impurities. Even with a relatively small amount of dirt remaining in the air, the quantity of such material carried into the machine during a year's time was enormous, particularly in machines in fuel burning stations. In the third and present stage of the development, the closed circuit ventilating system with the surface type of air cooler or heat exchanger is used. Since in this system the cooling air is recirculated, a relatively small amount of dirt is carried into the machine to clog ventilating passages,

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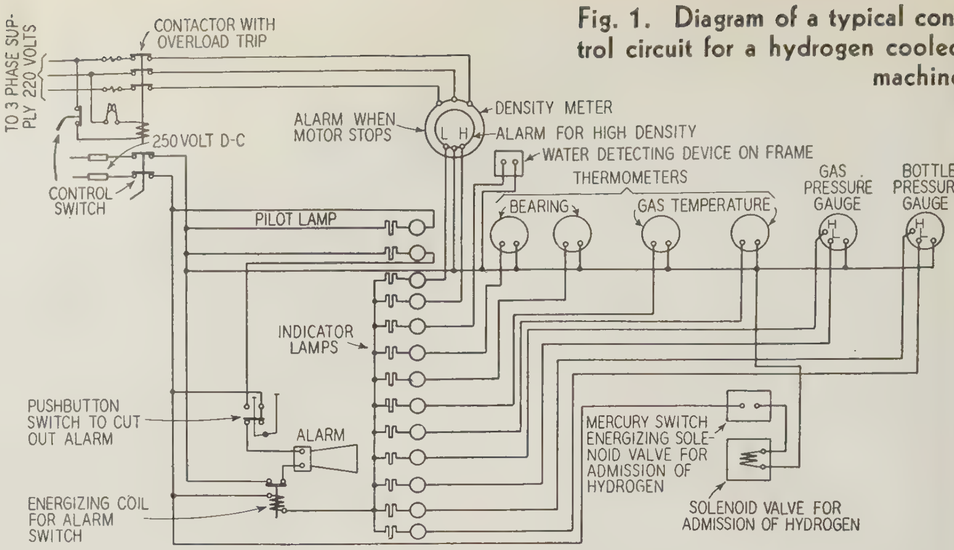

Fig. 1. Diagram of a typical control circuit for a hydrogen cooled machine

coat ventilating surfaces and reduce heat transfer coefficients, and be deposited on the surfaces of the coils. A further refinement of this system was obtained by introducing a make-up air filter at a low pressure zone in the ventilating system, for filtering the air required to compensate for the outward leakage at the sections above atmospheric pressure. The use of the closed circuit ventilating system, with air coolers and make-up air filters, is considered to be a development of far reaching importance in improving the performance of both stator and rotor windings, and in reducing the maintenance expense and fire hazard due to winding failures. Another improvement could be effected by making the enclosure essentially air-tight, so that no make-up air would be required, and by keeping the inside of the machine as clean as possible. Probably this is the optimum condition to be obtained with air cooling, but it is questionable whether the benefits obtained would justify the added investment costs.

Cooling With Gases Other Than Air

The 3 most important factors that are closely dependent on the ventilation of electrical machines are: output per unit weight, or volume of material; loss per unit of output; and maintenance cost per unit of time. The use of gases other than air has been investigated by many during the past several years, to determine their effect on these factors. Since it is possible to eliminate completely the item of cleanliness by adopting a gas-tight housing structure, all gases can be placed on the same relative basis from this standpoint. In general, the use of gases having approximately the same densities as air, or greater densities, contributes a negligible amount in increasing either the efficiency or the unit output, but may have a great influence on maintenance costs. Noninflammable gases such as nitrogen, carbondioxide, and flue gas would eliminate completely the oxidation effects on organic materials, and the fire hazard. In most cases the evaluation of the gains from this factor alone would not justify the use of these gases as coolants of rotating electrical machines.

Gases Lighter Than Air

Hydrogen and helium are the 2 common gases that are appreciably lighter than air. Helium is inert and noninflammable, and from these considerations would be an ideal medium for ventilation purposes. Because of its scarcity and cost, however, it cannot be considered as an available cooling medium.

On the contrary, hydrogen can be obtained in unlimited quantities at relatively low cost, and is a more desirable cooling medium than helium because of its lower density and better thermal characteristics. Moreover, commercial hydrogen has the degree of purity desired and needed for cooling purposes; it is inert, nonexplosive, and will not support combustion. In general, it may be stated that at present hydrogen is the most desirable gas that can be used as the cooling medium for certain types of rotating electrical machines. The relative values of the principal characteristics of air and hydrogen that intimately concern the ventilation problem are given in table I.

Actual operating experience with hydrogen cooling of 2 types of synchronous machines fully demonstrates that the output can be increased approximately $1/4$ without exceeding the temperature limits specified for air cooling. In general, however, this gain in rating cannot be realized without sacrificing other important operating characteristics of the unit.

A clear visualization of the possible reduction in losses may be obtained by considering a particular type of machine operating under the same load conditions in air and in hydrogen, with the volume and pressure of the gas under static conditions maintained the same for both conditions. In the machine, losses will be $(a + b)$ per cent of the output, where the term b represents the windage, friction, and ventilation losses, and the term a represents all other losses. Since the windage, friction, and ventilation losses depend directly upon the density of the cooling medium, these losses will be reduced 90 per cent when the machine is operated in hydrogen, because the gas density is 10 per cent of that of air. The remaining losses will be modified slightly by the lower temperatures of the machine parts, but the change is small and may be neglected. This is a fundamental relation that applies to any type of machine. Greater reduction in losses can be obtained only by increasing the weights and dimensions of the active parts to lower the generated losses per unit of material; however, this defeats one of the main purposes of using hydrogen, and the added expense would be justified only in exceptional cases.

Since a mixture of hydrogen and air is explosive over a wide range of proportions, the design of the machine and the operating procedure are such that explosive mixtures are not possible under any condition of operation. In order to provide for some unforeseen condition brought about by the failure to follow the defined procedure, it has been deemed advisable to design hydrogen cooled machines to be explosion safe. In this connection it should be noted that "explosion safe" is interpreted to mean that the housing will not fail and result in damage to life and property external to the machine. No guarantee can be made that the explosion will not result in damage or dislocation of internal parts.

On the basis of the most favorable mixture for explosions, the pressure developed inside the enclosure would not become more than $^1/_2$ the maximum theoretical value, because of the large heat storage capacity and the exposed surfaces of the large mass of both active and and structural parts of the machine. Analysis and experience both indicate that the actual explosive pressure will not exceed 85 pounds per square inch; consequently, it has become accepted practice to design the housings of hydrogen cooled machines to withstand an internal pressure of 100 pounds per square inch without failure, and to withstand a pressure of 50 pounds per square inch without permanent distortion.

Hydrogen Control

It is necessary, in fixing the design features and operating procedure of hydrogen cooled machines, to follow conservative and safe practices. The 4 principal requirements of the hydrogen control equipment are determined by the running, standstill, gas filling, and gas emptying conditions. In filling or emptying the housing, it is considered necessary to use an inert gas such as carbon dioxide or nitrogen to make the initial displacement, so that there will not be any mixing of hydrogen and air in performing these operations. The primary functions of the hydrogen control equipment are to provide for manual scavenging and filling the machine housing with hydrogen, to maintain a predetermined purity and pressure of hydrogen in the machine housing by admitting hydrogen to the equipment, and to give warning of improper operation or failure.

Each unit is provided with a control and alarm system that is simple to operate, and is adequate from the standpoint of reliability and protection. A diagram of typical control equipment is given in figure 1.

Summary of Advantages of Hydrogen Cooling

The principal advantages resulting from the use of hydrogen as the coolant are:

1. Reduced windage, friction, and ventilating losses, because of the low density of the gas.
2. Increased output per unit volume of active material, because of the high heat-storage capacity, thermal conductivity, and heat transfer coefficients of hydrogen.
3. Reduced maintenance expense due to the freedom from dirt and moisture.
4. Increased life of the insulation on the stator winding because of the absence of oxygen and moisture in the presence of corona.
5. Reduced noise due to low gas density.

Classes of Machines Suitable for Hydrogen Cooling

Electrical machines that operate at high speeds and have high windage, friction, and ventilation losses are suitable inherently for cooling with hydrogen. This classification includes synchronous condensers, frequency converters, and turbine generators. From the construction standpoint, these types of machines can be arranged in 2 groups, one in which the complete unit can be enclosed in a metallic housing, and another in which it is necessary for shafts to extend through the housing. Synchronous condensers and frequency converters are complete units in themselves, and can be completely enclosed, whereas turbine generators require shaft extensions for mechanical connection to prime movers, and have entirely different cooling requirements.

Synchronous Condensers

Hydrogen cooling has been used for synchronous condensers for several years, and the units now giving satisfactory service have a total capacity of several hundred thousand kilovolt-amperes. With this class of equipment, in which there is no energy transformation, the principal stability requirement is that it maintain synchronous relations with the system during operating conditions, and under reasonable transient system disturbances. It is not necessary

Table I—Relative Values of the Characteristics of Air and Hydrogen as Coolants

Cooling Gas	Air	Hydrogen
Weight density	1.0	0.07
Thermal conductivity	1.0	7.00
Heat transfer coefficient from surfaces to gas	1.0	1.35
Specific heat	1.0	0.98
Support combustion	Yes	No
Oxidizing agent	Yes	No

to build such machines with high synchronizing or pull-out capacity. Hence, synchronous condensers offer the greatest possibilities with hydrogen cooling for obtaining maximum output per unit of material and minimum losses per unit of output. The full gain in rating cannot be obtained if an underexcited machine is required to deliver a relatively high output.

Actual experience in the design, manufacture, and operation of hydrogen cooled synchronous condensers has demonstrated that a housing structure can be fabricated from good quality steel plate with permanent joints made gas-tight by electric welding. The cylindrical housing member with dome-shaped end-heads is the most suitable type of structure to withstand the explosion pressure that might occur as a result of some unforeseen condition.

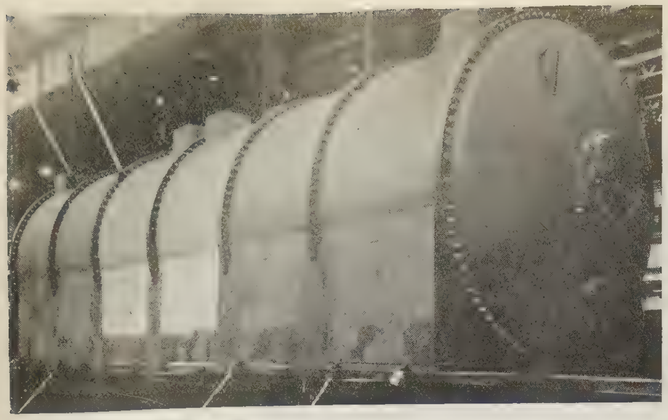

Fig. 2. Housing of a 60,000-kva hydrogen-cooled frequency converter

Frequency Converters

The frequency converter gradually is supplanting direct generation as the method of obtaining special frequencies for transportation and industrial applications. The principal frequency converter application has been the conversion from 60 to 25 cycles per second, for which a synchronous speed of 300 rpm is required. At this speed the windage, friction, and ventilation losses with air cooling are lower than for the synchronous condenser that can be built for higher speeds, and the use of hydrogen results in a smaller saving in losses. The characteristics of frequency converters differ from those of condensers, for a high pull-out torque usually is required to keep the systems from pulling apart under normal fault conditions. Since the short-circuit ratio and pull-out torque must be kept the same for both air and hydrogen cooling, the reduction in weight and dimensions for hydrogen cooling is not as large as it is for synchronous condensers, in which no direct or indirect restriction is placed on the pull-out characteristics. In meeting this requirement, it is necessary to utilize approximately $^1/_2$ the gain in rotor output to maintain stability, and the remainder to increase the output. As a result, the net reduction in physical size and weight of active parts is 12 to 15 per cent.

The economic factors that justify the use of hydrogen cooling for frequency converters are the same as for synchronous condensers. Although the gains from using hydrogen are not as large as they are for synchronous condensers, it is believed that an evaluation of all factors, such as the elimination of an external housing for outdoor applications, reduced maintenance, and simplicity of operation, will fully justify its use for 60 to 25 cycle frequency converters. At the present time there is only one hydrogen cooled frequency converter of this type in service, but it is expected that the use of hydrogen cooling will become standard practice for large units of this type in the future.

Although the electric utility industry has standardized on a frequency of 60 cycles per second for the generation, transmission, distribution, and utilization of electric energy, a portion of the Pacific Coast section of California uses a frequency of 50 cycles per second. The interconnection of adjacent 50 and 60 cycle systems requires 50 to 60 cycle frequency converters. Frequency converters for this application can be built for any desired rating at 600 rpm, and thus are well adapted for hydrogen cooling.

The hydrogen-cooled 60,000-kva frequency converter set now on order for the Bureau of Power and Light of the city of Los Angeles is interesting because of its appplication, rating, and speed.

The Bureau of Power and Light has a project under way at the present time for changing the frequency of its distribution and generating systems from 50 to 60 cycles per second. The city's Boulder Dam generators, now being installed, will be the only source of supply for the 60 cycle load; consequently, frequency conversion equipment is needed to interconnect the 60 cycle Boulder Dam supply and distribution systems with the 50 cycle distribution system. During this change-over period, a portion of the 60 cycle energy from Boulder Dam normally will go directly into the Bureau's 60 cycle system, and the remainder will go through the frequency converter into the 50 cycle system, thus utilizing the full output from the Boulder Dam generating station as it becomes available. Should there be an interruption of the supply from Boulder Dam, the 60 cycle load will be supplied through the frequency changer from the 50 cycle system. After the frequency change-over is completed, the need for frequency conversion internal to the Bureau's system will have disappeared. When this time is reached, the units can function either as frequency changers in connection with adjacent 50 cycle systems, or the rotors may be uncoupled and the 2 elements operated as synchronous condensers on the city system.

Design and Construction Problems

The design of a 60,000-kva, 600-rpm, 60-to-50-cycle frequency converter set involves numerous mechanical problems irrespective of the cooling medium used. The major mechanical problem of this particular set was introduced by the contract requirement that the 50 cycle motor element be capable of operating as a 60 cycle synchronous condenser at 720 rpm at some later date when the 2 units are separated. Both units are required to withstand an overspeed test of 25 per cent, based upon the 60 cycle operating speed, and the stresses in the rotor materials are not to exceed a specified percentage of

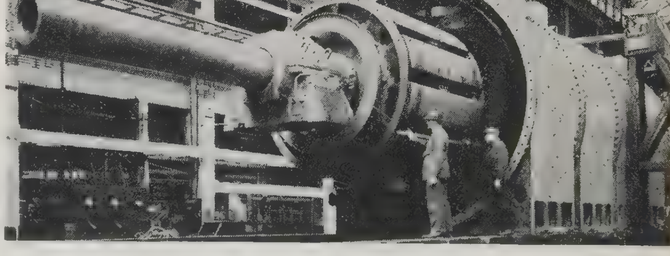

Fig. 3. Rotor assembly of 60,000 kva frequency converter

the yield points of the materials. Since the speeds of the generator and motor ends are 750 rpm and 900 rpm, respectively, it was necessary, in meeting the stress requirements, to use different dimensions, peripheral speeds, proportions, and quality of materials in the construction of the rotors.

In designing the frequency converter, special consideration was given to the problems of shipping, erection, and maintenance. It is particularly desirable that hydrogen cooled machines be built and assembled at the manufacturer's works, so that the necessary tests and corrective changes, if needed, can be made to insure that the housing structure is gastight and the leakage of hydrogen is kept to a minimum. In order to meet these requirements, the housing structure of the complete unit was arranged in 6 cylindrical transverse main sections and 2 end-head sections. The frame members for the motor and generator elements are formed by 2 sections, and each of the remaining 4 sections houses a bearing, pedestal, and hydrogen cooler sections. Separate end-head members are provided to facilitate erection, inspection and maintenance, and dismantling operations. The dimensions of the sections are such that each can be assembled with its associated apparatus and equipment, and shipped as a complete unit. A manhole is provided for each of the main and end-head sections, so that the internal parts readily can be inspected and maintained at any time. The end-heads are made dome-shaped in order to obtain maximum strength against internal pressures with the minimum weight of material.

The frequency converter unit is designed for outdoor operation, and when installed it will have the general appearance of figure 2.

The increasing use of high frequency energy for induction furnace applications offers another possibility for the hydrogen cooled frequency converter. For the ratings now required for this application, the most economical speed is 3,600 rpm. The cheapest set consists of a squirrel cage induction motor driving an inductor-type single-phase generator. Since neither element requires collector rings, this is the simplest and most suitable type of equipment for hydrogen cooling Hydrogen cooling of the high frequency converter would result in an increase in efficiency, reduced weights and dimensions, and reduced maintenance of the unit. Such frequency converters could be operated at a relatively high internal gas pressure, and the pressure could be permitted to vary over a relatively wide range. The only control equipment required would be a pressure gauge provided with high and low pressure alarm relay contacts. The need for the addition of hydrogen would be infrequent, and it could be introduced through valves operated either manually or automatically.

Turbine Generators

Interest in the application of high-temperature high-pressure turbines superimposed on existing generating units has increased rapidly during the past few years. This type of steam turbine is easier to build for a speed of 3,600 rpm than for a speed of 1,800 rpm, because smaller masses and dimensions can be used. As a result, development of this type of turbine has made it necessary to build 3,600 rpm generators for ratings much larger than previously were considered feasible. The largest 3,600 rpm turbine generator unit now in service in the United States is rated at 22,500 kva, 18,000 kw at 0.8 power factor. Careful analysis of fundamental test data and operating experience, and a concentrated effort to co-ordinate properly the proportions of both active and structural parts and to utilize materials more effectively, have made it feasible to build air cooled 3,600 rpm generators for ratings up to 62,500 kva, 50,000 kw at 0.8 power factor, which will be comparable in cost and reliability with similar large 1,800 rpm units. The rotor of a large 3,600 rpm generator necessarily operates at very high peripheral speeds; consequently, the windage, friction, and ventilation losses are inherently high, and they form a large proportion of the total losses. It is apparent that the 3,600 rpm turbine generator offers the greatest possibilities for reduction in losses when using hydrogen as the cooling medium.

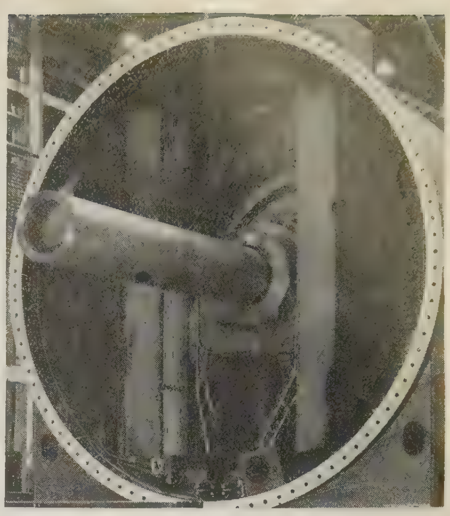

Fig. 4. End view of the housing of 60,000 kva frequency converter, showing a part of the control wiring and piping

One of the first turbine generators designed for, and operated with, hydrogen cooling was rated at 9,375 kva, 0.8 power factor and 3,600 rpm. This machine was built by the Westinghouse Company and was described by M. D. Ross.[1] Until the latter part of 1935 and the early part of 1936, however, no actual orders were placed for the construction of such machines. Outstanding among 3,600-rpm hydrogen-cooled turbine generators recently purchased are the 58,725-kva, 50,000-kw, 0.85-power factor unit of the New York Edison Company and the 43,750-kva, 35,000-kw, 0.80-power factor unit of the Union Gas and Electric Company, of Cincinnati, Ohio, and the 50,000-kva, 0.90 and 1.0-power factor unit of the West Penn Power Company. Generators that have been built, or are being built, with ratings of 18,750 kva or more, are listed in tables II and III.

Construction Problems

The only feature involved in the construction of hydrogen cooled turbine generators that is not common to synchronous condensers and frequency

1. For all numbered references, see list at end of paper.

converters is the sealing gland between the end members of the stator housing and the rotor shaft ends. Oil is the most suitable liquid available at present for use as the sealing medium. The oil type of gland shown in figure 5 has been tried experimentally with success, and has been used on the 9,375 kva hydrogen cooled turbine generator previously mentioned, with satisfactory results. The gland consists of a housing mounted rigidly on the bearing support and attached flexibly to the stationary end-housing parts. Annular grooves are provided in the housing for segmental-spring supported sealing rings. The floating rings are not free to rotate, but are free to expand radially, and thus permit a film of oil to flow between their inner surfaces and the shaft. Since minimum clearances are provided between the rings and the sides of the annular grooves in the housing, and are automatically maintained between the shaft and the rings, only a relatively small amount of oil is required to flow through the gland to provide adequate sealing. It is apparent that the oil passing through the gland acts as a mechanical vehicle for carrying hydrogen out of the housing and for carrying air into it. The magnitude of the exchange of gases depends upon the turbulence of the oil in contact with the gases. With the gland properly designed and constructed, and the flow of oil restricted to the requirements, a minimum amount of churning and foaming will occur.

Actual experience has demonstrated that the lubricating system for the bearings can be used with minor modification to supply the gland oil, and will require the introduction of only a nominal amount of fresh hydrogen to maintain the desired gas density.

In this class of machines, it is necessary to circulate large quantities of oil through the bearings to dissipate the bearing losses. At present, the purchasers believe it advisable, although not necessary, to have the generator bearings accessible from the outside, for inspection, maintenance, or replacement, without removing the hydrogen from the enclosure. This makes it necessary for the sealing glands to be located between the bearing supports and the rotor body, which results in an increase in the length of the span between bearing centers.

The rotors of 3,600 rpm generators, which are necessarily of relatively small diameter and great length, operate above the first critical speed and below the next higher critical speed, corresponding to an S-shaped curve of rotor deflection. It is essential that the distance between bearing centers be kept as short as possible so that this second critical speed will be well above the operating speed. It is feasible to build such generators, with hydrogen cooling for ratings up to 62,500 kva with internal blowers supported on the rotor shaft ends and the sealing glands located between the bearing supports and the rotor body.

Table II—3,600 RPM Turbine Generators Manufactured by the Westinghouse Company

Kva	Kw	Power Factor	Cooling Gas	Number of Units	Date Installed	Purchaser
18,750	15,000	0.8	Air	3	1930	Louisiana Steam Generating Corp.
18,750	15,000	0.8	Air	2	1931	Virginia Public Service Co.
22,500	18,000	0.8	Air	1	1931	Public Service Elec. & Gas Co. (N. J.)
29,400	25,000	0.85	Air	1	On order	Connecticut Light & Power Co.
43,750	35,000	0.8	Hydrogen	1	On order	Union Gas & Elec. Co., Cincinnati, O.
50,000	45,000 50,000	0.9 1.0	Hydrogen	1	On order	West Penn Power Co.
58,725	50,000	0.85	Hydrogen	1	On order	New York Edison Co.

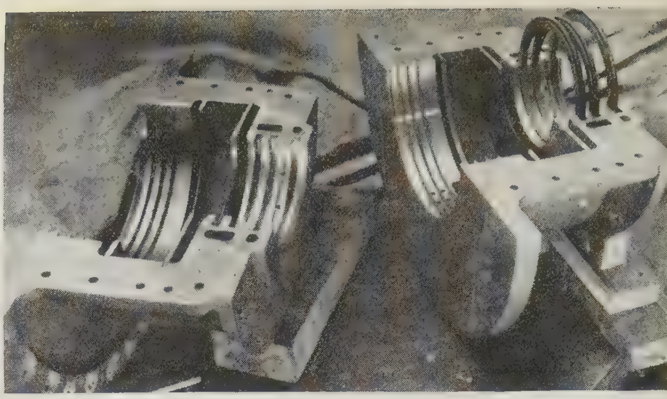

Fig. 5. Parts of oil type of sealing gland used on hydrogen cooled turbine generators

With the requirement that the bearings be accessible from the outside, the gas coolers must be located at sections around the housing periphery. The dimensions and proportions of the coolers can be controlled so that the coolers can be placed at right angles to the axis of the machine, and built into the top portion of the main cylindrical housing. This arrangement provides for the maximum accessibility for cooler maintenance and replacement. Although drainage channels are provided to divert cooler leakage and condensation around the stator core to the bottom of the housing, it is desirable to take further precaution against tube failure and its consequences by using only pure water in the cooler. If such water is not available, it is preferred, though not absolutely necessary, that distilled water be circulated through the cooler tube and the heat energy from it be transferred to another source of water supply by means of a heat exchanger.

For ratings greater than 62,500 kva, it will be necessary to omit the internal blower and place the sealing glands outside the bearing supports in order to obtain the minimum distance between bearing centers. With the present reliable performance record of high speed bearings, no particular difficulty should be experienced with the bearings placed inside the generator housing. A complete and reliable record of bearing performance can be obtained by means of remote indicating and recording instruments, and bearing replacements or maintenance will be required at infrequent intervals. The coolers may be located vertically or horizontally at the ends of the unit, and direct driven blowers may be used to circulate the gas. This arrangement automatically would produce an appreciable distance between the turbine and generator bearing supports at the

Table III—3,600 RPM Turbine Generators Manufactured by the General Electric Company

Kva	Kw	Power Factor	Cooling Gas	Number of Units	Date Installed	Purchaser
18,750	15,000	0.8	Air	1	Being installed	Ford Motor Co., Detroit, Mich.
25,000	20,000	0.8	Air	1	On order	United Light & Power Co., Chicago, Ill.
31,250	25,000	0.8	Air	1	On order	Public Service Co. of Colorado
31,250	25,000	0.8	Hydrogen	1	On order	Dayton (Ohio) Power & Light Co.
50,000	40,000	0.8	Hydrogen	1	On order	Appalachian Elec. Pwr. Co., Roanoke, Va.

coupling, and would furnish a desirable shaft flexibility to permit the vertical displacement of the turbine bearing, due to high temperatures, without disturbing the division of the loading on the 2 respective bearings.

Large 3,600 RPM Generators

Hydrogen cooling of 3,600 rpm turbine generators is of outstanding importance because of the present need for ratings larger than can be obtained with air cooling. Hydrogen cooling, together with its improvement in cleanliness, introduces possibilities for making significant improvements in construction and ventilation. A development program that promises to increase materially the rotor ampere-turn output is in progress now. In order to utilize properly the expected increase in rotor output, it will be necessary to make intensive design studies and investigations to determine the actual mechanical, magnetic, electrical, and thermal conditions that exist in each incremental part of the machine under both full-load and no-load conditions. Gains resulting from improvements in ventilation necessitate corresponding improvements in materials and design of the electric and magnetic circuits.

With respect to the rotor, it is interesting that the physical properties of the alloy steel forgings have been improved materially, but their magnetic permeability is appreciably lower than the permeability obtained in steel forgings with poorer physical properties. Although there has been a real improvement in the magnetic properties of sheet steel for use in building stator cores during the past decade, a further reduction in losses and an increase in permeability are desirable in order to permit higher working flux densities in the stator cores of hydrogen cooled machines.

An analysis of fundamental data obtained from time-temperature tests on actual machines has led to important changes in machine construction and has made it possible to determine the magnetic and electrical characteristics of materials for structural parts.

The radial depth of the stator laminations back of the slots is determined by magnetic conditions. A study of the losses in the stationary parts revealed the fact that the flux density in the armature core back of the slots could not be increased materially above the existing limits, because certain losses in the frame parts surrounding the core increase rapidly with increasing flux density. At high magnetic flux densities a certain portion of the flux traveled through the frame, causing a high loss in the solid iron material. Recent tests of armature lamination samples at high flux densities indicated that the hysteresis component of the loss increased less rapidly near saturation density than at lower densities. Since it is possible to ventilate adequately the iron back of the slots, it was deemed desirable to find a way of designing the core for higher flux density and take advantage of the good properties of the iron without creating high losses in the frame. A copper damper winding of simple design was developed and applied on the frame of a 9,375 kva generator The damper winding consisted of copper straps, properly located with respect to the frame members under consideration, and short-circuited by a copper plate at each end of the core. Iron loss tests on this machine and on duplicate units without the frame damper winding indicated a reduction of $^1/_4$ in the iron loss at core flux densities suitable for modern machines. A satisfactory damper winding, as described here, will permit designs with higher core flux densities, and will result in more compact machines without a sacrifice in performance.

On the basis of the present status of hydrogen cooling for 3,600 rpm turbine generators, machines rated as high as 75,000 kva, 0.8 power factor and with unity short-circuit ratio can be built, and from investigations in progress, it is expected that it will be feasible to build a machine of any rating that may be required in the immediate future.

References

1. The Application of Hydrogen Cooling to Turbine Generators, M. D. Ross. A.I.E.E. Trans., v. 50, Mar. 1931, p. 381–86.

2. Hydrogen as a Cooling Medium for Electrical Machines, Edgar Knowlton, C. W. Rice, and E. H. Freiburghouse. A.I.E.E. Trans., v. 44, 1925, p. 922–34.

3. Windage Losses in Air, Hydrogen, and Carbon Dioxide, C. W. Rice. *G. E. Review*, v. 28, May 1925, p. 336–41.

4. Liquid Film Seal for Hydrogen-Cooled Machines, C. W. Rice. *G. E. Review*, v. 30, Nov. 1927, p. 516–30.

5. Outdoor Hydrogen-Ventilated Synchronous Condensers, R. W. Wiesseman. A.I.E.E. Trans., v. 48, Oct. 1929, p. 1221-29.

6. Hydrogen Cooling of Large Electrical Machines, C. J. Fechheimer. *Elec. Journal*, v. 26, Mar. 1929, p. 127–30.

7. Hydrogen—A Successor to Air, C. J. Fechheimer. *Elec. Journal*, v. 26, Sept. 1929, p. 405–07.

8. Fundamentals of Heating Calculations of Electrical Machines, Robert Pohl. *Archiv für Elektrotechnik*, v. 12, no. 4, 1923, p. 361–69.

9. Hydrogen as a Cooling Medium, Max Jakob. *Verein Deutscher Ingenieure*, v. 70, June 1926, p. 889–90.

10. Free and Forced Convection of Heat in Gases and Liquids, C. W. Rice. A.I.E.E. Trans., v. 42, p. 653–706; v. 43, 1924, p. 131–44.

11. Forced Convection of Heat in Gases and Liquids, C. W. Rice. *Indust. and Engg. Chemistry*, v. 16, May 1924, p. 460–67.

12. Motors for Operation in Rooms Filled with a Mixture of Hydrogen and Air, Philip Suter. Bulletin of the Schweizer Elektrotechnischer Verein, Jan. 2, 1930, p. 33.

13. The Explosion Limits of Gas Mixtures, E. Asch. *Zeitschrift für Technische Physik*, v. 4, no. 12, 1923, p. 468–71.

14. Hydrogen Cooling of Electrical Rotating Machinery, C. J. Fechheimer. *Elec. News and Engg.*, Mar. 15, 1931, p. 43–44.

15. Developments in Hydrogen Cooled Condensers, C. J. Fechheimer. *Elec. Journal*, v. 28, Mar. 1931, p. 165–69.

16. Interchanging Hydrogen and Air in Hydrogen-Cooled Machinery, C. J. Fechheimer. *Elec. Journal*, v. 28, June 1931, p. 361–62.

Special Tests on Impulse Circuit Breakers

A novel circuit arrangement that supplies a large current to a circuit breaker under test and then subjects the open terminals to a high recovery voltage has been used in testing the 2,500,000-kva 287.5-kv impulse oil circuit breakers built for the Boulder Dam-Los Angeles transmission line. A description of this arrangement, which makes available several times the apparent power of the testing apparatus, is given in this paper, together with oscillograms which show details of the arc extinction process not previously known, and the results of a study of voltage distribution.

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THE UNIQUE construction and the positive and reliable interrupting characteristics of the 287.5 kv impulse oil circuit breakers for the transmission line from Boulder Dam to Los Angeles made it possible to supplement the conventional tests with tests made by means of a novel circuit arrangement simulating faithfully the conditions present in an interruption at several times the apparent power available at the testing plant. By means of this circuit one of the 2 interrupting units of a single pole was subjected to arc currents as high as 8,300 amperes up to the time of interruption, when a recovery voltage of 132 kv was applied at a recovery rate of 2,500 volts per microsecond, and with the overshoot normally associated with circuit interruption. The circuit is not generally applicable because of certain requirements of breaker performance, but it has proved very useful in testing breakers of the impulse type.

The interrupting rating of this breaker is 5,000 amperes at 287.5 kv, with 3 cycle operating time. The maximum voltage which is anticipated across one unit in the worst type of service operation, however, is 146 kv. The test voltage was therefore within about 10 per cent of the maximum service voltage for the one unit. It was the highest available at the testing plant with the circuit arrangement used. It was not possible to duplicate the most unfavorable timing relationships which may occur, but reasonable assurance that these can be handled satisfactorily is given by the fact that the maximum current interrupted was 66 per cent higher than the interrupting rating of the breaker.

Several cathode ray oscillograms show the behavior of the breaker at contact separations on the border line for successful interruption. In such cases the arc path may break down several times in the course of the recovery transient although between breakdowns it is a good insulator.

A study of voltage distribution between the 2 sections of the breaker in the course of a normal interruption showed this distribution to be substantially in accordance with capacitance relationships.

Description of Circuit Breaker

The 287.5 kv impulse type of oil circuit breaker for the Boulder Dam-Los Angeles transmission line has already been described[1] so that a very brief description will suffice for the purpose of this paper.

A single phase unit is shown in figure 1. The interrupting units are the 2 horizontal elements encased in porcelain at the top of the structure. A cross-sectional view of one of these elements is shown in figure 2. Each of these elements consists of a tube of molded insulation divided into 2 portions by a horizontal board placed slightly above the center line of the tube. This board seals off the 2 portions of the tube except for ports immediately above the contacts. One pair of contacts and a portion of the underside of the board are shown in figure 3. The moving contact is carried on a bridging member between 2 horizontal wooden rods suspended from the underside of the board, and contact motion is horizontal. The location of the ports directly above the break near the stationary contact is shown in the figure. There are 4 such breaks in each horizontal element or 8 in the complete single pole unit. At the time of opening of the contacts, the piston in the

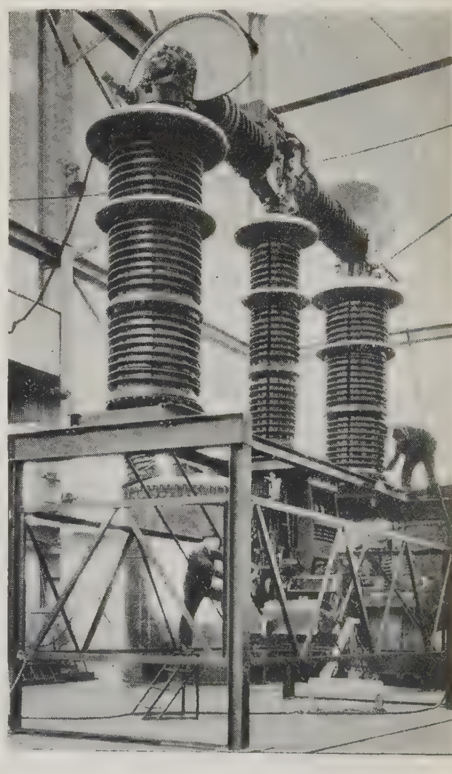

Fig. 1. One pole of 287.5 kv impulse oil circuit breaker

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1. For numbered references see list at end of paper.

large casting at the left of figure 2 places the oil below the board under pressure, which causes an upward blast of oil through the ports. This blast extinguishes the arc.

Conventional Tests of Interrupting Capacity

The tests of a more or less standard nature on this breaker have also been given in the previous paper and may be summarized briefly as follows:

1. Tests on a complete single pole unit (8 breaks) up to:
   15 per cent of rated interrupting current at 264 kv;
   136 per cent of rated interrupting current at 44 kv;
   186 per cent of rated interrupting current at 22 kv.
2. Tests on half of a single pole unit (4 breaks) up to:
   15 per cent of rated interrupting current at 264 kv;
   27 per cent of rated interrupting current at 154 kv.
   The nearest convenient approximation of 146 kv, the maximum voltage anticipated across half of a single pole on the basis of voltage distribution by capacitance in the clearing of a 3 phase ungrounded short circuit, was 154 kv.
3. Tests on a single break up to:
   86 per cent of rated interrupting current at 44 kv;
   46 per cent of rated interrupting current at 110 kv.
   The maximum voltage anticipated across a single break in the clearing of a 3 phase ungrounded short circuit is 44 kv. The performance at 110 kv across one break was not entirely consistent, indicating that this is approximately the limit of the voltage which may be applied across one break.

The justification of the rating of the breaker was based upon 2 lines of reasoning. In the first place, it has been shown[2] that in a breaker of the impulse type, ability to interrupt depends entirely upon the maintenance of oil velocity, properly directed, in the neighborhood of the arc. In these tests the oil velocity has been checked by careful measurements of piston velocity and found to be unaffected by current magnitude up to about 140 per cent of rated interrupting current, as shown in figure 4. Inasmuch as tests at voltages and recovery rates considerably beyond those required showed this velocity to be sufficient for their interruption, it is believed that the ability to clear the required

Fig. 3. Lower side of baffle showing contact and operating rod assembly and entrance to ports

voltage and recovery rate was present also up to at least 140 per cent of rated current, even though in tests at such currents it was not used. Moreover, inasmuch as the arc length and duration were nearly as great in the high current tests as in the high voltage tests, and since the test current considerably exceeded the rating, the mechanical stresses in the high current tests should exceed, by a reasonable margin of safety, those resulting from an interruption of rated current at rated voltage. The breaker has therefore demonstrated its ability both to interrupt rated current at rated voltage and recovery rate and to withstand the mechanical stresses incident thereto.

In the second place, the tests on a single break have demonstrated the ability of one break to interrupt practically full rated current at the voltage to be expected across the most severely stressed break in the worst type of service interruption, while tests up to $2^1/_2$ times this voltage, with a moderate reduction in current, indicate that a very wide factor of safety is present in this respect.

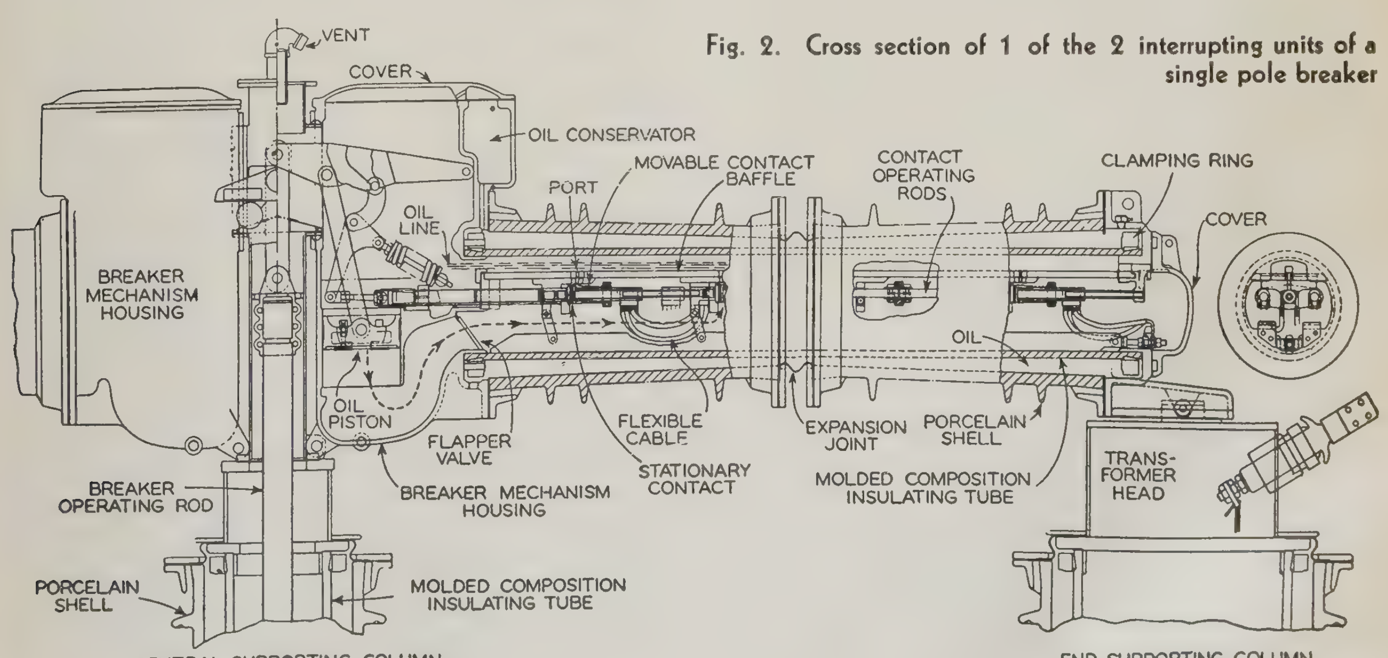

Fig. 2. Cross section of 1 of the 2 interrupting units of a single pole breaker

Although the designers felt full confidence in the justification of the rating by the 2 methods described, particularly in view of the margin of safety evidenced by each, there appeared a third test procedure which more nearly approached the operating condition of applying rated interrupting current to the breaker and following it so rapidly as to give a high voltage recovery rate with full voltage across its terminals.

The circuit for this test is shown in figure 5. Power is supplied from the generator, through reactors if it is desired to control the current by this means, to the low voltage winding of transformer 2. The windings of this transformer are arranged to deliver the current which it is desired to interrupt, which was obtained in this case at 44 kv. This transformer feeds current through the unit under test and an auxiliary unit in series. Connected across the terminals of transformer 2 is a winding of transformer 1 designed to operate at the same voltage. Transformer 1 then steps up this voltage to that at which the breaker is to be tested and this voltage is applied through a sphere gap to the point between the unit under test and the auxiliary unit. Thus the unit under test is subjected to the short-circuit current available from the testing plant at the voltage of transformer 2 and has applied, immediately after clearing, the voltage of transformer 1, which is several times as high. The resulting test duty corresponds to one at several times the power available from the testing plant on the conventional test basis.

It will be observed that in order to be effective this testing scheme requires that the breaker under test and the auxiliary unit interrupt their circuits simultaneously. Moreover, interruption on either unit before the time of interruption of the second unit is greatly facilitated by the fact that the restriking of the arc on the second unit also chops off the recovery characteristic on the first unit. It is therefore necessary that both the unit under test and the auxiliary unit have extremely positive interrupting characteristics in the sense that they are capable of interrupting on one current zero against a very much higher recovery voltage than on the previous current zero. The time of interruption of the breakers must also be subject to precise control in order to permit accurate synchronizing. These requirements prevent a general application of the scheme at the present time, but the positive performance of the

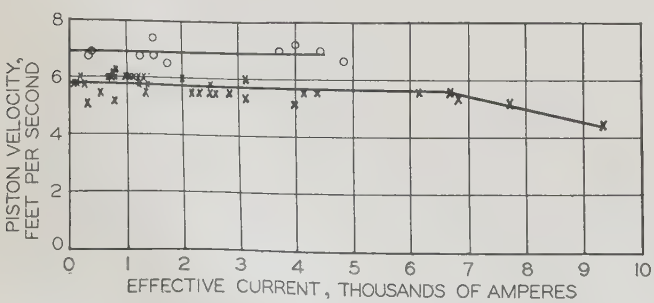

Fig. 4. Effect of arc current on piston velocity

Crosses from co tests; circles from oco tests

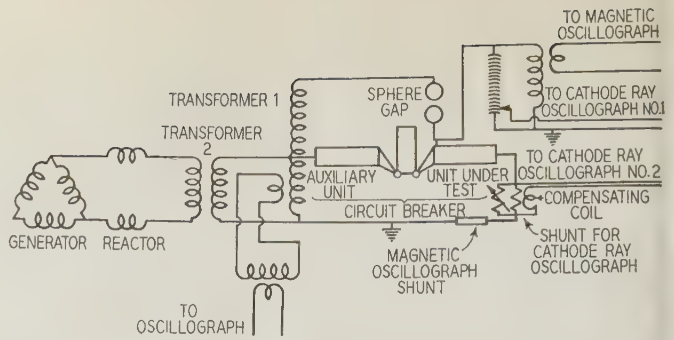

Fig. 5. Circuit for testing breakers beyond the power limits of the testing plant

high voltage impulse breaker on its earliest tests made it appear that with this breaker the scheme could be successfully used.

A convenient method of obtaining the precise synchronization of interruption by the 2 breaker units was furnished by the fact that live metal is normally exposed at the center of a single pole of this breaker, with identical units, one on either side, operated by a common mechanism. By making the intermediate connection to this point, half of the single pole breaker was made to serve as the unit under test with the other half as the auxiliary unit. This method of testing was desirable also from the standpoint that the maximum voltage which could be obtained from transformer 1 was 132 kv, which compared favorably with 146 kv, the maximum anticipated voltage across half of a single pole unit, but less favorably with the voltage to be anticipated across the entire pole.*

The function of the sphere gap was concerned with precision control of synchronization. Each unit will, of course, accomplish the final interruption at a time when its current is instantaneously equal, very nearly, to zero, and these times will be very nearly the same for the 2 units provided no current flows into the intermediate point from the high voltage winding of transformer 1. Although under ideal conditions such current would not flow, it is evident that the arc voltage of both units appears across the 44 kv winding of transformer 1 and that this will cause current to flow in the high voltage winding of this transformer if the circuit through that winding is closed. The sphere gap, set at such a value as not to be broken down by arc voltage but to be readily broken down on the rise of the voltage recovery curve immediately after interruption, prevents current flow in this circuit during the short circuit, but makes the circuit available for application of voltage very promptly upon the cessation of short-circuit current.

Cathode Ray Oscillograph Equipment

In order to obtain complete data on the performance both of the breaker and of the circuit, in addition to the usual magnetic oscillograph equipment

* The 146 kv which may appear across the terminals of one unit in service lies between +20 kv and +166 kv, so that the voltage distribution among its 4 breaks will be somewhat worse than with the application of 146 kv across the unit with one terminal at ground potential, as in the present tests. On the basis of obtaining the same voltage across the most highly stressed break, a value of about 150 kv should be used. On this basis the voltage used in test was about 12 per cent low instead of 9.6 per cent.

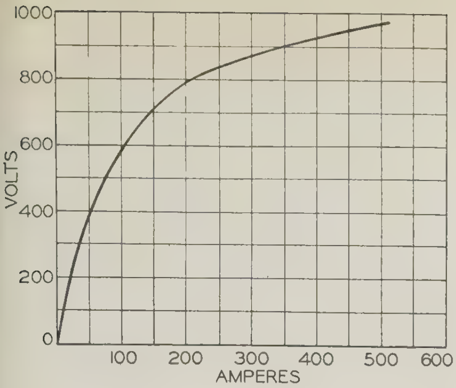

Fig. 6. Calibration curve of shunt for current measurement by cathode ray oscillograph

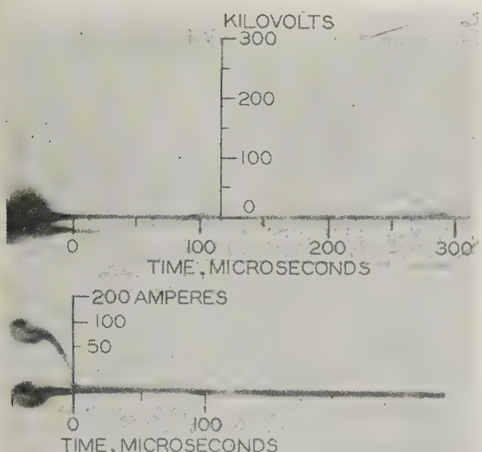

Fig. 7. Cathode ray oscillograms of current and recovery voltage on a test at 99 kv and 2,700 amperes

2 cathode ray oscillographs were employed, one to record the voltage to ground at the center casting of the breaker, which is the voltage across the unit under test, and the second for the most part to record the current through the unit under test, although it was used in some cases to record the voltage at the high voltage terminal of transformer 1.

It was desired to obtain a sensitive record of the current so that its behavior in the neighborhood of current zero could be conveniently studied. For this purpose, recording electrostatically, it was necessary to limit the voltage applied to the oscillograph at the current crest for the sake of safety as well as to avoid generation of a high inphase voltage in the power circuits. To accomplish this, a shunt was used which consisted of 2 units in parallel; and ohmic shunt in the form of a water box in order to give a straight line characteristic in the neighborhood of zero current, and a unit of a material whose resistance depends upon the applied voltage in order to hold down the voltage at crest current to a safe value. An arrangement was developed which gave a calibration curve (figure 6) with a deviation of less than 20 per cent from a straight line up to 40 amperes, applying 400 volts to the oscillograph deflection plates, but which was capable of passing 10,000 amperes with an estimated drop of only about 2,500 volts. As a precaution against the damage which might be caused by the flow of power currents inside the oscillograph house, a capacitor of about 1 microfarad was placed in series with the leads to the oscillograph just outside the oscillograph house.

In order to avoid errors resulting from voltages induced electromagnetically in the leads from the shunt to the deflection plates, a 10 turn coil was formed in these leads and so oriented as to induce a voltage opposite in sign to that induced in the remainder of the circuit. By this means the mutual inductance of the power circuit and the leads to the oscillograph was reduced to less than 5 microhenries, which would shift the record of a current passing through zero at a constant rate of change by only a half microsecond. This was sufficiently accurate for the current normally associated with the recovery transient, although breakdown of the circuit breaker gap brought about oscillations of very high frequency which were probably not correctly recorded.

Oscillograms of Circuit Performance

Figures 7 and 8 show cathode ray oscillograms obtained on this circuit and verifying its effectiveness in simulating an interruption at higher power. Figure 7 was taken at $^3/_4$ voltage or 99 kv. It shows a crest of 252 kv or 1.80 times normal crest appearing 144 microseconds after current zero, giving a voltage recovery rate of 1,750 volts per microsecond. Figure 8 was taken at full voltage, and the recovery oscillation was interrupted at 290 kv by the lightning arrester after 113 microseconds, corresponding to a recovery rate of 2,600 volts per microsecond.

In both of these figures, the curve of current as it approached zero on the last half cycle is shown. The apparent flat top form of the wave is caused by the bending of the calibration curve which has already been explained. It will be noted that the current ceases abruptly upon reaching zero, evidencing no conduction in the reverse direction.

In these 2 cases, the form of the curve of voltage rise across the unit under test was much the same. At current zero there was a negative voltage corresponding to the arc voltage. For a brief time after current zero, there was a slow rise caused presumably by power transmitted through the capacitance of the sphere gap. An abrupt rise in voltage then followed, caused by breakdown of the sphere gap, and from that time on the rate of rise was moderately rapid, keeping pace with the voltage at the transformer terminal.

Figure 9 shows a slightly different type of voltage

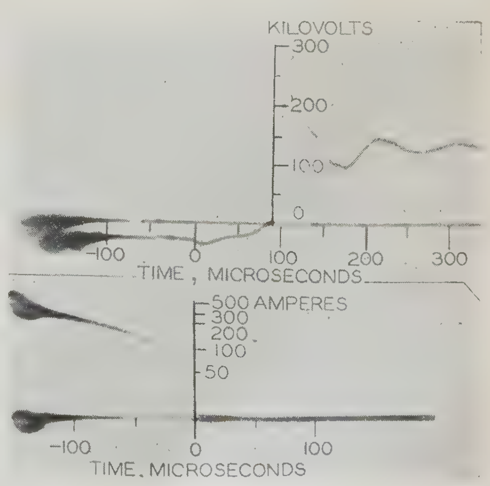

Fig. 8. Cathode ray oscillograms of current and recovery voltage on a test at 132 kv and 3,700 amperes

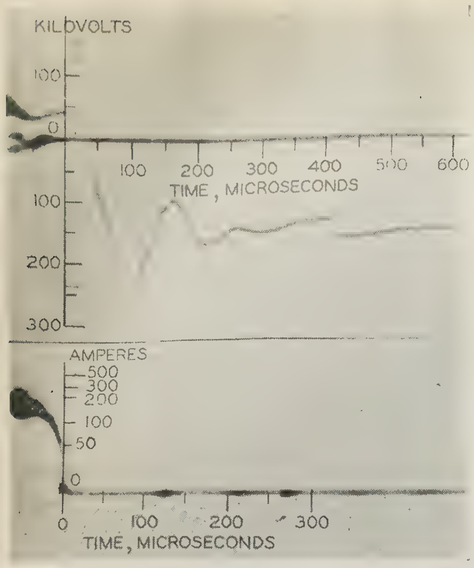

Fig. 9. Cathode ray oscillograms of current and recovery voltage on a test at 132 kv and 4,500 amperes

rise. Here the arc voltage rose high enough just before current zero to break down the gap with the result that there was no break in the rising portion of the voltage recovery curve.

Figure 10 shows a typical magnetic oscillogram. Its appearance conforms in every way with that corresponding to the condition which this test is intended to simulate, except for the breaks in the curve of voltage across the unit under test. These breaks appear because the voltage was supplied to the center point through a sphere gap, and the current through the gap was not at all times sufficient to maintain a good circuit. There seems to be no reason to believe, however, that a broken curve of this type should be less severe upon the breaker than a smooth sine wave provided the same crest values are maintained, which is in general the case.

Analysis of Performance at Current Zero

Current zeros during the arcing period may be divided into 5 groups in accordance with the behavior of the arcs at the current zero. These groups are:

1. *Both arcs restrike.* In this case, there is a subsequent current zero which falls in one of the other groups.

2. *One arc restrikes, but not both.* If the arc restrikes in the unit under test, a failure to clear at the existing contact separation is indicated. If it is the other arc which restrikes, the test should in general be regarded as inconclusive, but in this particular case, inasmuch as the 2 units were duplicates and operated simultaneously under conditions not greatly different, such tests may be considered in the same category as those in which the arc restruck in the test unit.

3. *Both arcs clear, but the auxiliary unit slightly ahead of the test unit.* This is caused by breakdown of the gap at the start of or during the last loop of current. In this case, the current flowing in the test unit at the time of interruption decays to zero much more slowly than the normal rate, as shown by the oscillogram of figure 11. There is also some loss of current in the last loop on account of that drained by the 44 kv winding of transformer 1. For these reasons, tests falling in this group are considered somewhat less conclusive than those of group 5.

4. *Both arcs clear simultaneously but the gap fails to operate promptly.* This occurred only once in the course of the 74 tests and was caused by high displacement of the current wave with a consequently low amplitude of the voltage recovery oscillation. Such a test is, of course, of no value.

5. *Both arcs clear simultaneously and the gap operates promptly.* This is the desired performance and constitutes a valid test under conditions of increased apparent power.

Results of Tests

Seventy tests were made with the current supplied at 44 kv and a potential of 132 kv applied at interruption, the current varying from 3,700 amperes to 5,700 amperes. Four more tests were taken on this same connection but with the excitation somewhat reduced. For these 74 tests, figure 12 shows the relative frequency of occurrence of cases belonging to each of the 5 performance groups as a function of contact separation. This figure shows that 49 tests, or 66 per cent of the total, fell in group 5, giving valid tests under conditions of increased apparent power.

It is also shown that the borderline value between clearing and restriking of the arc (between group 2 and group 5) is fixed within 0.1 inch at 0.9 inch per break, which is the same value as given by the conventional tests at 132 kv.

These tests, therefore, firmly establish the ability of one unit of the breaker to clear, without damage to itself, a current of 5,000 amperes at 132 kv with the same contact separation as that required in the tests within the capacity of the station on a conventional basis, and confirm the reasoning by means of which this ability was previously inferred.

With a contact separation of 0.9 inch required for normal interruption, the fact that one unit interrupts when a current zero occurs at a contact separation of 0.5 inch or more relieves the breaker somewhat in those tests which would normally produce the greatest arc lengths and so the greatest mechanical stress. Assurance that the distress will not be excessive in these cases must be obtained by increasing the cur-

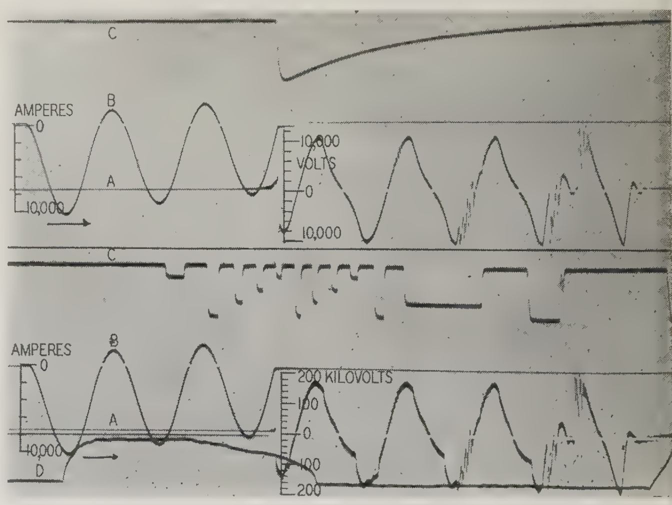

Fig. 10. Magnetic oscillograms of test at 132 kv and 4,900 amperes

Upper oscillogram: *A*—Primary voltage of transformer 1; *B*—Short-circuit current; *C*—Cathode ray oscillograph excitation current

Lower oscillogram: *A*—Voltage across test unit; *B*—Short-circuit current; *C*—Motion of operating rod, each step representing $^1/_2$ inch of travel; *D*—Total control current

rent to compensate for the difference in arc length.

For this reason, a few tests were made with the current supplied at 22 kv and a potential of 132 kv applied at interruption. Although a smaller percentage of valid tests was obtained with this connection than when current was supplied at 44 kv, as much as 8,300 amperes, or 166 per cent of the rated interrupting current, was cleared on such a test.

Borderline Behavior

The records obtained offer an opportunity to investigate the behavior of the breaker in the region of the limiting value of contact separation below which the arc will restrike for another loop.

A case in this region is shown in figure 13, where it may be noted the voltage rose across the breaker terminals, broke down, and rose again several times before finally rising, passing through the recovery oscillation, and going into the steady state restored voltage. This type of behavior took place for the most part when the contact separation was only slightly in excess of the 0.9 inch per break required to clear, 90 per cent of the cases in which it was observed occurring with a contact separation of 1.2 inch or less. On the other side of the borderline, the performance started out in the same way, but instead of the recovery oscillation being finally completed, the voltage dropped to zero and another half cycle of short-circuit current took place.

It may be noted from the current shown in figure 13 that an oscillation at very high frequency and involving high currents takes place at the time of each breakdown of the gap inside the breaker. This is an oscillation over a circuit comprising only the capacitance connected to the high voltage terminal of the breaker and the inductance of the circuit through the breaker to ground, the high values of current and frequency resulting from the fact that the inductance, as well as the capacitance, is very low. The shift in the zero line of the current curve following each of these oscillations is believed to result from the oscillograph circuit rather than to represent behavior of the current through the breaker.

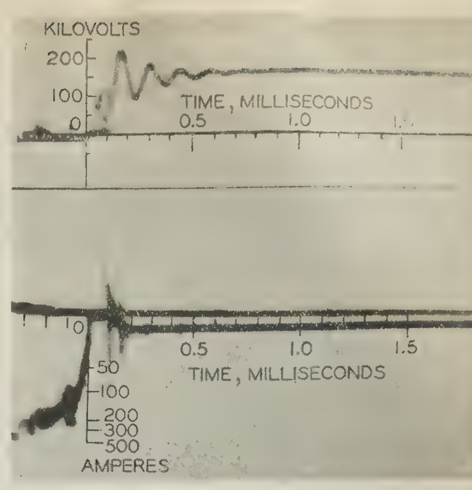

Fig. 13. Cathode ray oscillograms showing breaker behavior on current zero with contact separation barely sufficient for interruption

This type of behavior sets this breaker off from those in which conduction during the recovery transient has been observed previously. It suggests that the mechanism of interruption is different in the 2, perhaps that the plain break interrupts by deionization, whereas the impulse type operates by interposition of dielectric. The rise and comparatively gradual decay of currents of several amperes is more compatible with a process of deionization than with one of interposition of dielectric; but a sudden breakdown resulting in a low resistance path which permits an oscillation of very high frequency that is interrupted when it is partially damped out appears to be more consistent with interruption by the interposition of dielectric. In the borderline case, then, the rate of interposition would be not quite sufficient to avoid breakdown completely; an oscillatory breakdown, however, would provide current zeros on which subsequent interruptions could take place, and a breakdown of short duration would create only a small conducting hole in the interposed insulation which would be refilled rapidly by the same agencies which interposed the dielectric in the first place.

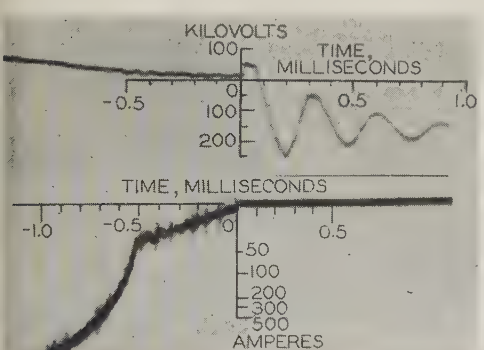

Fig. 11. Cathode ray oscillograms showing deformation of current wave as the result of premature operation of gap

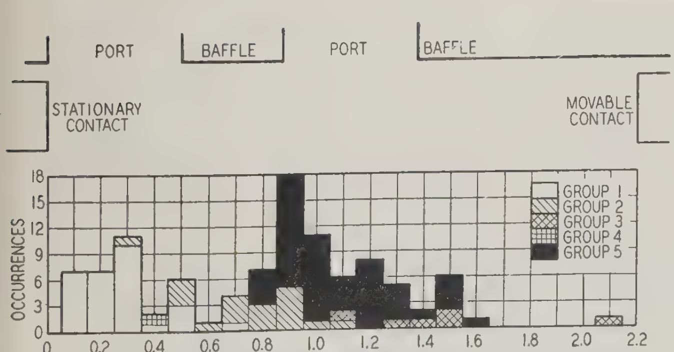

Fig. 12. Distribution of various types of breaker performance at current zeros with reference to contact separation

Determination of Voltage Distribution

Several cathode ray oscillograms have been taken which show the voltage distribution between the 2 sections of the high voltage impulse breaker interrupting a conventional 132 kv circuit. In order to obtain these oscillograms, the line from the high voltage transformer terminal was connected to one terminal of the breaker, the other being grounded, and connections were made from the high voltage terminal and from the exposed live metal at the top of the center column (the "center casting") to the

deflection plates of cathode ray oscillographs; thus one oscillograph measured the voltage across the entire breaker and the other the voltage across the section electrically adjacent to ground.

On account of the capacitance to ground of the voltage divider and the lead connecting it to the center casting of the breaker, the capacitance distribution in the breaker as tested was less favorable than the normal capacitance distribution of the breaker. In order to obtain records on an arrangement having moderately good potential distribution as well as on one having rather poor distribution, the grading shield at the upper joint of the middle porcelain column (see figure 1) was electrically connected to the high voltage terminal of the breaker and was raised about 15 inches above its normal position. Measurements of the potential distribution of each arrangement were made by a potentiometer method with 30 kv at 60 cycles, applied to the high voltage terminal. This showed 18 per cent of the voltage across the ground side of the breaker with the unimproved arrangement and 33 per cent with the improved arrangement, as compared to 25 per cent with no measuring equipment connected.

Theory of Voltage Distribution

It is commonly assumed that the distribution of voltage between breaks in a multibreak breaker is determined by capacitance relationships. Strictly speaking, this is true only of changes that take place after conduction in the arcs has ceased. While the modifications produced by other factors are very often of a minor nature, they occasionally become of considerable importance; hence they are listed here in order that the extent of their effect in the oscillograms presented may be analyzed. These other factors may be classified as follows:

I. Arc conduction during the recovery period

(*a*). If considerable, this factor will determine distribution completely; if very slight, it may only modify somewhat the capacitance distribution. In most cases, this will tend to improve distribution.

(*b*). It occasionally happens that one break becomes at least partially conducting, while another in series acts as a good insulator. In such a case the second break takes more than its share of the voltage, and in extreme cases carries the entire burden.

II. Arc voltage

(*a*). The voltage distribution at the instant of current zero will be determined by arc voltages and will therefore tend to be even rather than in accordance with capacitance. Assuming this to be the case, any inequalities in capacitance distribution may be considerably exaggerated when transposed into instantaneous values of voltage distribution. Suppose for instance, that at the instant of current zero a breaker has an arc voltage of −10 kv divided equally between 2 breaks and that at a time t_1 in the course of the recovery transient the voltage across the breaker rises to +30 kv, a change of 40 kv. If the capacitance distribution is 90 to 10, the voltage change across the ground side break will be only 4 kv, and the distribution at time t_1 will be 41 to 1; that is, one break will actually be more highly stressed than if the other break were not present.

(*b*). The analysis given in (*a*) will, of course, be somewhat modified if the arc voltages are not equal, or if one of the arcs is suddenly extinguished at a finite value of current. In the latter case, series arcs would, of course, be extinguished very quickly for lack of current, but this lack of current would not make itself felt until the voltage of the metal parts between the 2 arcs had been modified by the passage of a charge through the second arc.

There are 2 other considerations which must be borne in mind even when the voltage distribution is governed entirely by capacitance relationships. These are:

1. The part of the total voltage appearing across each break is a function of the relative magnitude of the voltages at the 2 terminals of the breaker with respect to ground. The distribution figures usually stated apply to the worst case, that is, where one terminal of the breaker is at ground potential, and this case is probably closely approximated by the vast majority of service operations. The other extreme is encountered when the voltages at the 2 terminals of the breaker are equal but of opposite polarity. In such a case, the potential will be equally distributed in a symmetrical 2 break breaker and there will be some improvement in the distribution of a breaker with more than 2 breaks, although the distribution in the latter case may remain very poor.

2. The capacitance relationships vary slightly with contact separation, being somewhat more favorable when the contacts are only slightly separated than when the breaker is fully open.

Oscillograms of Voltage Distribution

Figure 14 shows a number of the oscillograms obtained on this test. Parts *a* and *b* apply to the arrangement with the poorer capacitance distribution, and parts *c* and *d* to the improved arrangement. In all 4 cases, the most striking feature is the faithfulness with which the details of the voltage at the breaker terminals are reproduced in the voltage at

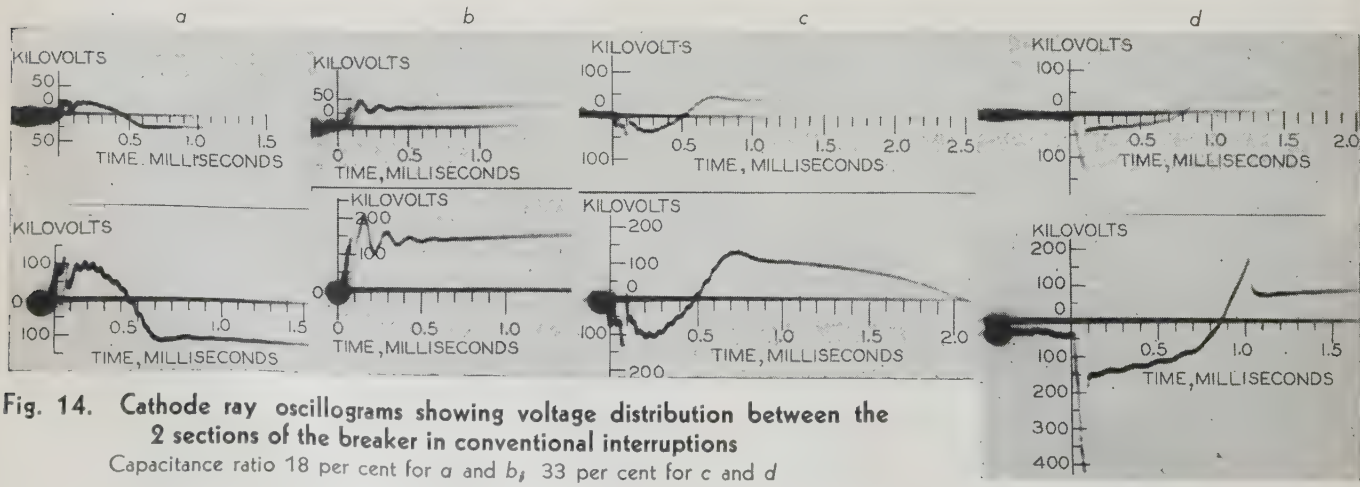

Fig. 14. Cathode ray oscillograms showing voltage distribution between the 2 sections of the breaker in conventional interruptions

Capacitance ratio 18 per cent for *a* and *b*, 33 per cent for *c* and *d*

the center casting. This in itself is presumptive evidence of voltage distribution in accordance with capacitance.

Confirming this presumption, while there is evidence of modification due to several of the causes listed, measurements at voltage peaks and other points which lend themselves to this purpose show deviations from the capacitance ratio varying from practically zero to about 10 kv, or $5^1/_2$ per cent of normal crest voltage. It may therefore be stated that on the breaker tested the capacitance ratio holds within a few kilovolts.

References

1. Circuit Breakers for Boulder Dam Line, D. C. Prince. Elec. Engg. (A.I.E.E. Trans.), v. 54, April 1935, p. 366–72.

2. Oil Blast Breaker Theory Proved Experimentally, D. C. Prince and E. J. Poitras. *Elec. Wld.*, v. 97, Feb. 28, 1931, p. 400–04.

Transformer Circuit Impedance Calculations

The purpose of this paper is to present a general method of impedance calculation equally applicable to all transformer circuits and all construction types. An effort is made to show how the "circuit method" can be used and to prove some of the circuit properties and relations that are pertinent to the problems involved.

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SHORT-CIRCUIT impedance is of fundamental importance in determining the performance of transformers. Recent advances in the art of transformer construction and the growing use of complicated circuits have necessitated, and in turn were made possible by, the introduction of new methods for calculating the short-circuit impedance.

Introduction

A general method for determining impedance of transformer circuits is presented in this paper. This method is applicable to all types of transformer circuits and to all types of transformer construction.

Circuits with mutual impedances usually can be replaced by *equivalent* circuits without mutual impedances. Such equivalent circuits have been adequately described elsewhere;[1,2,3] they are not particularly suitable for the purposes of, and will not be employed in, this paper.

For the sake of generalization and completeness, basic formulas are expressed in terms of impedances, although it is recognized that in most of the cases dealt with in practice it is quite sufficient to calculate reactances rather than impedances of transformer circuits.

Field and Circuit Methods. A few years ago a typical transformer short-circuit reactance problem consisted of calculation of leakage reactance between 2 windings only.

The problem consisted essentially of determination of the reluctance of the leakage flux path and of the linkage of the flux with windings. In other words, the problem required the determination of effective leakage flux *field*. For its solution, therefore, the knowledge of turn distribution of both windings, and of dimensions of windings and core were necessary.

Introduction of multiwinding transformers, load ratio control, and phase shift control into practice gave birth to many opportunities for the design and distribution of windings that would better perform the new operating functions, provided that reactance between various windings and their parts could be calculated.

It was found, however, that calculation of reactances by the field method was too cumbersome in many cases, and often was absolutely impractical. Furthermore, the field method, of course, is entirely inapplicable to the calculation of effective reactance of a group of interconnected transformers, as, for example, used for some types of load ratio control.

Therefore, for solutions of problems of such categories, the "circuit" method, presented here, has been developed. Its use in transformer design department over a period of years has demonstrated its practical value.

The circuit method consists of 3 steps:

1. The actual complicated windings are subdivided into relatively simple parts or represented as the results of superposition of relatively simple, actual, and fictitious windings.

2. Reactances between all parts of actual or fictitious windings are calculated by field method.

3. The values so obtained are substituted in proper circuit formulas for securing the effective reactance at specified transformer terminals of actual windings.

Formulas of the circuit method establish the relation between the effective reactance at given trans-

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1. For all numbered references see list at end of paper.

former terminals and reactances, polarities, connections, numbers of turns, and current phase of the interconnected parts of all windings.

Generally the precise solution by field method, even of a relatively simple reactance problem, is quite laborious. On this account field method formulas contain semi-empirical constants which depend on transformer construction. However, circuit-method formulas are mathematically exact, contain no empirical constants, and are independent of transformer construction. The circuit method allows the use of the field method to be limited to calculation of reactance between the simple parts of windings where its highest accuracy is obtained. As final results are secured by putting these (relatively accurate) values of component reactances into precise formulas of the circuit method, the accuracy of the final answer in many cases is considerably bettered.

From the designer's point of view, the circuit method has the following other advantages.

For any change in connections or turn distribution between parts of windings, a complete new calculation must be performed where the field method is employed, even where physical dimensions of windings are not affected. When in such cases, the circuit method is used the only change is in the choice of formulas or even only in the coefficients of the formulas.

The turn distribution giving the minimum reactance for a given configuration and dimensions of windings can easily be obtained from reactance formulas of the circuit method.

The 2 components of impedance (resistance and reactance) usually are dealt with independently in transformer calculations. Inasmuch as in most cases they are of a different order of magnitude, no appreciable error is introduced by this simplification, even when such treatment is incorrect theoretically. Physically, resistance and reactance may have very little in common. In the field methods of solution they retain their reciprocal aloofness. In the circuit methods the introduction of the concept of impedance, considered as a complex quantity, permits operation with both components simultaneously, whenever desired.

Types of Transformer Circuits. The great variety of transformer circuits encountered in practice will be appreciated better from the following, more or less natural, classification:

1. *By the number of phases,* transformer circuits can be divided into single phase and polyphase circuits. The polyphase circuits can be subdivided further into symmetrical and unsymmetrical circuits. In polyphase circuits, transformers can be either of single phase or of polyphase construction. Ordinarily the number of electrical phases and magnetic phases (whether obtained by single phase or polyphase transformers) is the same, but there are circuits in which the number of electrical phases is different from the number of magnetic phases. When the number of electrical and magnetic phases is the same they usually coincide; that is each electrical phase is restricted to a single magnetic phase. There are circuits, however, in which electrical and magnetic phases do not coincide. This happens usually when the number of electrical phases is different from the number of magnetic phases, as in various phase transformation connections; but it may happen even when the number of electrical and magnetic phases is the same, as for instance in zigzag, extended delta, and other connections. The term *phase interconnection* will be used to describe circuits in which electrical and magnetic phases do not coincide.

2. *By the number of transformers contained,* circuits can be divided into single and multitransformer circuits.

3. *By winding connections,* circuits located on each magnetic phase of each transformer may be classified as belonging to one or more of the following types: autotransformers, series, multiple, coupling, off-ratio, phase interconnection. Many complicated circuits can be resolved into the fundamental types, listed above, in more than one way; there are circuits, moreover, that do not belong clearly to any one of these types.

The classification of circuits just given will not be used as the basis for calculation, since the method presented here has the important advantage of being equally applicable to all types of transformer circuits.

Some Basic Properties of Circuits

A discussion of some fundamental properties of circuits in general will be helpful. Only a rather special and a very simple type of circuit need be considered; namely, a passive static circuit of constant bilateral parameters. Moreover, only the steady state condition under single frequency excitation need be considered.

Within these restrictions, the circuit may be of any degree of complexity and may be represented by the general network of figure 4, having any number of excited terminals connected by branches possessing not only self- but also mutual impedances of resistive, reactive, and capacitative type.

1. Conservation of Power and Reactive Volt-Amperes. Let us define the power and reactive volt-amperes as the readings of a wattmeter and a reactive volt-ampere meter, respectively. If readings of these meters are taken for all the terminals and all the branches of the network of figure 4, then as shown in appendix I:

The sum of power (or reactive) volt-amperes at external terminals of a network is equal to the sum of power (or reactive) volt-amperes in the branches of the network.

It follows that any complicated network can be subdivided, for convenience of calculation, into a number of subnetworks and the total power and reactive volt-ampere input obtained as the sum of power and reactive volt-ampere inputs to separate subnetworks. This method of calculation is particularly convenient when the network can be subdivided into subnetworks without mutual impedances between them ("Independent" subnetworks).

The voltages impressed on the terminals of the network of figure 4 are restricted in no way as to their relative phase angles. Thus, for instance, they may be the phase voltages of a polyphase system. The total power and reactive volt-ampere input may be expressed then as the sum of inputs to all phase terminals. Where the network is symmetrical with respect to the phase terminals and the impressed phase voltages are symmetrical, total power and reactive volt-ampere input obviously is given by the input to any one phase terminal multiplied by the number of phases.

However, even when a polyphase network as viewed from its terminals is symmetrical, it does not

necessarily follow that it can be subdivided in such a manner that each phase has its own independent subnetwork.

2. Relation Between Energy and Power and Reactive Volt-Amperes. Although it is sufficient for the purposes of this paper to consider the law of conservation of power and reactive volt-amperes as simply a necessary formal consequence of Kirchhoff's circuit laws, a "physical" interpretation may be helpful.

No simple physical significance can be attached to the power and reactive volt-ampere input to one branch of the network, as in the presence of mutual impedances these values obviously depend on the currents of other branches. However, when the inputs to all branches are added together they give net input to the circuit. Similarly, no simple physical interpretation can be given to the power and reactive volt-ampere input to one terminal of the network, as these values obviously depend on the zero reference potential. However, when the inputs to all terminals are added together, the result becomes independent of the reference potential and again gives the net input to the circuit.

It is shown in appendix II that:

1. Net power volt-ampere input to a network is equal to the average rate of energy dissipation in the network.

2. Net reactive volt-ampere input to a network is equal to the difference between the average values of electromagnetic and electrostatic energies stored in the network, multipled by twice the angular frequency.

Thus the law of conservation of power and reactive volt-amperes is not only analogous to the law of conservation of energy, but is actually its consequence. It is also clear now why the voltages impressed on the network were restricted to a single frequency.

3. Input Volt-Amperes Due to Orthogonal Components of Currents. Total power and reactive volt-ampere input to a network can be expressed in terms of self- and mutual impedances of all branches and of currents in all branches (see appendix II). Generally, the currents in various branches will be out of phase. If these currents are resolved into orthogonal components with respect to any arbitrary reference phase, then as shown in appendix III:

Total power (or reactive) volt-ampere input to a network is equal to the sum of power (or reactive) volt-ampere inputs calculated for the two orthogonal components of currents taken separately.

Thus the calculation of volt-ampere input to a network carrying out-of-phase currents in its branches is reduced to a simpler calculation, repeated twice, of volt-ampere input to the same network, but with all branch currents in phase.

4. Current Flow in a Network. The equilibrium conditions of a network are given by the current and voltage laws of Kirchhoff.

The number of branches that may be assigned arbitrary independent currents without violating the current law of Kirchhoff is known as the number of degrees of freedom of a network. With these currents assigned, the currents in all other branches are determined by the current law of Kirchhoff, as sums and differences of independent branch currents. Thus each independent branch current can be traced through a closed circuit within the network and becomes an independent mesh current.

To determine the independent mesh currents the voltage law of Kirchhoff must be invoked. This could be done in the usual manner by writing the voltage equations for the closed circuits described by the independent mesh currents. This additional and separate procedure is, however, entirely unnecessary. As shown in appendix IV, the power and reactive volt-ampere input formula itself contains the voltage law equations of Kirchhoff. These equations can be obtained in an explicit form by taking partial derivatives of the power and reactive components of the volt-ampere input formula with respect to the orthogonal components of independent mesh currents.

If the power and reactive volt-ampere input to a network is denoted by

$$P + jQ$$

and the independent mesh current of mesh A is denoted by

$$I_A = I_A' + jI_A''$$

the Kirchhoff voltage law equations for this mesh can be written as

$$\frac{\partial(P + jQ)}{\partial I_A'} + j\frac{\partial(P + jQ)}{\partial I_A''} = 0 \qquad (1)$$

or

$$\frac{\partial P}{\partial I_A'} - \frac{\partial Q}{\partial I_A''} = 0 \qquad (2)$$

$$\frac{\partial P}{\partial I_A''} + \frac{\partial Q}{\partial I_A'} = 0 \qquad (3)$$

The flow of currents in the network may be determined, therefore, in terms of orthogonal components of independent mesh currents by forming equations 1 or 2 and 3 for all independent mesh currents and solving the resultant system of simultaneous equations.

Equations 1 or 2 and 3 show that in a purely reactive 2-terminal network the flow of currents is such that the reactive volt-ampere input has its minimum possible value for a given input current.

5. Relation Between Input Volt-Amperes and Impedance. For a 2-terminal network the relation between the power and reactive volt-ampere input to the network and its impedance ("driving point impedance" in circuit terminology) is given by the simple equation

$$P + jQ = (R + jX) \ |I|^2 \qquad (4)$$

where $(R + jX)$ is the impedance and I is the effective value of the input current. The same relation may be extended to symmetrical polyphase networks, using per-phase values of volt-ampere input, impedance, and current.

With the exception of symmetrical polyphase net-

works, the unqualified term impedance becomes ambiguous when used for networks with more than 2 input terminals.

Application to Transformer Circuits

All the properties and methods of calculation applicable to the general network as described, apply to networks consisting of several interconnected transformers or of several windings on the same core. A transformer network has, however, some peculiarities of its own that may be taken advantage of in calculating the flow of currents, the volt-ampere input, and the impedance.

The expression volt-ampere input to a network was not likely to be misinterpreted. In what follows, the expression volt-ampere input to a transformer circuit or to a transformer must be given the same interpretation of *net* input, that is the difference between the input and the output.

There are no mutual impedances between windings placed on different cores. It is obvious, therefore, that in a transformer circuit each transformer forms an independent subcircuit. The total power and reactive volt-ampere input to the entire circuit is equal to the sum of power and reactive volt-ampere inputs to all transformers. Moreover, each magnetic phase of polyphase transformers forms an independent subcircuit, since the effects of mutual leakage flux can be neglected. Thus in polyphase transformer circuits with phase interconnections, magnetic phases of transformers, rather than electric phases of the circuit, are the natural basis of calculation.

Since each magnetic phase of a polyphase transformer is equivalent, for the purposes of this paper to a separate transformer, the word transformer will be used in what follows to designate both separate single phase transformers and separate magnetic phases of polyphase transformers.

In a transformer circuit the transformer windings take the places of the branches of the general network. Neglecting the exciting current, the vector sum of ampere-turns of all windings on each transformer must add up to zero. Thus, within a transformer, in addition to the Kirchhoff current law there is an analogous ampere-turn balance law.

The flow of currents in transformer circuits is not necessarily uniquely determined by the Kirchhoff current law and the ampere-turn balance law. In multiwinding transformers with some connections of windings the ampere-turns in only one winding may be given an arbitrary value, the ampere-turns in the remaining windings becoming fixed by the ampere-turn balance law. With other connections ampere-turns in several windings can be assigned arbitrary values without violating the ampere-turn balance law. By analogy with the general network, the number of windings that can have arbitrary independent ampere-turns without violating the ampere-turn balance law may be called the number of degrees of freedom.

In transformers having a degree of freedom of 1, the distribution of ampere-turns among windings is completely determined by connections and the number of turns in various windings and is independent of impedances. In transformers of a higher number of degrees of freedom the distribution of ampere-turns among windings depends on impedances, and can be determined by the same method as the flow of currents in the general network.

Tracing the flow of currents and the subsequent calculation of the circuit are facilitated by abandoning the volt-ampere-ohm system of units in favor of the per-unit system. Taking an arbitrary number of volt-amperes as reference volt-amperes, the per-unit value of current for every part of the circuit is found as follows:

Let

$$(\textit{Reference Volt-Amperes}) = (\textit{No-Load Voltage})(\textit{Reference Current}) \tag{5}$$

$$(\textit{Actual Volt-Amperes}) = (\textit{No-Load Voltage})(\textit{Actual Current}) \tag{6}$$

Then

$$(\textit{Per-Unit Current}) = \frac{(\textit{Actual Current})}{(\textit{Reference Current})} = \frac{(\textit{Actual Volt-Amperes})}{(\textit{Reference Volt-Amperes})} \tag{7}$$

Equation 7 is a general relation applying for transformer circuits of any complexity. For every transformer the ratio of volt-amperes of the right-hand side of equation 7 may be replaced by the corresponding ratio of ampere-turns, since the no-load volts per turn have the same value for all windings on one core. For every series-connected circuit on one transformer the ratio of ampere-turns may be replaced by the corresponding ratio of turns, since the same amperes must flow in all parts of the circuit.

With self- and mutual impedances of windings, as well as currents, expressed in the per-unit system the power and reactive input to a transformer circuit will also be expressed in per-unit, based on the same reference volt-amperes. Equation 4 then will give the impedance of the circuit in the per-unit system, based on the same reference volt-amperes. In practice, the rated kilovolt-amperes of the circuit (rated kilovolt-amperes per phase in symmetrical polyphase circuits) is usually taken as the reference volt-amperes. Where the input current is taken as unity, equation 4 becomes:

$$P + jQ = R + jX \tag{8}$$

that is, per-unit power and reactive input is equal to per-unit impedance.

In transformers, short-circuit impedance between 2 windings can be both calculated and tested with greater accuracy and ease than the self- and mutual impedances of windings. Moreover, while the self- and mutual impedances of windings are subject to saturation, the short-circuit impedance is not. Fortunately, a simple formula ties together the short-circuit, the self- and the mutual impedances of 2 transformer windings. Utilizing this formula, the power and reactive input expression in terms of self- and mutual impedances of windings can be rewritten in terms of short-circuit impedances of pairs of windings, as shown in appendix V.

Finally, if per cent instead of per-unit values of short-circuit impedances are used, the power and reactive input to the circuit and the impedance of the circuit will also be in per cent, all based on the same reference kilovolt-amperes.

Calculation of Impedances of Transformer Circuits

The procedure to be followed in calculating the impedance of transformer circuits may be outlined as follows:

1. Draw a complete diagram of the circuit, including all phases if the circuit is polyphase. In a symmetrical polyphase circuit the calculation may be limited to one transformer out of every bank of transformers.

2. Select the reference kilovolt-amperes on which to base the calculation. Usually the rated kilovolt-amperes per phase of the circuit is taken as reference and the input current is taken as unity.

3. Utilizing the Kirchhoff current law and the ampere-turn balance law trace the flow of input current through the network, indicating the direction of flow in every winding. If the circuit is of degree of freedom 1, the per-unit values of current in all windings of all transformers are immediately ascertainable by equation 7. In the absence of phase interconnections the currents in all windings of every transformer will be in phase or in phase opposition. The phase displacement between currents in different transformers of a group, such as may be caused by Y/Δ connections, need not be taken into account. In transformers containing phase interconnections the currents in windings will be out of phase with respect to one another and must be resolved into orthogonal components. The reference vector is arbitrary, but it usually is convenient to resolve currents in all windings into "in-phase" and "quadrature" components with respect to the input current to the transformer.

4. Where the circuit is of several degrees of freedom the currents that cannot be ascertained by the Kirchhoff current law and the ampere-turn balance law are denoted by algebraic symbols representing the yet undetermined per-unit values of currents. Each current must be denoted by 2 symbols to represent its 2 orthogonal components, except that in circuits of negligible resistance without phase interconnections the currents in all windings of every transformer will be in phase or in phase opposition, so that the second symbol is superfluous. Where it is used, however, correct answer will be obtained just the same.

Occasionally the circuit for which the impedance is to be determined contains isolated closed circuits. The safe procedure is always to assume that currents flow in these isolated circuits.

5. Express the short-circuit impedances of all pairs of windings of each transformer in per cent, based on the reference kilovolt-amperes selected under item 2 of this list. The conversion formula from ohms to per cent is:

$$\textit{Per cent } Z = \frac{1}{10}\frac{(\textit{Ohms})\ (\textit{Reference Kva})}{(Kv)^2} \tag{9}$$

Where Kv = no-load voltage in kilovolts of winding at the terminals of which the impedance in ohms was determined.

Equation 9 shows that the per cent value of impedance is directly proportional to the reference kilovolt-amperes and inversely proportional to the square of excitation on transformer. This is, therefore, the rule for converting to the desired basis per cent impedance values when given on a different basis. In some circuits transformers are used at excitation differing from their normal excitation. In such cases it is essential to verify that the same value of voltage has been used in determining the per-unit current and the per cent impedance.

6. Find the power and reactive input to each transformer in the circuit as the sum of inputs due to the 2 orthogonal components of

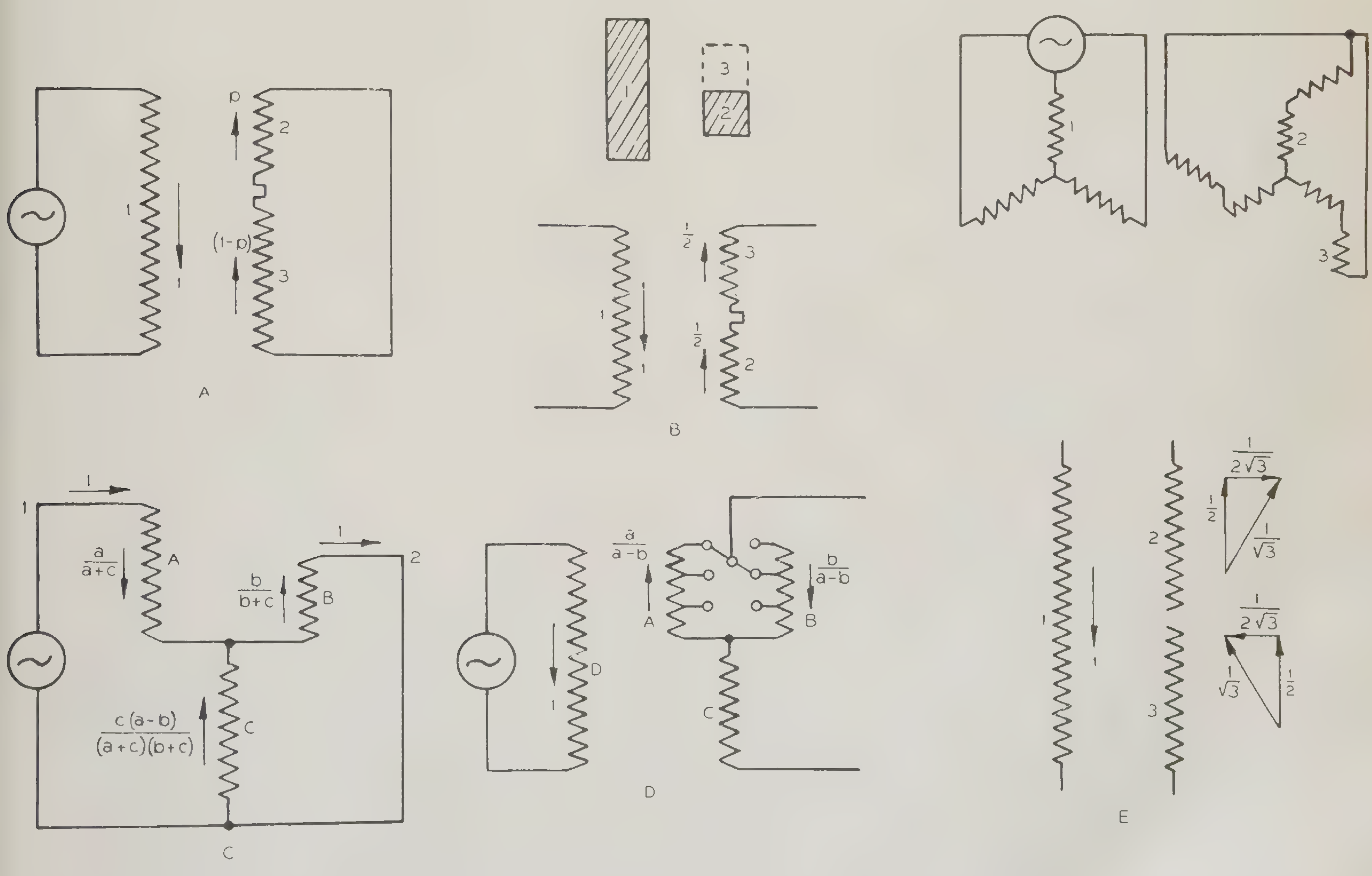


Fig. 1. Transformer circuits; one transformer; number of degrees of freedom = 1

A—Series connected winding; B—Fictitious winding; C—"Fork" autotransformer; D—Off-ratio connected windings; E—Y/zigzag transformer

currents. For each component the input is obtained as follows (see equation E4 of appendix V):
Multiply the short-circuit impedance for each pair of windings by the currents in the 2 windings, thus forming $n(n-1)/2$ triple products for an n-winding transformer. Assign plus sign to products in which the 2 currents have opposite directions and minus sign to products in which the 2 currents have the same direction. Calculate the sum of the products.

7. Find the impedance of the complete circuit by taking the sum of power and reactive inputs to all transformers. For circuits of degree of freedom 1, this is the complete solution.

8. For circuits of several degrees of freedom the expression obtained under item 7 contains one or more undetermined currents and cannot be evaluated numerically until these currents have been determined. Find these currents by solving the system of simultaneous equations obtained by writing equations 1, or 2 and 3 for all undetermined currents. In circuits of negligible resistance this is equivalent to finding the current distribution giving the minimum reactance.

The procedure described in the foregoing list of items gives the power and reactive input to a transformer circuit; it can be used, therefore, equally well when the term impedance does not apply. The rules given appear complicated and may suggest that the calculation itself is lengthy and involved. This mistaken impression will be corrected by a study of the following examples.

Illustrative Examples

The technique of application of the general method of calculation of impedance of transformer circuits is illustrated by the following examples. Simple circuits of representative types have been selected, as circuits of greater complexity are solved by exactly the same procedure. It should be remembered that in practice resistances can usually be neglected; this greatly simplifies the calculation of circuits of several degrees of freedom. A further saving in time and effort will be obtained if calculations are performed, whenever possible, numerically rather than algebraically.

1. *One Transformer; Number of Degrees of Freedom = 1.* Figure 1A shows a 2 winding transformer in which the secondary consists of 2 distinct windings (2 and 3) connected in series. The ratio of turns in winding 2 to total secondary turns is p. The transformer is shown connected for impedance test. The impedance from the primary winding 1 to the complete secondary winding is given by:

$$\begin{aligned} Z_{1-23} &= 1 \times pZ_{1-2} + 1 \times (1-p)Z_{1-3} - p \times (1-p)Z_{2-3} \\ &= pZ_{1-2} + (1-p)Z_{1-3} - p(1-p)Z_{2-3} \end{aligned} \quad (10)$$

Impedance expressions for cases of primary and secondary each consisting of several windings connected in series are obtained with equal ease.

Figure 1B is an example of the use of fictitious windings. Assume that reactance between windings 1 and 2, shown in cross section and diagrammatically, is difficult to calculate by "field" methods because of the unsymmetrical location of winding 2 with respect to winding 1. If a fictitious winding 3, a duplicate of winding 2, is placed on top of winding 2 and is connected in series with it, it often is possible to calculate by "field" methods both the reactance X_{1-23} from winding 1 to windings 2 and 3, connected in series, and the reactance X_{2-3} between winding 2 and 3. Then to find the reactance X_{1-2} between the actual windings:

$$\begin{aligned} X_{1-23} &= {}^1/_2X_{1-2} + {}^1/_2X_{1-3} - {}^1/_4X_{2-3} \\ &= X_{1-2} - {}^1/_4X_{2-3} \\ X_{1-2} &= X_{1-23} + {}^1/_4X_{2-3} \end{aligned} \quad (11)$$

Figure 1C shows a so-called "fork" autotransformer, that is an autotransformer with 2 series windings A and B and one common winding C, connected for the impedance test. Let the turns in these windings be a, b, and c, respectively. In terms of line kilovolt-amperes taken as unity the kilovolt-amperes in the windings are as shown on the figure. The effective impedance introduced in the system by the "fork" autotransformer is given, therefore, by:

$$Z_{1-2} = \frac{a}{a+c}\frac{c(a-b)}{(a+b)(b+c)}Z_{A-C} + \frac{a}{a+c}\frac{b}{b+c}Z_{A-B} - \frac{b}{b+c}\frac{c(a-b)}{(a+c)(b+c)}Z_{B-C} \quad (12)$$

Figure 1D represents diagrammatically a load ratio control transformer under excitation, but at no load.

In load ratio control transformers the process of tap changing under load always necessitates operation with windings of unequal turns connected in multiple. In some designs this connection is only a momentary condition, in others it is used for continuous operation. The physical arrangement of windings may be such as to conceal the presence of off-ratio connected windings, but their use is unavoidable so long as the present methods of load ratio control are employed.

On figure 1D the winding equipped with load ratio control consists of 3 parts. The parts A and B, with turns a and b, respectively, are connected in multiple (off-ratio). The part C, containing c turns, is connected in series with A and B. At no load with full voltage impressed on winding D a circulating current will flow in windings A and B. The ampere-turns of A and B will be of opposite direction and of different magnitude, the net ampere-turns will be reflected on the primary side, so that winding D will draw current from the line. This current will be practically wattless and will add directly to the exciting current of the transformer. For unit line current, the ampere-turns in windings A, B, D are as shown on the figure. The effective reactance limiting the no-load current, exclusive of the exciting current, is given, therefore, by:

$$X_{eff} = \frac{a}{a-b}X_{D-A} - \frac{b}{a-b}X_{D-B} + \frac{ab}{(a-b)^2}X_{A-B} \quad (13)$$

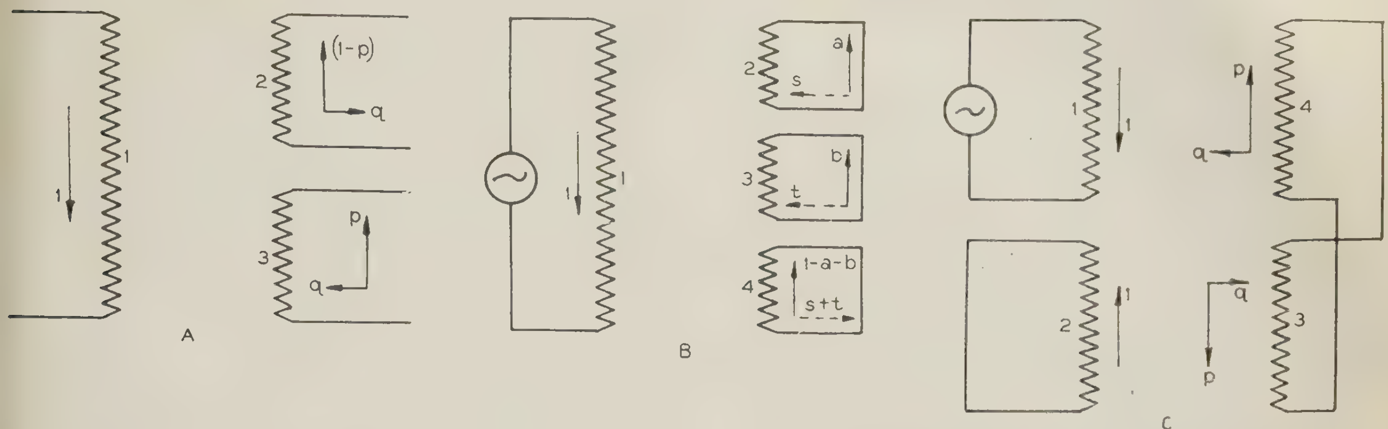
Figure 1E shows a Y/zigzag transformer on impedance test. By virtue of symmetry, only one magnetic phase need be considered. If the ampere-turns in winding 1 are taken as unity, the ampere-turns in windings 2 and 3 become as shown on the figure. The impedance is given, therefore, by:

$$\begin{aligned} Z_{eff} &= [{}^1/_2Z_{1-2} + {}^1/_2Z_{1-3} - {}^1/_4Z_{2-3}] + \frac{1}{12}Z_{2-3} \\ &= {}^1/_2Z_{1-2} + {}^1/_2Z_{1-3} - {}^1/_6Z_{2-3} \end{aligned} \quad (14)$$

Impedance formulas for other connections involving phase interconnection such as, for instance, the ex-

tended delta, inscribed delta, and hexagon connections, can be obtained in a similar manner.

2. *One Transformer; Several Degrees of Freedom.* Figure 2*A* shows a 3-winding transformer with 3 windings carrying simultaneous out-of-phase loads, so that the load losses are the sum of losses in high voltage and low voltage windings due to unity power factor load and the losses in low voltage and tertiary windings due to zero power factor load. Only 2 2-winding impedances must be known if this method of calculation is employed, while all 3 2-winding



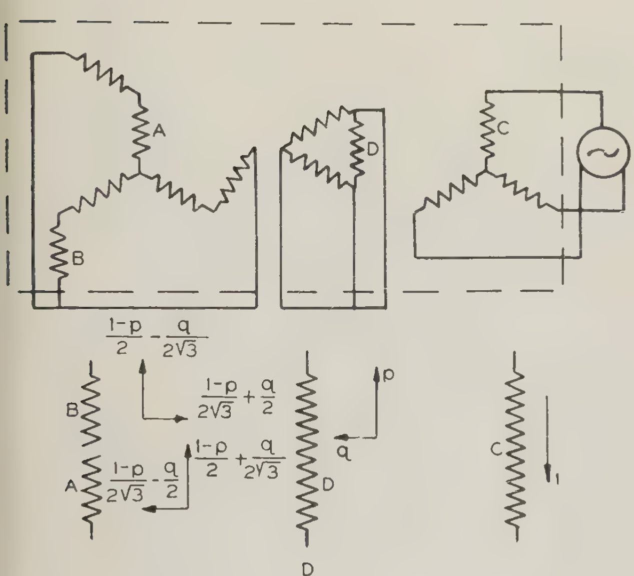


Fig. 2. Transformer circuits; one transformer; several degrees of freedom

A—Three winding transformer
B—Four winding transformer
C—Transformer with coupling winding
D—Y/delta/zigzag transformer

impedances must be known if the standard equivalent circuit method is used.

as shown on the figure where the kilovolt-ampere load on winding 1 is taken as unity. The net power and reactive input to the primary is given by:

$$P + jQ = pZ_{1-3} + (1-p)Z_{1-2} - p(1-p)Z_{2-3} + q^2Z_{2-3} \quad (15)$$

So that the load losses in the transformer are

$$P = pR_{1-3} + (1-p)R_{1-2} - p(1-p)R_{2-3} + q^2R_{2-3} \quad (16)$$

Frequently the ampere-turns of 2 windings of a 3 winding transformer are in time quadrature. Winding 2, for instance, may be the low voltage winding of a step-down transformer carrying a lagging power factor load, corrected to unity power factor by a synchronous condenser connected to the tertiary winding 3. In this case

$$p = 0$$

and consequently

$$P = R_{1-2} + q^2R_{2-3} \quad (17)$$

Consider the same transformer with winding 1 excited and windings 2 and 3 simultaneously short-circuited. The effective impedance is given by:

$$Z_{1-2//3} = pZ_{1-3} + (1-p)Z_{1-2} - p(1-p)Z_{2-3} + q^2Z_{2-3} \quad (18)$$

where p and q are to be determined from the simultaneous equations:

$$\frac{\partial R_{1-2//3}}{\partial p} - \frac{\partial X_{1-2//3}}{\partial q} = 0 \quad (19)$$

$$\frac{\partial R_{1-2//3}}{\partial q} + \frac{\partial X_{1-2//3}}{\partial p} = 0 \quad (20)$$

Performing the partial differentiations indicated by these equations:

$$R_{1-3} - R_{1-2} + (2p-1)R_{2-3} - 2qX_{2-3} = 0 \quad (19a)$$

$$2qR_{2-3} + X_{1-3} - X_{1-2} + (2p-1)X_{2-3} = 0 \quad (20a)$$

Wherefrom:

$$p = \frac{R_{2-3}(R_{1-2} + R_{2-3} - R_{1-3}) + X_{2-3}(X_{1-2} + X_{2-3} - X_{1-3})}{2(R^2_{2-3} + X^2_{2-3})} \quad (21)$$

$$q = \frac{R_{2-3}(X_{1-2} + X_{2-3} - X_{1-3}) - X_{2-3}(R_{1-2} + R_{2-3} - R_{1-3})}{2(R_{2-3}^2 + X_{2-3}^2)} \quad (22)$$

When resistances are negligible as compared to reactances:

$$p = \frac{X_{1-2} + X_{2-3} - X_{1-3}}{2X_{2-3}} \quad (21a)$$

$$q = 0 \quad (22a)$$

However, if resistances are recognized as negligible, the calculation can be carried throughout in terms of reactances:

$$X_{1-2//3} = pX_{1-3} + (1-p)X_{1-2} - p(1-p)X_{2-3} + q^2X_{2-3} \quad \text{(18a)}$$

Where p and q are to be determined from the simultaneous equations:

$$\frac{\partial X_{1-2//3}}{\partial p} = 0 \quad \text{(19b)}$$

$$\frac{\partial X_{1-2//3}}{\partial q} = 0 \quad \text{(20b)}$$

that is:

$$X_{1-3} - X_{1-2} + (2p-1)X_{2-3} = 0 \quad \text{(19c)}$$

$$2qX_{2-3} = 0 \quad \text{(20c)}$$

Wherefrom:

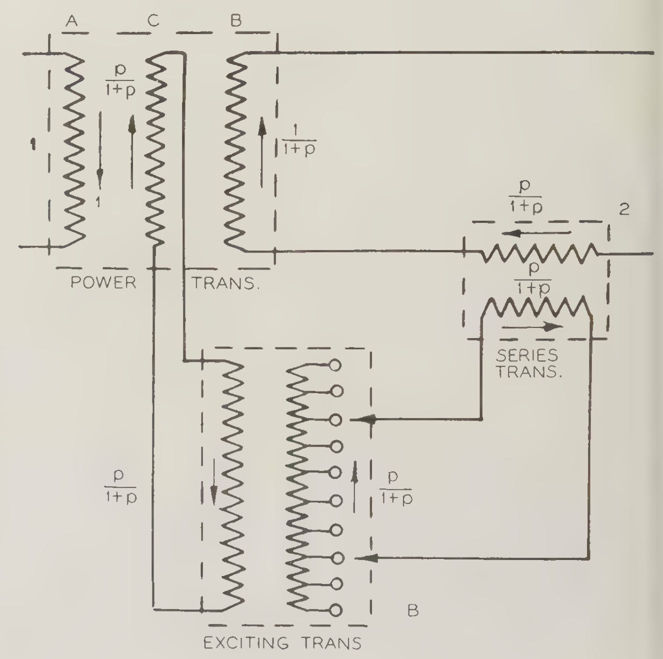
$$p = \frac{X_{1-2} + X_{2-3} - X_{1-3}}{2X_{2-3}} \quad \text{(21a)}$$

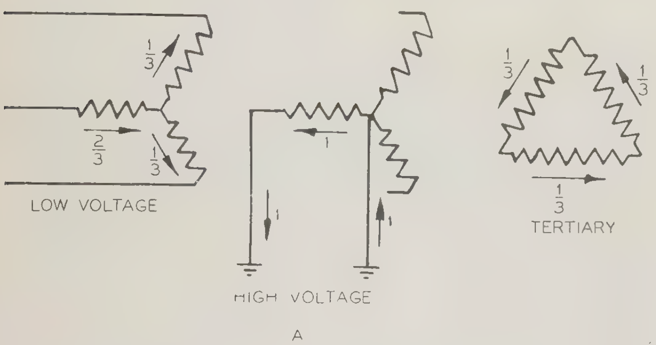
$$q = 0 \quad \text{(22a)}$$

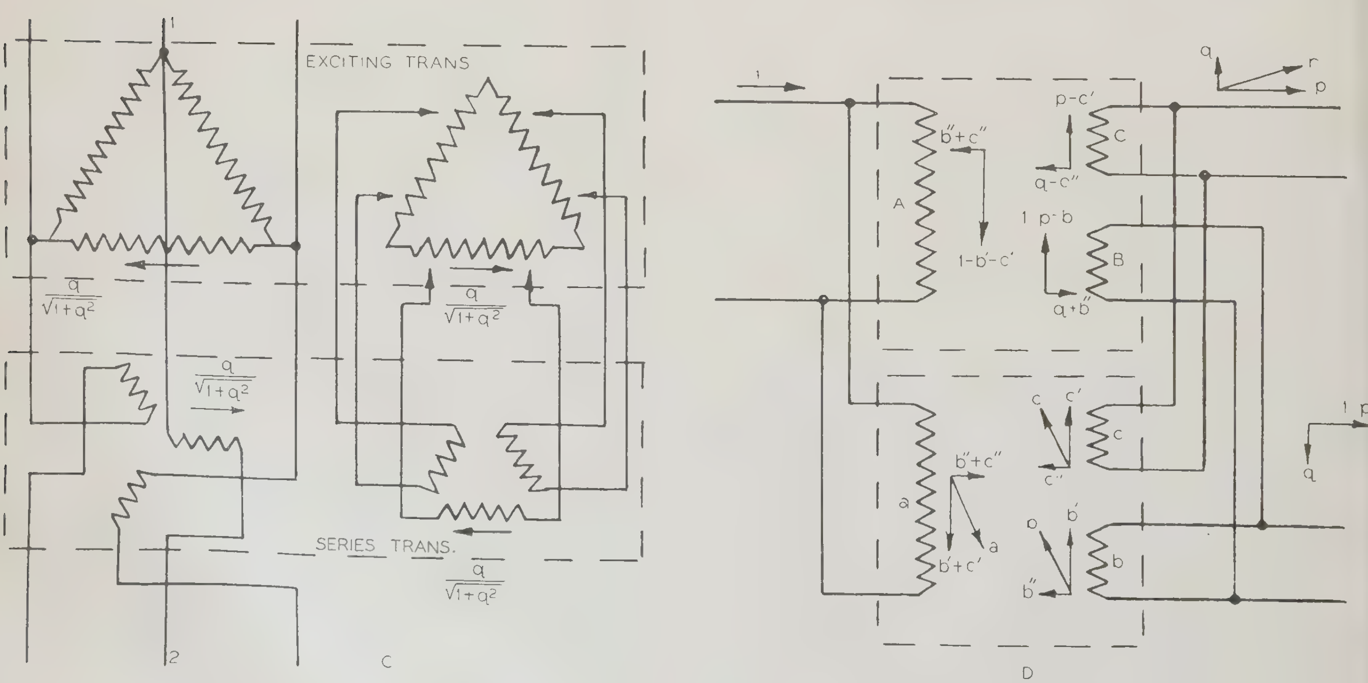
The impedance of a multiwinding transformer having any number of windings excited and the remaining windings short-circuited can be determined in a similar manner. Thus figure 2B shows a 4 winding transformer with winding 1 excited and windings 2, 3, 4 short-circuited. The general solution in terms of impedances can be carried out in the same manner as for the 3 winding transformer on figure 2A, except that the flow of currents will be given now by 4 simultaneous equations. The currents in the short-circuited windings will be found

Fig. 3. Transformer circuits; several transformers; one or more degrees of freedom

A—Line-to-ground fault on a Y/Y/delta bank
B—Load ratio control circuit
C—Load phase control circuit
D—Parallel operation of 2 3-winding transformers







to contain quadrature components with respect to the primary current.

In practice, resistances usually can be neglected. On this basis:

$$X_{1-2//3//4} = aX_{1-2} + bX_{1-3} + (1-a-b)X_{1-4} - abX_{2-3} - a(1-a-b)X_{2-4} - b(1-a-b)X_{3-4} \quad (23)$$

where a and b are given by:

$$\frac{\partial X_{1-2//3//4}}{\partial a} = 0 \quad (24)$$

$$\frac{\partial X_{1-2//3//4}}{\partial b} = 0 \quad (25)$$

The solution of these simultaneous equations gives:

$$a = \frac{2X_{3-4}(X_{1-4}+X_{2-4}-X_{1-2}) - (X_{2-4}+X_{3-4}-X_{2-3})(X_{1-4}+X_{3-4}-X_{1-3})}{4X_{2-4}X_{3-4} - (X_{2-4}+X_{3-4}-X_{2-3})^2} \quad (26)$$

$$b = \frac{2X_{2-4}(X_{1-4}+X_{3-4}-X_{1-3}) - (X_{2-4}+X_{3-4}-X_{2-3})(X_{1-4}+X_{2-4}-X_{1-2})}{4X_{2-4}X_{3-4} - (X_{2-4}+X_{3-4}-X_{2-3})^2} \quad (27)$$

Note that in addition to a and b currents, quadrature currents s and t are permitted by the ampere-turn balance law. Since it is clear on physical grounds that in the absence of phase interconnections and resistance all currents must be in phase or in phase opposition, the reactance formula (23) may be written as shown omitting the contribution of quadrature currents. If quadrature currents are included, the condition that $X_{1-2//3//4}$ be minimum will reduce them to zero, as was found for $X_{1-2//3}$ of the 3 winding transformer.

Figure 2*C* shows a transformer with a primary winding 1, a secondary winding 2, and a coupling winding consisting of equal turn coils 3 and 4 connected in parallel. On impedance test, the effective impedance is given by:

$$Z_{1-2(3//4)} = Z_{1-2} - p(Z_{1-3} + Z_{2-4} - Z_{1-4} - Z_{2-3}) + (p^2 + q^2)Z_{3-4} \quad (28)$$

If the solution is carried out in complex notation throughout, p and q are given by:

$$\frac{\partial Z_{1-2(3//4)}}{\partial p} + j\frac{\partial Z_{1-2(3//4)}}{\partial q} = 0 \quad (29)$$

that is:

$$2pZ_{3-4} - (Z_{1-3} + Z_{2-4} - Z_{1-4} - Z_{2-3}) + 2jqZ_{3-4} = 0$$

Wherefrom:

$$p + jq = \frac{Z_{1-3} + Z_{2-4} - Z_{1-4} - Z_{2-3}}{2Z_{3-4}} \quad (30)$$

Equations 28 and 30 taken together give a complete solution of the circuit. The algebraic substitution of equation 30 into equation 28 may be performed as follows:

$$p = \frac{1}{2}[(p + jq) + (p - jq)]$$

$$p^2 + q^2 = (p + jq)(p - jq)$$

and $(p - jq)$ is obtained by replacing all impedances in equation 30 by their conjugates. The result of the algebraic substitution is the final formula:

$$Z_{1-2(3//4)} = Z_{1-2} - \frac{(Z_{1-3} + Z_{2-4} - Z_{1-4} - Z_{2-3})^2}{4Z_{3-4}} = Z_{1-2} - (p + jq)^2Z_{3-4} \quad (31)$$

When resistances are negligible the quadrature component of the current circulating in the coupling winding disappears, as shown by equation 30, and as in previous examples the currents in all windings are either in phase or in phase opposition.

Impedance formulas for transformers with several coupling windings and with coupling windings consisting of more than 2 coils can be obtained in a similar manner.

Figure 2*D* shows a Y/Δ/zigzag transformer. The Y connected winding is excited, the delta and zigzag windings are short-circuited. The 3 magnetic phases are symmetrical. The windings and ampere-turns of one magnetic phase are shown on the figure.

Let:

$$Z_1 = \frac{1}{2}(Z_{A-C} + Z_{B-C}) - \frac{1}{6}Z_{A-B}$$

$$Z_2 = \frac{1}{2}(Z_{A-D} + Z_{B-D}) - \frac{1}{6}Z_{A-B}$$

$$Z_3 = Z_1 + Z_2 - Z_{D-C}$$

$$Z_4 = \frac{1}{2\sqrt{3}}(Z_{A-D} + Z_{B-C} - Z_{A-C} - Z_{B-D})$$

Then the effective impedance is given by:

$$Z = Z_1 - pZ_3 \mp qZ_4 + (p^2 + q^2)Z_2 \quad (32)$$

Where p and q have the values determined by:

$$\frac{\partial Z}{\partial p} + j\frac{\partial Z}{\partial q} = -Z_3 \mp jZ_4 + 2Z_2(p + jq) = 0$$

that is:

$$p + jq = \frac{Z_3 \pm jZ_4}{2Z_2} \quad (33)$$

Substituting equation 33 into equation 32

$$Z = Z_1 - \frac{Z_3^2 + Z_4^2}{4Z_2} \quad (34)$$

Wherever double signs appear in these equations the upper sign applies for phase rotation shown on figure 2*D*. If any 2 leads from the generator to the Y connected winding are interchanged the phase rotation will be reversed. On figure 2*D* the Y will retain its present appearance, the delta and the zigzag will be reversed, and wherever double signs appear in the above equations the lower sign will apply. Thus the current flow in short-circuited windings changes when phase rotation is reversed. This is characteristic of all phase interconnection circuits of several degrees of freedom. The effective impedance has the same value, regardless of phase rotation.

Equation 33 shows that even when resistances are negligible the line currents of the 2 short-circuited windings are not in time quadrature with respect to

the open circuit line voltages of these windings but contain in phase components, equal and opposite in the 2 lines, and reversing with a reversal of phase rotation. This is also characteristic of all phase interconnection circuits of several degrees of freedom. If, however, the presence of these in phase components is objectionable they may be made to disappear by a suitable arrangement of the windings. For the circuit of figure 2*D* the required condition is:

$$X_4 = \frac{1}{2\sqrt{3}}(X_{A-D} + X_{B-C} - X_{A-C} - X_{B-D}) = 0 \tag{35}$$

3. *Several Transformers; One or More Degrees of Freedom.* Figure 3*A* shows a bank of 3-winding transformers. The high voltage winding is Y connected with neutral grounded, it is assumed to be disconnected from the line. The low voltage winding is Y connected with isolated neutral, and is assumed to be connected to the system. The tertiary is delta connected and isolated from external circuits. A line-to-ground fault is assumed at one terminal of the high voltage winding. This is a circuit of degree of freedom 1. The ampere-turns in various windings are shown on figure 3*A*, where the fault current is taken as unity. On this basis:

$$X_{eff} = \left[\frac{2}{3} X_{H-L} + \frac{1}{3} X_{H-T} - \frac{2}{9} X_{L\ T}\right] + \left[\frac{1}{9} X_{L-T}\right] + \left[\frac{1}{9} X_{L-T}\right] \tag{36}$$

where brackets separate the contributions of the 3 transformers. If the 3 transformers are alike:

$$X_{eff} = \frac{2}{3} X_{H-L} + \frac{1}{3} X_{H-T} \tag{36a}$$

Figure 3*B* shows a load ratio control circuit tying together systems 1 and 2 and consisting of a power transformer, an exciting transformer, and a series transformer. The power transformer is a 3-winding unit, with primary winding A, secondary winding B, connected in series with the high voltage winding of the series transformer, and tertiary winding C, providing excitation for the exciting transformer. The exciting transformer is shown connected to give $p \times 100$ per cent "boost" to the secondary voltage. The number of degrees of freedom is 1 and the kilovolt-amperes in all windings are shown on the figure in terms of the system kilovolt-amperes. The over-all reactance from system 1 to system 2 is given by:

$$X_{1-2} = \left[\frac{1}{1+p} X_{A-B} + \frac{p}{1+p} X_{A-C} - \frac{p}{(1+p)^2} X_{B-C}\right] + \left(\frac{p}{1+p}\right)^2 X_E + \left(\frac{p}{1+p}\right)^2 X_S \tag{37}$$

Where X_E is the reactance of the exciting transformer on this particular tap and X_S is the reactance of the series transformer at excitation corresponding to this particular tap. Equation 37 shows that over-all reactance, including the exciting and the series transformers, may be smaller than the reactance between the primary and the secondary windings of the power transformer.

Figure 3*C* shows a load phase control circuit tying together systems 1 and 2 and consisting of an exciting transformer and a series transformer. The exciting transformer is shown connected to introduce a quadrature voltage of $q \times 100$ per cent of the line-to-neutral voltage. The number of degrees of freedom is 1 and the kilovolt-amperes in all windings are shown on the figure in terms of the system kilovolt-amperes per phase. The over-all reactance from system 1 to system 2 is given by:

$$X_{1-2} = \frac{q^2}{1+q^2}(X_E + X_S) \tag{38}$$

where X_E is the reactance of the exciting transformer on this particular tap and X_S is the reactance of the series transformer at excitation corresponding to this particular tap.

Figure 3*D* shows 2 3-winding transformers operating in parallel. The load current division between the 2 transformers may be found as follows:

Using the familiar 3-winding notation let:

$$Z_A = \frac{1}{2}(Z_{A-B} + Z_{A-C} - Z_{B-C}) \qquad Z_a = \frac{1}{2}(Z_{a-b} + Z_{a-c} - Z_{b-c})$$

$$Z_B = \frac{1}{2}(Z_{A-B} + Z_{B-C} - Z_{A-C}) \qquad Z_b = \frac{1}{2}(Z_{a-b} + Z_{b-c} - Z_{a-c})$$

$$Z_C = \frac{1}{2}(Z_{A-C} + Z_{B-C} - Z_{A-B}) \qquad Z_c = \frac{1}{2}(Z_{a-c} + Z_{b-c} - Z_{a-b})$$

Then the total net power and reactive input to the 2 transformers is given by:

$$P + jQ = Z_A + [(1-p)^2 + q^2]Z_B + (p^2 + q^2)Z_C - 2b'[Z_A + (1-p)Z_B] - 2c'[Z_A + pZ_C] + 2b''qZ_B - 2c''qZ_C + 2(b'c' + b''c'')[Z_A + Z_a] + (b'^2 + b''^2)[Z_A + Z_B + Z_a + Z_b] + (c'^2 + c''^2)[Z_A + Z_C + Z_a + Z_c] \tag{39}$$

Solving the simultaneous equations:

$$\frac{\partial(P + jQ)}{\partial b'} + j\frac{\partial(P + jQ)}{\partial b''} = 0$$

$$\frac{\partial(P + jQ)}{\partial c'} + j\frac{\partial(P + jQ)}{\partial c''} = 0$$

Currents in windings b and c are found to be:

$$b = \frac{[Z_A + (1-r)Z_B](Z_A + Z_C + Z_a + Z_c) - [Z_A + rZ_C](Z_A + Z_a)}{(Z_A + Z_B + Z_a + Z_b)(Z_A + Z_C + Z_a + Z_c) - (Z_A + Z_a)^2} \tag{40}$$

$$c = \frac{[Z_A + rZ_C](Z_A + Z_B + Z_a + Z_b) - [Z_A + (1-r)Z_B](Z_A + Z_a)}{(Z_A + Z_B + Z_a + Z_b)(Z_A + Z_C + Z_a + Z_c) - (Z_A + Z_a)^2} \tag{41}$$

The currents in all other windings can now be determined from r, b, and c.

Equations 40 and 41 apply only for parallel operation of 2 3-winding transformers with Y or delta connected windings or in single phase systems. The somewhat more complicated formulas applying for parallel operation of transformers with phase interconnections, as well as formulas for parallel operation of any number of transformers having any

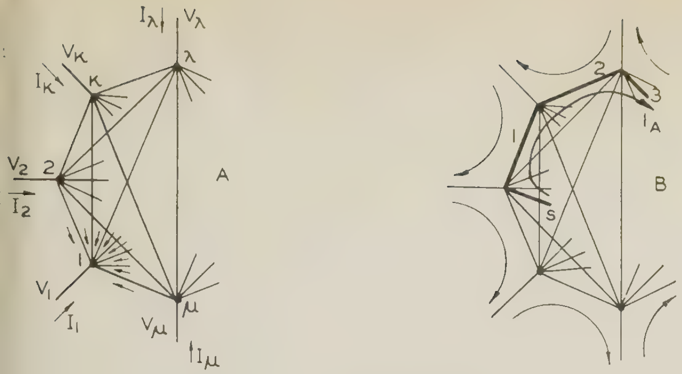


Fig. 4. General network

A—Showing branch currents   *B*—Showing mesh currents

number of windings can be derived in exactly the same manner.

Appendix I—Conservation of Power and Reactive Volt-Amperes[4]

The well-known relation

$$P + jQ = \check{E}\hat{I} = (R + jX)\;\;|I|^2 \tag{A1}$$

where $\hat{I}$ is the conjugate and $|I|$ is the absolute value of $\check{I}$, gives the power and reactive volt-ampere input as it would be measured by a wattmeter and a reactive volt-ampere meter actuated by an effective voltage $\check{E}$ and an effective current $\check{I}$.

Consider a network of μ points (figure 4*A*) in which every point is connected to all other points so that the number of branches is

$$n = \frac{\mu(\mu - 1)}{2} \tag{A2}$$

Assume this network to be a passive static network of constant bilateral parameters in steady state under single frequency excitation. All voltages and currents will be represented as complex quantities by their effective values. Assume potentials $\check{V}_1, \check{V}_2 \ldots \check{V}_\mu$ applied to points 1, 2, . . . μ the corresponding currents entering the network being denoted by $\check{I}_1, \check{I}_2 \ldots \check{I}_\mu$. As a special case, $\check{V}_1, \check{V}_2 \ldots \check{V}_\mu$ may be the phase voltages of a balanced μ-phase system.

The equilibrium condition of the network will be determined by the current and voltage laws of Kirchhoff.

The current law permits us to write (replacing currents by their conjugates and observing that for any branch $\kappa\lambda$ the current entering point κ must be leaving point λ):

$$\begin{aligned}
\hat{I}_1 &= \phantom{+\hat{I}_{21}} - \hat{I}_{21} - \hat{I}_{31} - \hat{I}_{41} - \ldots - \hat{I}_{\mu 1}\\
\hat{I}_2 &= +\hat{I}_{21} \phantom{+\hat{I}_{32}} - \hat{I}_{32} - \hat{I}_{42} - \ldots - \hat{I}_{\mu 2}\\
\hat{I}_3 &= +\hat{I}_{31} + \hat{I}_{32} \phantom{+\hat{I}_{43}} - \hat{I}_{43} - \ldots - \hat{I}_{\mu 3}\\
\hat{I}_4 &= +\hat{I}_{41} + \hat{I}_{42} + \hat{I}_{43} \phantom{-\hat{I}} - \ldots - \hat{I}_{\mu 4}\\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
\hat{I}_\mu &= +\hat{I}_{\mu 1} + \hat{I}_{\mu 2} + \hat{I}_{\mu 3} + \hat{I}_{\mu 4} + \ldots
\end{aligned}$$

Multiplying the first equation by $\check{V}_1$, the second by $\check{V}_2$, etc., and adding:

$$\begin{aligned}\check{V}_1\hat{I}_1 + \check{V}_2\hat{I}_2 + \ldots + \check{V}_\mu\hat{I}_\mu = (\check{V}_2 - \check{V}_1)\hat{I}_{21} + \\ (\check{V}_3 - \check{V}_1)\hat{I}_{31} + \ldots + (\check{V}_3 - \check{V}_2)\hat{I}_{32} + \text{etc.}\end{aligned}$$

Let:

$$\begin{aligned}\check{V}_\kappa - \check{V}_\lambda &= \check{E}_l = \text{voltage across branch } l\\ \check{I}_{\kappa\lambda} &= \check{I}_l = \text{current in branch } l\end{aligned}$$

Then the above expression can be rewritten as

$$\sum_{\lambda=1}^{\lambda=\mu}\check{V}_\lambda\hat{I}_\lambda = \sum_{l=1}^{l=n}\check{E}_l\hat{I}_l \tag{A3}$$

Appendix II—Relation Between Power and Reactive Volt-Ampere Input and the Energy Dissipation and Storage[4,5]

Combining equation *A*1 and *A*3

$$P + jQ = \sum_{\lambda=1}^{\lambda=\mu}\check{V}_\lambda\hat{I}_\lambda = \sum_{l=1}^{l=n}\check{E}_l\hat{I}_l \tag{B1}$$

Substituting

$$\check{E}_l = \sum_{k=1}^{k=n}\check{Z}_{kl}\check{I}_k \tag{B2}$$

into equation *B*1

$$\begin{aligned}P + jQ &= \sum_{l=1}^{=n}\sum_{k=1}^{k=n}\check{Z}_{kl}\check{I}_k\hat{I}_l \\ &= \frac{1}{2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}(R_{kl} + jX_{kl})(\check{I}_k\hat{I}_l + \hat{I}_k\check{I}_l)\end{aligned} \tag{B3}$$

The expression

$(\check{I}_k\hat{I}_l + \hat{I}_k\check{I}_l)$

is a pure real since the product of conjugates is the conjugate of the product and the sum of conjugates is real. Consequently equation *B*3 can be rewritten as

$$P + jQ = \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}R_{kl}\check{I}_k\hat{I}_l + j\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}X_{kl}\check{I}_k\hat{I}_l \tag{B4}$$

Furthermore:

$$\begin{aligned}Q &= \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}X_{kl}\check{I}_k\hat{I}_l \\ &= \sum_{l=1}^{l=n}\sum_{k=n}^{k=n}\left(\omega L_{kl} - \frac{1}{\omega C_{kl}}\right)\check{I}_k\hat{I}_l \\ &= \omega\left[\sum_{l=1}^{l=n}\sum_{k=n}^{k=n}L_{kl}\check{I}_k\hat{I}_l - \frac{1}{\omega^2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\frac{1}{C_{kl}}\check{I}_k\hat{I}_l\right]\end{aligned} \tag{B5}$$

But:

$$\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}R_{kl}\check{I}_k\hat{I}_l = \text{average rate of energy dissipation in network} = D$$

$$\frac{1}{2}\sum_{l=n}^{l=1}\sum_{k=1}^{k=1}L_{kl}\check{I}_k\hat{I}_l = \text{average electromagnetic energy stored in the network} = T_m$$

$$\frac{1}{2\omega^2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\frac{1}{C_{kl}}\check{I}_k\hat{I}_l = \text{average electrostatic energy stored in the network} = T_e$$

Equation *B*4 can now be rewritten as:

$$P + jQ = D + j2\omega(T_m - T_e) \tag{B6}$$

Appendix III—Resolution of Input Volt-Amperes Into Input Volt-Amperes of Orthogonal Components of Currents

Resolve all currents into orthogonal components with respect to an arbitrary reference vector, so that

$$I_k = I_k' + jI_k'' \tag{C1}$$

It follows that

$$(\check{I}_k\hat{I}_l + \hat{I}_k\check{I}_l) = 2(I_k'I_l' + I_k''I_l'')$$

Equation $B3$ can now be rewritten as:

$$P + jQ = \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}(I_k'I_l' + I_k''I_l'')$$

$$= \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'I_l' + \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k''I_l''$$

$$= \left[\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}R_{kl}I_k'I_l' + \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}R_{kl}I_k''I_l''\right] +$$

$$j\left[\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}X_{kl}I_k'I_l' + \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}X_{kl}I_k''I_l''\right] \quad \textbf{(C2)}$$

Appendix IV—Current Flow in a Network

The current law of Kirchhoff does not establish definitely the flow of currents in the network of figure 4. Several closed circuits (called meshes) can be traced within the network. Currents circulating in these meshes (called mesh currents) automatically satisfy the current law of Kirchhoff. An independent mesh is a mesh containing at least one branch not contained in other meshes. The minimum number of independent mesh currents capable of representing by their superposition all the branch currents of a network is equal to the number of degrees of freedom of the network and is given by

$$N = n - \mu + 1 = 1 + \frac{\mu(\mu - 3)}{2} \quad \textbf{(D1)}$$

The selection of independent meshes is arbitrary, provided that their number is as given by equation $D1$ and that all branches of the network are included. Suppose that independent meshes, $A, B, C, \ldots N$ have been selected, carrying $\check{I}_A, \check{I}_B, \check{I}_C, \ldots \check{I}_N$ independent mesh currents, such that the superposition of these mesh currents gives the branch currents $\check{I}_1, \check{I}_2, \ldots \check{I}_j, \check{I}_k, \check{I}_l, \ldots \check{I}_n$. One of these meshes, mesh A, consisting of branches $1, 2, \ldots j, k, l, \ldots s$ is shown on figure $4B$.

The relation between branch and mesh currents for the branches of this mesh is given by:

$$\check{I}_j = \check{I}_A + \Delta\check{I}_j \text{ for } j = 1, 2 \ldots s \quad \textbf{(D2)}$$

where $\Delta\check{I}_j$ is the contribution of mesh currents of all other independent meshes having branch j of mesh A as a common branch. Equation $D2$ obviously holds for orthogonal components of branch and mesh currents:

$$I_j' = I_A' + \Delta I_j' \text{ for } j = 1, 2 \ldots s \quad \textbf{(D3)}$$

$$I_j'' = I_A'' + \Delta I_j'' \text{ for } j = 1, 2 \ldots s \quad \textbf{(D4)}$$

Utilizing equations $B2$ and $C1$ the voltage law of Kirchhoff applied to mesh A becomes:

$$\sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}\check{I}_k = \sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}(I_k' + jI_k'') = 0 \quad \textbf{(D5)}$$

The flow of currents in the network may be determined by writing equation $D5$ in terms of mesh currents for all independent meshes and solving the resultant systems of simultaneous equations.

It will be shown now how equation $D5$ can be obtained in terms of orthogonal components of mesh currents from the power and reactive volt-ampere input formula through the process of partial differentiation with respect to I_A' and I_A''. From equations $C2$, $D3$, and $D4$:

$$\frac{\partial(P + jQ)}{\partial I_A'} = \frac{\partial\displaystyle\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'I_l'}{\partial I_A'}$$

The expression to be differentiated may be rewritten as:

$$\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'I_l' = \sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'I_l' +$$

$$\sum_{l=s+1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'I_l' = \sum_{l=1}^{l=s}\sum_{k=1}^{k=s}\check{Z}_{kl}I_k'I_l' +$$

$$\sum_{l=1}^{l=s}\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'I_l' + \sum_{l=s+1}^{l=n}\sum_{k=1}^{k=s}\check{Z}_{kl}I_k'I_l' +$$

$$\sum_{l=s+1}^{l=n}\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'I_l'$$

The second and the third terms of the last expression are equal, the last term is independent of I_A'. The expression to be differentiated becomes therefore:

$$\sum_{l=1}^{l=s}\sum_{k=1}^{k=s}\check{Z}_{kl}I_k'I_l' + 2\sum_{l=1}^{l=s}\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'I_l'$$

Substituting equation $D3$:

$$\sum_{l=1}^{l=s}\left[\sum_{k=1}^{k=s}\check{Z}_{kl}(I_A' + \Delta I_k')(I_A' + \Delta I_l') + 2\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'(I_A' + \Delta I_l')\right]$$

Dropping terms without I_A':

$$\sum_{l=1}^{l=s}\left[\sum_{k=1}^{k=s}\check{Z}_{kl}\{I_A'^2 + (\Delta I_k' + \Delta I_l')I_A'\} + 2\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'I_A'\right]$$

Differentiating with respect to I_A':

$$\sum_{l=1}^{l=s}\left[\sum_{k=1}^{k=s}\check{Z}_{kl}(2I_A' + \Delta I_k' + \Delta I_l') + 2\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'\right]$$

Using once more equation $D3$, this can be rewritten as:

$$\sum_{l=1}^{l=s}\left[\sum_{k=1}^{k=s}\check{Z}_{kl}(I_k' + I_l') + 2\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'\right] =$$

$$\sum_{l=1}^{l=s}\left[2\sum_{k=1}^{k=s}\check{Z}_{kl}I_k' + 2\sum_{k=s+1}^{k=n}\check{Z}_{kl}I_k'\right] = 2\sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'$$

It has been shown, therefore, that

$$\frac{\partial(P + jQ)}{\partial I_A'} = 2\sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k' \quad \textbf{(D6)}$$

and similarly:

$$\frac{\partial(P + jQ)}{\partial I_A''} = 2\sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}I_k'' \quad \textbf{(D7)}$$

Substituting equations $D6$ and $D7$ into equation $D5$:

$$\frac{\partial(P + jQ)}{\partial I_A'} + j\frac{\partial(P + jQ)}{\partial I_A''} = 0 \quad \textbf{(D8)}$$

The flow of currents in the network of figure 4 may be determined, therefore, in terms of orthogonal components of mesh currents by forming expressions $D8$ for all independent meshes and solving the resultant system of simultaneous equations.

Separating the real from the imaginary terms, equation $D8$ may be rewritten as:

$$\frac{\partial P}{\partial I_A'} - \frac{\partial Q}{\partial I_A''} = 0 \quad \textbf{(D9)}$$

$$\frac{\partial P}{\partial I_A''} + \frac{\partial Q}{\partial I_A'} = 0 \quad \textbf{(D10)}$$

These expressions are easily remembered as they have the form of Cauchy-Riemann equations. Since, however, $(P + jQ)$ is not an analytic function of $(I_A' + jI_A'')$ simultaneous equations $D9$ and $D10$ yield a definite solution.

It will be observed that for a circuit without resistance equations $D9$ and $D10$ become

$$\frac{\partial Q}{\partial I_A''} = 0 \tag{D11}$$

$$\frac{\partial Q}{\partial I_A'} = 0 \tag{D12}$$

so that in a network without resistance of figure 4 the flow of currents must be such as to give the reactive volt-ampere input to the network an extremal value. See also appendix VI.

Appendix V—Net Input to a Multiwinding Transformer in Terms of Short-Circuit Impedances Between Its Windings

The power and reactive volt-ampere input formulas derived for the general network of figure 4 are directly applicable to the multiwinding transformer of figure 5. Windings $1 \ldots k, l, \ldots n$ of figure 5, which are all located on one magnetic phase of a transformer, correspond to branches $1 \ldots k, l, \ldots n$ of the network of figure 4.

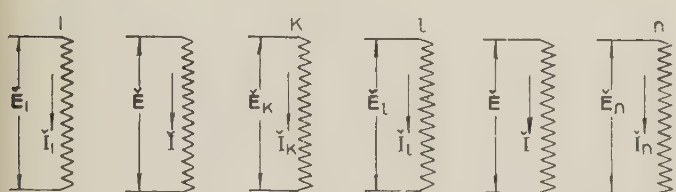


Fig. 5. One magnetic phase of a multiwinding transformer

Expressing all quantities in per-unit the power and reactive input to windings on one magnetic phase of a multiwinding transformer is given, therefore, by equation $B3$ which may be written as:

$$P + jQ = -\frac{1}{2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}(-2\check{Z}_{kl}\hat{I}_l\check{I}_k) \tag{E1}$$

Neglecting the exciting current:

$$\sum_{k=1}^{k=n}\check{I}_k = 0 \qquad \sum_{l=1}^{l=n}\hat{I}_l = 0$$

Consequently:

$$-\frac{1}{2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{ll}\hat{I}_l\check{I}_k = -\frac{1}{2}\sum_{l=1}^{l=n}\check{Z}_{ll}\hat{I}_l\sum_{k=1}^{k=n}\check{I}_k = 0$$

$$-\frac{1}{2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kk}\hat{I}_l\check{I}_k = -\frac{1}{2}\sum_{k=1}^{k=n}\check{Z}_{kk}\check{I}_k\sum_{l=1}^{l=n}\hat{I}_l = 0$$

Equation $E1$ may be rewritten, therefore, as

$$P + jQ = -\frac{1}{2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}(\check{Z}_{kk} + \check{Z}_{ll} - 2Z_{kl})\hat{I}_l\check{I}_k \tag{E2}$$

But on the assumption of negligible exciting current

$$\check{Z}_{kk} + \check{Z}_{ll} - 2\check{Z}_{kl} = \check{Z}_{k-l} \tag{E3}$$

where $\check{Z}_{k-l}$ is the short-circuit impedance between windings k and l. Substituting equation $E3$ into equation $E2$:

$$P + jQ = -\frac{1}{2}\sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{k-l}\check{I}_k\hat{I}_l \tag{E4}$$

This expression can be resolved into its power and reactive components and into contributions of orthogonal components of currents exactly in the same manner as the original expression $B3$.

Appendix VI

In appendix IV the current flow in a network of several degrees of freedom was established from the voltage law of Kirchhoff expressed in terms of partial derivatives:

$$\frac{\partial(P + jQ)}{\partial I_A'} \text{ and } \frac{\partial(P + jQ)}{\partial I_A''}$$

The equilibrium condition of a network may also be expressed as follows: Referring to appendix I, instead of starting with equation $A1$ define a complex function $(U + jW)$ by the following equation:

$$(U + jW) = \check{E}\check{I} \tag{F1}$$

Following exactly the method of appendix I, it may be shown that

$$\sum_{\lambda=1}^{\lambda=\mu}\check{V}_\lambda\check{I}_\lambda = \sum_{l=1}^{l=n}\check{E}_l\check{I}_l \tag{F2}$$

For the special case of a network in which for all points, except point 1, either the potential of the point is zero or the current fed into the network at the point is zero:

$$U + jW = \check{V}_1\check{I}_1 = \sum_{l=1}^{l=n}\check{E}_l\check{I}_l \tag{F3}$$

By equation $A3$ the power and reactive volt-ampere input to this network is given by:

$$P + jQ = \check{V}_1\hat{I}_1 = \sum_{l=1}^{l=n}\check{E}_l\hat{I}_l \tag{F4}$$

If, therefore, for such a network I_1 is taken as reference vector,

$$\check{I}_1 = \hat{I}_1$$

and hence

$$U + jW = P + jQ$$

Thus when these restricting conditions are satisfied the function $(U + jW)$ may be used to calculate the impedance of transformer circuits.

Following the procedure of appendix II, it can be shown that:

$$U + jW = \sum_{l=1}^{l=n}\sum_{k=1}^{k=n}\check{Z}_{kl}\check{I}_k\check{I}_l \tag{F5}$$

To establish the current flow in a network of several degrees of freedom a procedure similar to appendix IV may be followed, except that voltages and currents need not be resolved into orthogonal components. The equilibrium conditions are given by:

$$\frac{\partial(U + jW)}{\partial\check{I}_A} = 2\sum_{l=1}^{l=s}\sum_{k=1}^{k=n}\check{Z}_{kl}\check{I}_k = 0 \tag{F6}$$

where the differentiation is justified by the fact that $(U + jW)$ is an analytic complex function of the complex variable $\check{I}_A$.

References

1. Theory of Three-Circuit Transformers, A. Boyajian. A.I.E.E. Trans., v. 43, 1924, p. 508–28.

2. Progress In Three Circuit Theory, A. Boyajian. A.I.E.E. Trans., v. 52, Sept.-Dec. 1933, p. 914–17.

3. Equivalent Circuits—I, F. M. Starr. A.I.E.E. Trans., v. 51, June 1932, p. 287–98.

4. Inherent Limitations on Transformations Possible by Stationary Apparatus, J. Slepian. A.I.E.E. Trans., v. 38, pt. 2, 1919, p. 1697–1711.

5. Communication Networks (a book), E. A. Guillemin, v. I and II. John Wiley & Sons, Inc., New York, N. Y.

6. Allgemeine Methode der Berechnung der Streuung von Transformatoren, G. Petrow. *E.u.M.*, v. 51, June 18, 1933, p. 345–50.

Engineering Education

— Opinions and Influencing Factors

Engineers are concerned with the preparation of young men to serve a civilization and culture based largely upon the results of technological developments. This fact makes necessary a multisided, better educated, and more roundly interested individual. Combined with the increasing body of fundamental technical knowledge and high standards of technical education, the adequacy of a 4 year professional educational program has been challenged. Opinions regarding the situation are summarized herein, together with the activities of the Engineers' Council for Professional Development, through whose united program there is hope for a fuller understanding leading to the consummation of an accepted philosophy and effective procedure.

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FOR 15 years the insistent demand in the engineering colleges has been for a broader training than was heretofore the case. The situation is summed up admirably by Professor H. W. Bibber of Ohio State University who says:[1] "From earliest times up to about 10 years ago, engineers have always been concerned primarily with production, transportation or communication. There has never been a sufficient supply of manufactured goods or raw materials in the places they were wanted, and the engineer's work of directing the great sources of power in nature have been primarily that of providing more goods or services, such as transportation or electric power. There has always been a demand for a greater capacity to produce than existed at the time. Usually this has been because the technical means of production were inadequate. Up to recent years, therefore, the engineer has devoted himself largely to the conquest of the physical difficulties associated with the application of natural science. The part of his work dealing with 'the use and convenience of man' in many cases has been minor. Nevertheless, the social aspects of the application of power always have been within the engineer's domain.

"As many of the technical problems that years ago confronted engineers have been approaching a complete solution, more of their work has concerned itself with the economic and social aspects of power application and communication. The result of this trend is shown in the opinion of practicing engineers as to what the nature of engineering education should be, as exhibited in the report of the Society for the Promotion of Engineering Education on its investigation of engineering education. Other studies of similar character have been made by individual colleges and universities. All of these indicate that more and more attention should be given in the schools to the social sciences such as economics, business organization, history, psychology and sociology."

President C. C. Williams of Lehigh University speaking before the last annual convention of the Society for the Promotion of Engineering Education stated:[2] "It seems reasonable to expect engineering education to expand gradually ... first to strengthen engineering administration wherein it now includes these matters, and second to offer preparation for assuming those functions wherein other agencies have been proved incompetent. There lies with engineering the responsibility for the economic control and stability of its own affairs and if that responsibility is not recognized and accepted, engineering education to that extent must be adjudged wanting."

Another point of view, advanced by Webster N. Jones, a former executive of the Goodrich Rubber Company states:[3] "Much thought is being given to selection and guidance of prospective engineering students. There is a decided tendency toward the extension of the teaching of basic cultural subjects to prepare graduates to meet ever-changing economic conditions. Engineering colleges are realizing the fundamental importance of research, especially for students with a decided bent toward creative work. The human side of engineering is being stressed and will lead to greater life enrichment. Teachers of engineering are aware of their grave responsibility in encouraging young men to search diligently for knowledge and wisdom and in stimulating them to aspire to a higher order of citizenship. The ultimate

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1. For all numbered references, see list at end of paper.

goal of engineering colleges is to produce men of superior qualifications, especially initiative, creative instinct, breadth of vision and capacity for hard work: men who possess physical fitness to enable them to carry on energetically, tenacity of purpose to compel them to stick to the end, resourcefulness to direct them out of the beaten path into unexplored regions, personality to enable them to live amicably in their environment, and knowledge in their particular fields to render them capable of creative work."

In looking, recently, into the history of 70 years of engineering at Lafayette College, it is interesting to note how the making of curricula swings like a pendulum. In 1889 the electrical engineering course included such so-called cultural subjects as French, German, history, English, philology, rhetoric, botany, astronomy, public speaking, Bible, and political economy. By 1920 the desire to include the maximum of technical subjects had reduced the list to English, Bible, economics, and business law. Since then, of course, the pendulum again has swung in favor of more of the so-called cultural subjects.

As a means of carrying out these ideals for engineering education, the colleges and universities have adopted, quite generally, the 4 year undergraduate course. It is agreed almost universally that the time allotted is insufficient to give adequately the training desired.

To acquaint the student with the technological advances that research is constantly making, to have the student take enough social science, literature, and language to provide the background that a professional man should possess, and to engage in the worth-while extracurricular activities that an engineering student needs, is obviously quite beyond the scope of the 4 year course.

Institutions which have had the courage to insist on a longer period have also had sufficient endowment to carry the cost with the falling registration that inevitably has resulted.

The task, then, in these 4 year institutions has been a difficult one, but is being met by eliminating those technical courses which cannot be justified on the basis of a necessity for developing the powers of analysis of the student, and by introducing those social science studies which lead to a solution of the problem of man's duties in the social organization. The underlying function of engineering education is not to acquire a vast store of facts and technical information, but to learn how to reason and to solve problems. At first these problems will be in the physical realm where the validity of the data is not in question, where the variables are easily expressed, and where there is, in general, but one answer to the problem. As President R. E. Doherty of Carnegie Institute of Technology has so well put it:[4] "After the ability to solve physical problems has been developed, the student then may turn with more profit to problems in the social sciences where the variables are much more numerous, the reliability of the data is frequently in question, and more than one correct solution is not uncommon." One institution has definitely laid down the principle that $^2/_3$ of the student's time shall be devoted to "the science stem" and $^1/_3$ to "the social and economic stem."

Even with these changes, the time is so short that the Society for the Promotion of Engineering Education after having declared in the report of its investigation of engineering education[5] that "the normal length of the undergraduate curriculum should remain 4 academic years," directed at its annual meeting in 1933 that a committee be appointed to study the question of the length of the engineering curriculum. The majority report of the committee seemed to favor no definite length of time, but left the question open in order that further development and experimentation might be carried on. The minority report declared that the engineering curriculum should be lengthened.

Several institutions are now offering 5 year courses, of which Columbia University, Massachusetts Institute of Technology, and the University of California are notable examples. California's justification for offering the 5 year course is contained in the statement that the plan is "based on the recognition of the fact that a 4 year period of study is inadequate to give satisfactorily the combination of culture, basic scientific, and engineering studies essential to the highest type of engineering and to afford at the same time leisure for the development of the physical well being and human interest of the student." The Massachusetts Institute of Technology announces that the 5 year course is "designed to meet the demand for engineers with a thorough understanding of the social and economic implications of their profession. This course will include the same professional studies as are offered at present in any one of the departments of engineering or science, but will also include an increasing program of more advanced studies in the fields of economics and the social sciences running through the last 3 years of the 5 year course."

Columbia University offers 2 programs of study beginning with enrollment in Columbia College. The first curriculum includes 2 years in the college preparatory to study in the professional school of engineering. The second specifies a 3 year program in the college. The latter program includes in addition to engineering subjects those subjects which are required for every candidate for the bachelor of arts degree. Upon the completion of the latter program and the first 2 years' work in the school of engineering, the student is awarded the degree of bachelor of arts.

A third type of engineering instruction deserving attention is that which has been offered by Haverford College for several years and is now the announced plan at Harvard University. The student enters the college as a candidate for the bachelor of arts degree at Haverford and for the bachelor of science degree at Harvard, and may elect to take this major work in the "engineering sciences," just as he might elect a major in any other department. If he wishes to take professional work in engineering, it will be necessary for him to enter a graduate school in engineering. This system more nearly follows the method employed for candidates for professions such as the law and medicine.

Another possible solution of the problem has been proposed by L. W. W. Morrow, editor of *Electrical World*, as follows:[6] "If engineering educators agree

to follow the demands of industry, the specifications for 2 types of courses to give are as follows:

"*Course 1.* A course for the mass of students that is based upon a broad education developed largely from science, engineering and economics and taught chiefly by teachers using engineering methods and having both business and engineering knowledge, so as to train men to staff and operate mechanized industry.

"*Course 2.* A course with very high standards in science and engineering to be taught by scientific and engineering specialists to be given a selected number of students so as to train men to supply the specialized technical needs of industry—the future professional engineers.

"Undoubtedly some schools can and should develop both of these courses, but a large number will find it wiser to offer only one. Since the demand for the greatest number of men is filled by course 1 and since only a small number of men are needed by industry for course 2, the implications are that a majority decision will favor course 1. Moreover, only a few schools can obtain and sustain the faculty and facilities needed to carry out the objectives of course 2, while most of the schools could readily adapt themselves to the specifications of course 1.

"To be successful in completely meeting industry demands, it would be necessary to make a complete revision of the typical university organization and to form a separate college for course 1, and another college for course 2. This, of course, would involve a radical change of the present university organizations and would require recombinations of the existing colleges and courses."

It is apparent that these current views of curricular problems are based upon broad concepts of the social purposes which engineering serves. The fact that different conceptions of the social service rendered by engineering education do exist, in itself justifies differences in objectives and curricula. Its importance is illustrated further in the conflicting views that, on the one hand, engineers should be limited in number to a strictly professional group whose professional practices are sanctioned by law, and, on the other hand, that engineering education in the present era is most suitable to individual preparation for industrial life and as such, a wider diffusion of technical knowledge and method is favored. The latter viewpoint is quite general within the electrical industries where the majority of graduates are not engaged in occupations of the more advanced technical nature, but rather in activities difficult or impossible to correlate with the traditionally professional divisions of engineering education. Nevertheless, these occupations require a knowledge of engineering facts and an understanding of the fundamentals underlying applied science.

While it would be wrong to emphasize unduly the service function of the engineering schools in supplying industrial needs for technical and leadership talent, nevertheless the hopes and ambitions of most of the boys who choose engineering careers are related unquestionably to the standards sought by the profession as a whole. That the requirements must be widely different for the 2 viewpoints expressed is evident. It is also apparent that, carried to their conclusion, the requirements for strictly professional activities alone would halt the entry of many highly desirable technical administrators and make acute the question of how many should enter the profession. In this connection, it is of interest to note (figure 1) what has happened in the recent 15 years or the postwar period. Taking the broader social view, it may be observed that the percentage increase in supply over this period does not exceed the average rate of growth of the population as a whole. It also may be observed that considerable increase in graduate study has been manifested in recent years. The committee on graduate study of the Society for the Promotion of Engineering Education notes this by the fact that in 1921–22, 178 advanced degrees were granted in engineering. In 1933–34 there were 1,197. So far as the doctorate is concerned, less than 10 were granted in engineering in 1928; in 1934, 126 were earned, putting engineering in second place in the sciences—exceeded by chemistry only, and ahead of physics. While the influence of the depression is apparent, there is no doubt that graduate study is on the increase, also that the popularity and relative need for 5 and 6 year courses remains to be determined.

Considerable thought is being given to the question of length of curricula because of the desire to provide the broader service function of engineering education and at the same time preserve an acceptable technical content. Reflected in the problem of time limitation in providing an ideal program is also the economic limitation of individual institutions. While at present opinion may well look with disfavor on attempts at standardization, the successful adoption of a 5 or 6 year curriculum appears to require some uniformity of action which is not likely to eventuate since the majority of the 150 engineering schools in the United States have yet to celebrate their fiftieth anniversaries and even in recent years may be regarded as educational laboratories, individually and collectively seeking the best answer to the question of what is the best way of meeting the needs of young men of varying talents. This is the fundamental concern in which the colleges, industry, and the profession have a common responsibility. The problem is further broken down into the 3 questions: first, who can hope to achieve rewards in engineering practice that will sustain their enthusiasm and enhance their growth; second, what flexible educational program is best to stimulate continuous development; and third, what recognitions are suitable to various stages of achievement.

The organization, in 1932, of the Engineers' Council for Professional Development was the most constructive step ever taken toward the co-operative solution of this problem. This Council is a conference of 7 engineering bodies with representation from the 5 great national engineering societies. The latter represent the professional aspects of the engineer's life—civil, mining, mechanical, electrical, and chemical engineering. The educational phase which is discussed here is represented by the Society for the Promotion of Engineering Education, and the legal phase by the National Council of State Boards of Engineering Examiners.

Those engineers who were concerned in the organization of E.C.P.D. have felt that the purposes

of the council can be served only by the whole-hearted co-operation of the engineering bodies principally concerned. In other words, all those interested in the welfare of engineers and engineering should work toward a common end.

The 4 major committees through which the E. C. P. D. functions are: the committee on student selection and guidance, the committee on engineering schools, the committee on professional training, and the committee on professional recognition. Considered in chronological order, they cover a period of 10 of the most constructive years in a young man's life. Thus the union of the profession and the preparatory institutions becomes an agency of collective responsibility for giving expression to their ideas, and should add encouragement to show initiative and supply wisdom in the development of these ideas. Since full accounts of the yearly activities of E.C.P.D. are published in the annual reports of the Council for 1933, 1934, and 1935, no detail is necessary here. It may be helpful, however, to list some of its activities as illustrations of concerted effort given to educational problems.

As related to the problem of selection, counciling, and self-determination of interest and ability, the committee on selection and guidance has:

1. Urged the organization of a nationwide counselor service, with the schools and national societies co-operating.

2. Prepared a suggested guide for interviewers, and an interest and aptitude analysis form suitable for both high school students and college freshmen.

3. Revised and circulated copies of the pamphlet, "Engineering—A Career, A Culture."

4. Interested itself in the experiments of certain schools conducting guidance conferences at summer camps.

5. Studied the results of predictive tests to determine scholastic aptitude.

As related to undergraduate instruction, the committee on engineering schools has thus far concentrated its efforts on the immediate problem of accrediting, concerning which there is a later discussion.

As related to the problem of continued education and the stimulation of individual effort to organize a postcollege educational program, the committee on professional training has:

1. Prepared a pamphlet, "Suggestions to Junior Engineers," including a self-analysis guide for the formulation of an indvidual program of development and a suggested general reading list.

2. Undertaken the preparation of a selected bibliography of engineering texts for both graduates and students.

3. Made a survey of educational facilities throughout the country, with particular emphasis upon courses available to men at work.

4. Considered the possibilities of an operating program for professional development and initiated trial procedure in selected communities.

As related to consistency and uniformity in recognizing achievement, the committee on professional recognition is studying the 3 avenues through which recognition is now accorded, that is: the engineering schools, through their charter function of granting degrees; the corporate membership of the national engineering societies, through their membership grades; and the legal sanction afforded only by professional registration in which the specification of minimum standards for public protection is an important factor.

This concerted action of the profession, within which are embodied many factors bearing directly upon curricula, is not without precedent in educational history for it is similar to the purpose served by the Council on Medical Education organized

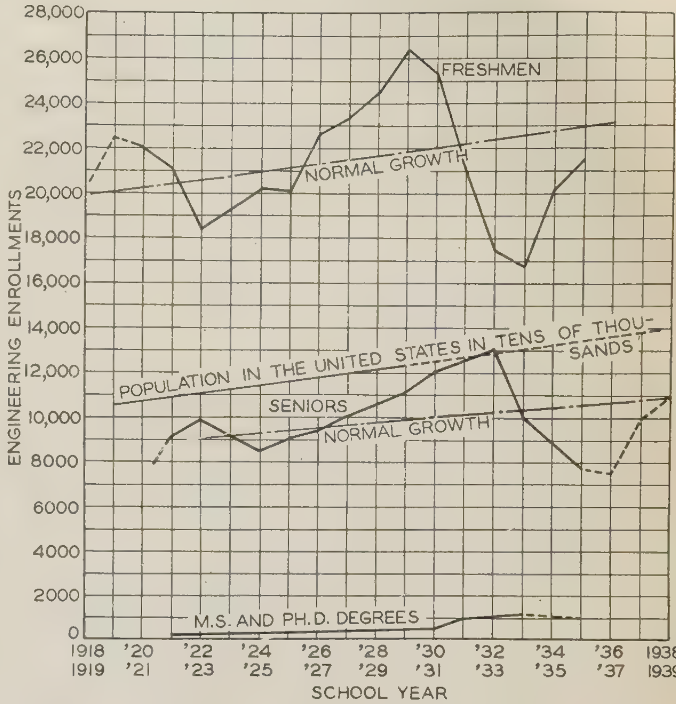

Fig. 1. Engineering college enrollments and advanced engineering degrees granted in colleges and universities in the United States since 1918

30 years ago. Many of the problems are similar in social and legal aspects. There are likewise very striking characteristic differences. For example, whereas one deals with highly personalized service in which public health and protection are of major importance, the other deals with groups working collectively in rendering service to a corporate institution in which protection of life and property is often not a factor. The lack of intensified individual importance in the engineering profession perhaps has influenced too much the popular concept of engineering as a profession, or rather, an engineer as a professional individual. Recent years, however, have shown a rapidly developing mass of law requiring the measurement of engineers and their licensing. The majority of States now have some form of licensing procedure. Consideration of the qualifications engineers should meet has precipitated the necessity for the accrediting of undergraduate schools, and a method has been tried out in schools of the New England and Middle Atlantic States with the idea of applying it to all schools. The immediate purpose is to establish an accredited list, acceptable to such agencies as feel the need of it.

Inasmuch as the committee on engineering schools has undertaken this work, it is pertinent to introduce

here a discussion of the procedure which could, if not wisely conceived, have a detrimental effect upon educational liberty or freedom of development. One thing is evident: An era in engineering education has begun which will demand more courage and vision than have been required by any previous generation. The action of E.C.P.D. in assuming the responsibility of accrediting is largely based upon the premise that the profession is best able through the engineering faculties represented by the Society for the Promotion of Engineering Education to do the job. The plan adopted divides the United States into 7 districts. The work of accrediting was carried out during the last year in 2 regions. The plan involved an invitation to the institutions of these 2 areas to apply for accrediting, a report submitted by those applying, an inspection by a delegatory committee, a report by the members of the delegatory committee, a conference of all delegates in these regions with chairmen in other regions and a final recommendation to be submitted to the main committee and in the fall to E.C.P.D.

It is obvious that the thoroughness and the quality of the job done depends in large measure upon the character of the inspection and the qualifications of the inspectors. The selection of qualified educators should insure sympathetic consideration of the objectives of individual institutions and protection of their liberties. In past experience this element has been considered of foremost importance. It must be recognized, however, that the final judgment is after all the composite opinion of capable judges, and that the basis of judgment must consider qualitative as well as quantitative aspects. While it is customary to judge institutions in terms of their financial stability, physical plant, semester credit hours, and faculty, these criteria are after all merely someone's opinion, and at the best are not wholly satisfactory measures. While the judgment exercised may vary considerably, in the final analysis the decision should indicate whether or not the instruction in an institution is consistent with the ideals of the profession and whether the physical facilities and financial stability are adequate to maintain minimum standards.

While it is too early to formalize the helpful possibilities of accrediting procedure, where there is understanding and confidence there is hope of achievement.

Bearing upon the subject of inspection, George F. Zook, president of the American Council on Education, made the following significant statement in an address before the Annual Congress on Medical Education and Licensure, Chicago, Ill., February 17, 1936:[7]

"There is another process on which all accrediting agencies have placed great reliance, namely, that of inspection. There can be no doubt that an outside inspector, or inspectors who are known to be thoroughly competent and sympathetic, can do more to jar an institution loose from a spirit of dull complacency than almost anything one can think of. I am convinced, therefore, that for purposes both of accrediting and stimulation the device of visitation is highly desirable. But there is a wide difference in carrying on the 2 functions which thus far has not been sufficiently recognized in the practices of any accrediting agency. After all, one has a very different attitude toward the policeman who knocks at his door than he does toward the doctor. One comes to compel an individual to meet an arbitrary legal standard; the other to help him through education and advice.

"I do not wish to carry this comparison too far. I realize that it has not been easy for inspectors to play the dual rôle of policeman and doctor. Furthermore, the Council on Medical Education has wisely chosen people out of the medical schools themselves who are known to be thoroughly competent. Nevertheless, the inspectors have represented essentially an outside organization. They have been received with a certain amount of trepidation almost everywhere. With some hesitation I venture the opinion that even in the better institutions the inspections have been too much in the nature of detailed criticisms. In other words, there is not enough of the element of friendly stimulation which, as I endeavored to say earlier, should be the keynote for the visitation at all institutions, save those which may properly be regarded as marginal in character.

"My reason for including these observations, it will be remembered, is merely to support the thesis which I have advanced, namely, that the accrediting and stimulation of higher institutions are properly aspects of the same function and that both can best be carried on by any agency which directly represents the institutions themselves.

"My fourth and last observation is that accrediting agencies in higher education should eliminate quantitative standards and go over completely to a frank attempt to evaluate, qualitatively, the processes and product of higher institutions."

Only when we have come to an understanding of the needs and situations which confront the engineering profession today can we hope to achieve that unity within our ranks which, combined with the high personal integrity and technical skill of its individuals, will be instrumental in solving, at least in part, the technical, social, and economic problems we have created. It is not so much methods, material, and product which needs standardization as excellency in the jobs which are worth doing. It is with these things in mind that the curricula of our schools and colleges must be designed.

References

1. Engineering Education Is Meeting the Challenge, Harold W. Bibber. Elec. Engg. (A.I.E.E. Trans.), v. 53, 1934, p. 1356–9.

2. The New Epoch in Engineering Education, C. C. Williams. Elec. Engg. (A.I.E.E. Trans.), v. 55, 1936, p. 132–6.

3. What Are the Trends in Engineering Education? Webster N. Jones. *Chemical and Metallurgical Engineering*, Sept. 1934, v. 41, p. 462.

4. Report, 1934, Dean Robert E. Doherty, School of Engineering, Yale University, to President Angel.

5. Report of the Investigation of Engineering Education, S.P.E.E., v. 1, 1923–29, p. 85.

6. Industry Demands and Engineering Education, L. W. W. Morrow. Elec. Engg. (A.I.E.E. Trans.), v. 53, 1934, p. 518.

7. Address, Annual Congress on Medical Education, George F. Zook. *School and Society*, April 25, 1936.

Highway Lighting—Principles and Sources

Linking the first studies of highway lighting with the present studies, this paper enumerates some aspects of the optical problem involved in producing satisfactory levels of brightness on typical pavements to insure adequate visibility under various weather conditions. Characteristics of incandescent, sodium vapor, and high intensity mercury vapor lamps are discussed, together with the designs of luminaires, mounting heights, and spacings necessary to provide optimum conditions for vision.

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THE STREET LIGHTING committee of the Illuminating Engineering Society, after a prolonged investigation of the optical principles involved, published in 1929 the Code of Street Lighting. The descriptive text accompanying this code contains a wealth of technical description and explanation of many of the factors affecting vision and visibility with artificial illumination on both streets and highways.[1] The code includes minimum recommendations for the lighting of highways but with certain modifications where the requirements are unusual or the lighting conditions exceedingly difficult. However, because of the different requirements for highway lighting, a subcommittee was appointed in 1935 to draft a code of highway lighting; this important work is well under way. Furthermore, because of the relatively great importance of the matter, a subcommittee was appointed several years ago to study pavement brightness and to co-operate with the manufacturers of road building materials and designers of highways to improve the reflecting characteristics of their products and finished pavements.

Early Accomplishments of the Electrical Industry in Highway Lighting

The first organized effort by the electrical industry to secure recognition of the necessity for highway lighting, the problems concerning visibility, and the desirability of systematically developing this class of lighting service occurred with the appointment of the committee on the lighting of interurban highways by the executive committee of the commercial section of the National Electric Light Association. This committee rendered a comprehensive report at the national convention at Philadelphia in June 1914.[2] Excerpts read as follows:

"The lighting of interurban highways presents to the engineer an interesting and difficult problem, due to the large number of diverse factors encountered in arriving at a satisfactory solution, not only from the point of view of the illuminating specialist but of the commercial man as well. In general, the requirements of but 2 classes need be considered, drivers of horse-drawn vehicles and motorists.

"We are continually reading of serious night accidents involving the automobile, usually with a horse-drawn vehicle or a motorcycle, which statistics show could have been avoided had adequate illumination been provided."

With the silhouette principle of lighting, the ability of the eye to see objects clearly a sufficient distance ahead to avoid collision is greater than with any other type of illumination.[3]

"In silhouette lighting, seeing is accomplished by the discernment of objects in contrast with a lighted background which is usually the street surface.

"A serious objection to low intensity lighting on roadways is the glare of approaching automobile headlights which tends to destroy the adaptability of the eye to low intensities. Therefore it is desirable to obtain the elimination of the glare from the automobile headlights on those boulevards and highways which are used greatly at night for pleasure driving or otherwise, and this can be accomplished only by illumination of the kind described above. Then with the general adoption of the electric automobile headlight with its ease of control, and the enactment of suitable legislation regulating the use of powerful headlights, great improvement along these lines may be expected.[4]

"The character of the country through which the highway passes and the nature of the road surface determine largely the standard of illumination which should be adopted. If a low standard is adopted, the casual motorist will necessarily have to depend on his headlights for roadway illumination.

"On the other hand, improved roads, such as state highways, which run through many of the states, are to be considered and since the principal user is the motorist, a higher standard of illumination must be adopted and proper surface illumination must be provided so that the use of headlights may be carefully restricted, thus contributing greatly to the comfort and safety of all users of the highway."

Modern Requirements of Highway Lighting

From the report just quoted it will be seen that the present-day requirements of highway lighting are not greatly different in qualitative respects but that quantitatively the requirements for good vision have increased enormously. Car registrations were 460,000 in 1910 and in 1930 registrations were issued for 26,000,000 vehicles. Average speeds were 15 miles per hour in 1910 whereas today 50 to 60 miles per hour is not uncommon. Moreover, car miles per vehicle per annum have increased enormously.[5] Bus and freight lines operate, winter and summer, at high speeds on fixed schedules despite hazardous road conditions. Judging from accident reports and records of fatalities there are more pedestrians abroad now than formerly, probably as the result of the extension of mushroom settlements along the highways. In the design of modern highways, great changes in construction have taken place; superhighways with separated lanes of travel, overpasses, underpasses,

A paper recommended for publication by the A.I.E.E. committee on the production and application of light, and scheduled for discussion at the A.I.E.E. summer convention, Pasadena, Calif., June 22–26, 1936. Manuscript submitted April 17, 1936; released for publication May 6, 1936. An address embracing the substance of this paper was presented at the A.I.E.E. North Eastern District meeting, New Haven, Conn., May 6–8, 1936.

1. For all numbered references see list at end of paper.

Fig. 1. New bituminous macadam pavement with egg size crushed stone

clover-leaf intersections, bridges, and traffic circles have come into the problem, and all require special study and treatment from the lighting point of view. In the pavement itself many improvements have been made in the interests of safety so far as skidding is concerned, but the increasing use of the bituminous macadam surface employing bluish gray or reddish brown stones or crushed rock of egg size is disquieting, to say the least, to the lighting engineer because of the increased difficulty and cost of lighting such dark diffusing surfaces (figure 1). Usually such surfaces are at their best from the fixed highway lighting viewpoint when they are wet and reflect the images of the light sources from tiny pools of water and wet surfaces at all upward angles, so that the entire roadway appears bright and in prime condition to produce silhouettes. On the contrary, the old time smooth surfaced asphalt with its inherently high sheen is rapidly being replaced because when wet it tends to permit skidding (figure 2). Such a pavement when it is wet presents the most difficult problem of all in highway lighting, although it is comparatively easy to light when dry. On this asphalt pavement the reflections of the light sources form narrow streaks of intense brightness which in themselves constitute sources of glaring brightness in contrast with the darker portions of the pavement.

Probably because of the rapidly mounting need for hard surfaced roads for expediting transportation over the nation's highways, public officials have apparently devoted but little attention to the proportion of night accidents which result from inadequate illumination. Therefore it appears to the lighting engineer that the highways have been designed wholly with regard to the safe movement of vehicular traffic by day, the general assumption being that the lighting requirements can be met adequately by the illumination provided by the headlights, tail lights, colored running lights, and illuminated direction signals on trucks and busses.

The effectiveness of reflecting devices on the rear of vehicles and of direction signs and markings along the highway also depends on light provided by the headlights, although in some states stop-and-go traffic signals and also flashing beacons are installed and maintained at dangerous intersections by the departments of public works. In several states notable progress has been made in lighting "danger spots" and in the installation of trial highway lighting demonstrations employing incandescent, high intensity mercury vapor, and sodium vapor luminaires. However, in the large majority of these installations of planned highway lighting the expense of installation and maintenance is borne wholly by the utilities and municipalities or the counties, as the case may be. Generally speaking, highway construction and highway lighting today are in about the same relative positions as building construction and scientific interior lighting were some 25 years ago. Then the last thing considered by the architect was light. Today, the architect contemplates lighting as a primary element in building design and construction. Perhaps highway authorities will realize in the future the importance of the fixed illumination factor and take it into consideration when the highway is planned and specifications drawn.

The assumption by many people that the transient illumination provided by the vehicles themselves is adequate is obviously erroneous in that many vital considerations affecting the eye itself are entirely disregarded. Even the cause and effects of headlight glare seem to be but little understood.

Dr. Harry R. DeSilva, psychologist at Massachusetts State College, has conducted reaction tests on hundreds of drivers and determined many interesting facts concerning vision; he writes in part as follows:[6]

"Of all the senses involved in driving an automobile, vision is of paramount importance.

"When the ordinary person thinks of tests for vision, he usually calls to mind a color-vision test or an acuity test. While we include these among our visual tests, we consider them of less significance than our other tests, which measure glare blindness, movement threshold, depth perception, and tunnel vision.

Fig. 2. Asphalt pavement with uniform brightness from regular or specular reflection; this surface when wet forms virtual images of luminaires

Fig. 3. Extremes in pavement surface reflectivity may not result in similar extremes of brightness; specular asphalt and diffuse concrete show great variations under natural daylight

Left—Facing toward sun; *Center*—Facing away from sun; *Right*—Looking directly down

"Driving an automobile at night is more difficult than day driving because of the shorter range of road visible in front. But driving along in the face of the blinding headlights of oncoming cars is doubly precarious. What makes matters worse is that some people are blinded much more by headlights than others; in fact, their disability is serious enough to be designated as glare blindness. Upon questioning some of these glare-blind people, we found they realized they did not see well at night in the face of glare, but that they thought that they experienced no more difficulty in this respect than did the average person. In other words, many people who are glare blind do not recognize their defect.

"The average driver was favorably disposed to our glare test. Many of our glare-blind subjects remarked, after the test, that they intended henceforth to do as little night driving as possible. If this apparatus should be made available in all of the testing offices of our state motor registry, it would in time do much to teach glare-sensitive people to be more careful about their night driving. The test could also be used with advantage in educational work among people, such as bus or truck drivers, who *have* to drive at night."

Vision at Night

Considering the foregoing, how are objects seen at night under highway lighting? Stated simply, vision is accomplished primarily in one of 2 ways or in a combination of the 2 ways. Objects, fixed or moving, are seen either in silhouette against a lighted background of the highway pavement or contiguous surfaces or areas, or else by the direct light from the lamps themselves.[7] When objects are seen in silhouette, their bulk or outline is observed; when objects are seen in perspective or by direct light, their details or features are distinguished by means of the light reflected from the object to the eye. All persons and objects are seen by day or night either in silhouette or in perspective or by a combination of both. From this it is evident that in order to see at a considerable distance by direct light at night a relatively high intensity of illumination on the vertical surfaces of objects must be employed. Until certain recent developments occurred in vapor lamp lighting and the more efficient utilization of light from incandescent lamps, high intensities have been impractical from an economic standpoint except in white way or intensive lighting as in main business streets.[8]

Fig. 4. The correct application of light on a highway can overcome great differences in the reflectivity of different parts of the pavement by producing uniform brightness on the traffic lanes

The foregoing fundamental optical principles should be understood by all for they apply also to visibility on roads without adequate highway lighting where the driver has to depend solely upon his headlights. Everything considered, it is probably safe to state that 80 per cent of all objects on the highway at night are revealed by silhouette vision regardless of the type of lighting, whether by headlights or from fixed highway luminaires.[9]

Moreover, it is probably safe to assert that when natural conditions for visibility are poor, as in mists or fog, vision of distant objects is impossible except by the silhouette method. Under such conditions a driver's own headlights can be of little use to him since the fog particles diffuse the light and reflect it back at the eye level and thus form an illuminated veil through which he is unable to see any appreciable distance. Conversely, light from the opposite direction from high mounted luminaires designed to give a strong directional effect at angles well below the horizontal illuminates the atmosphere in such a manner that a deep background of relatively low brightness is formed which is actually advantageous in the production of silhouettes. Studies by the author of highway lighting systems installed along the eastern coast where dense fogs are sometimes encountered have yielded much valuable information on this phase of the subject.

Glare

Another important phase to be considered fundamentally is that of glare. What is or what consti-

bring developments which will affect the whole situation, but the subject is too involved to discuss in detail in limited space.

Fig. 5. Miniature highway for comparing methods of lighting

Scale model of a 30 foot road, 900 feet long, with dark bituminous diffusing pavement. Note objects at 300 feet and 500 feet respectively; lighting of 10 foot candles intensity is from 50 foot towers placed 30 feet from edge of road

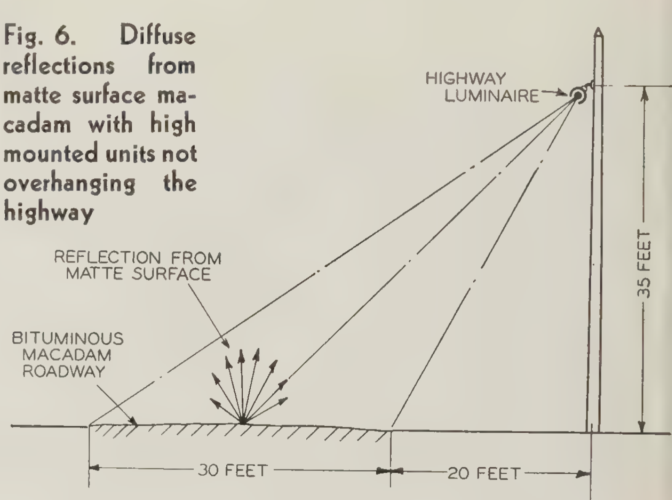
tutes glare? There still is a great difference of opinion among authorities. Perhaps glare can be defined best as "light out of place." Certainly it cannot be defined wholly in terms of candle power; for instance, a headlight that may appear glaring and interfere with vision at night on a dark country road will hardly affect vision in bright daylight. In other words, glare is largely a matter of contrast. Well defined and drastic headlight laws were put into effect in many states in 1921 and have accomplished some remarkably good results. But, even with the headlight equipment in perfect condition and the car on a perfectly level pavement with the main beams depressed, at least 800 to 2,000 candle power is permitted by law in horizontal directions toward the eyes of the oncoming driver.[10] This value is higher than the intensities from well designed highway luminaires at or slightly below the horizontal, but in the case of the latter, the high mountings usually employed (25 feet) lift the light source well out of the operator's field of vision.

Headlight beams compared with light from highway luminaires are much higher in intensity; the candle power from the former may be of the order of 50,000 maximum while from the latter 4,000 is about the maximum employed. However, in highway lighting the light sources are fixed while candle power intensities of headlights may vary from 800 to 50,000 at a very rapid rate because of the constant rocking or pitching of the car. Terrific eye strain and physical fatigue are the results.

Some investigators are working on the matter of polarization of the headlight beams of automobiles as a possible glare reducing device. None of the work as yet has reached a practical state. The future may bring developments which will affect the whole situation, but the subject is too involved to discuss in detail in limited space.

With adequate highway illumination, bright headlights are unnecessary. The control of glare from highway luminaires is in the expert hands of design engineers trained to handle such problems rather than in the inexpert hands of the car owner. Reid and Chanon from their miniature street tests have found that even the 800 candle power intensity from depressed headlight beams under certain conditions results in a loss of visibility equivalent to about 50 per cent of the illumination of the highway from fixed lighting.[11] Other quantitative determinations made by them show that certain types of existing highway lighting equipment causes a similar loss from the inherent glare of the luminaires themselves. However, the glare effect of headlights under practical conditions seems not to have been reported.

Fig. 6. Diffuse reflections from matte surface macadam with high mounted units not overhanging the highway



Major Considerations in Highway Lighting Design

In general, there are 4 major considerations in the successful illumination of highways:

1. The analysis of the reflection characteristics and visibility conditions of typical road surfaces.

2. The characteristics of light sources and the design of lighting auxiliaries to secure the most efficient application of the generated light to produce the maximum visual effectiveness on the highway.

3. The type of circuit and electrical distribution in view of the differences in operating characteristics of the various sources of light now available for highway lighting.

4. The mechanical design of the lighting system to give the best possible placement, mounting, and maintenance of the lamps and luminaires.

Lighting Characteristics of Road Surfaces

There are several basically different types of road surfaces in common use: concrete, bituminous macadam, brick, and asphalt. Each of these in turn has wide variations in its "skin" composition under practical conditions of use which may greatly affect and change the individual appearance and effective-

ness for best visibility under various artificial lighting systems. Fog and rain introduce still further complications because the reflection characteristics and appearance of even damp pavements may be entirely changed for better or worse.

The efficiencies of various pavements in terms of total light reflected in all directions varies from approximately 4 per cent to 30 per cent, the former value applying to the darker bituminous and asphalt surfaces and the latter value to ordinary uncolored concrete. This wide range of reflection values would be very disconcerting from the economic viewpoint if all pavement surfaces rigorously followed Lambert's cosine law of emission,[12] that is, if the surfaces were perfectly diffusing, for this would mean that the brightness of the pavements would be directly proportional to their respective reflectivities in terms of colorless body radiation, a white body reflecting 100 per cent of the light falling upon it. To put this matter in a practical way is to say that a concrete roadway of 30 per cent reflectivity and illuminated to a satisfactory degree of uniform brightness would require less than 14 per cent of the light required by the bituminous or asphalt pavements to appear equally bright. Fortunately, however, it frequently happens with the darker pavements that regular reflection may take place to a considerable extent so that the resultant brightness of the pavement toward the eye with low angles of incident light may greatly exceed that of concrete with equal illumination. When the pavement surface is worn smooth or contains bright particles which reflect light as a mirror reflects, it is said to have regular or specular reflection. Conversely, if a pavement surface is rough, such as cross-brushed concrete or crushed rock bound in tar, it is said to reflect light diffusely or to have irregular reflection. Most pavement surfaces are made of materials or are formed such that to a greater or lesser degree they combine both types of reflection.[13] In figure 3 are shown 2 types of pavement, asphalt and concrete, comparing the action of daylight upon them. The concrete has a reflection factor of about 30 per cent while the asphalt reflects approximately 4 per cent of the light striking it. The 2 pavements appear to merge as one in the direction from which the light is coming, but are distinct when viewed with the light behind the observer. The application of these characteristics on a lighted highway is shown in figure 4. In addition it may be observed in the figure that the traveled lanes marked by oil dripping from automobiles, although reflecting the same amount of light in each case, appear brighter than the concrete when the observer is facing the light and darker when the observer is facing away from the light.

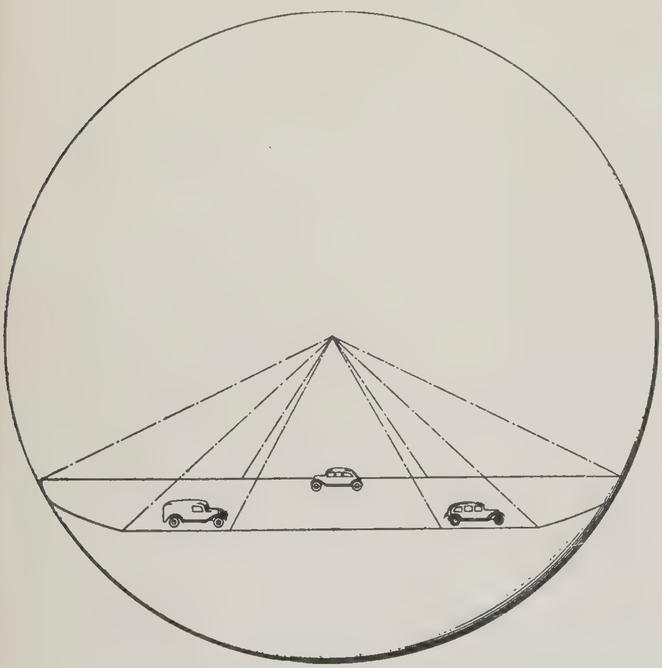


Fig. 7. Diagram showing that with light sources radiating uniformly in all directions and mounted 30 feet to light center above edge of 30 foot pavement and 130 feet apart only 12.5 per cent of total generated light flux falls on pavement within the hypothetical sphere

For many years the problems of uniform surface brightness, glare, and the correct distribution of the light from the luminaire over the pavement have been subjects of much discussion among street lighting engineers. Years ago the author and his associates designed a highway in miniature to represent to scale typical conditions as found on highways (figure 5). It is completely equipped with facilities for studying the diversified problems of highway illumination from the various viewpoints presented in this paper. The highway itself is a 4 sided member arranged for rotation and having on each side a surface simulating one of the basic surfaces as mentioned above (brick excepted). By simply turning a hand wheel 90 degrees the different surfaces in turn are quickly positioned in the same relation to any selected type of highway illumination. By this means the visibility of various objects can be compared quantitatively on the different road surfaces with different degrees of illumination, different placement of the luminaires lengthwise or transversely, etc. Among the most important studies were those concerned with asymmetric light distributions, pavement brightness, and glare. It was found, for example, that the high brightness of the asphalt pavement of the specular type permits higher candle power intensities from the

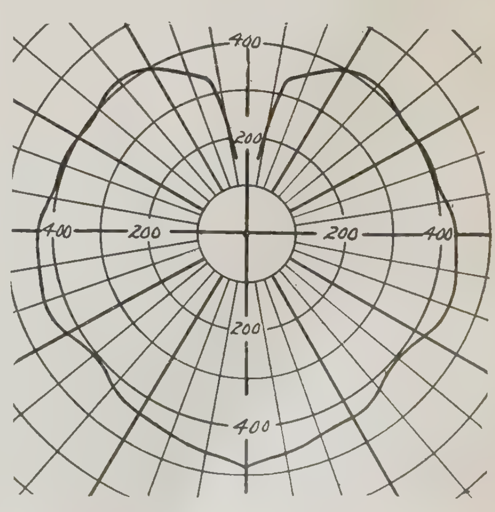


Fig. 8. Iso-candle distribution curve of light from 300 watt multiple type C mazda lamp

luminaire toward the eye than does bituminous macadam reflecting diffusely; that is, the brightness of the former more nearly approaches the brightness of the source, thus reducing glare, but in the latter case the pavement is of such a low order of brightness that the conditions for high contrast between pavement and light source are increased and glare, with inability to see, is the result. Another important finding was that the light sources gave better results over wet or specular pavements if they were placed in rows over the traffic lanes but with alternate lamps in staggered formation, as by this means each lane has its own path of brightness. The ideal method of lighting pavements having a perpetual sheen would be to employ sources of light of great linear dimensions and of relatively low brightness mounted transversely over the roadway. This scheme was tried by the author in miniature and to some extent in practice with the first installation of sodium vapor lamps in the United States. The lamps were 24 inches in length, mounted well out over the pavement, and on wet nights the pavement reflections were remarkably effective. Although the present 10,000 lumen sodium vapor lamp is only 12 inches in length, some advantage is gained by its transverse horizontal arrangement.

The ideal road surface for artificial illumination would be a perfectly diffusing white surface because the direction of the incident light would be immaterial, for with uniform illumination the area would appear of maximum uniform brightness no matter from what direction the view was taken. Thus, the luminaires could be at one side and away from the pavement or at a great height above the pavement (figure 6). Bituminous and other matte pavements reflecting diffusely could be improved greatly in reflectivity if the surfaces were given a wash or heavy spray of light colored concrete or cement. Compared with the cost of producing satisfactory brightness with increased intensities of illumination, road surface treatments would be relatively inexpensive. On concrete pavements darkened and polished with wear the desirable matte characteristics could be restored and the reflectivity improved by heavy sand blasting. It is frequently claimed that dark pavements are easier on the eye under natural lighting conditions. This probably is true to the extent that conditions for visibility on the highways are generally improved when the sky is overcast. However, it is clear from figure 3 that the brightness may be painfully high, under sunlight, even with a pavement of only 4 per cent reflectivity if the bright sky or the sun happens to be in the field of view and the light falls on the roadway at low angles of incidence.

Light Sources and Luminaire Design

The primary problem of illumination design is to conserve as much as possible of the generated light at or near the source, and redistribute it over the roadway under consideration to produce reasonably uniform brightness throughout the pavement surface, and to accomplish the desired result with a minimum of glare. From 1914 to 1933 the most practical source of light available was the highly concentrated filament of an incandescent lamp. The initial distribution of light from the conventional loop-type series street lighting filaments of 2,500 lumens output and up is usually pictured by the lighting engineer as being virtually spherical in form but having conical depressions at the poles, that is, the filament for practical purposes is considered as a

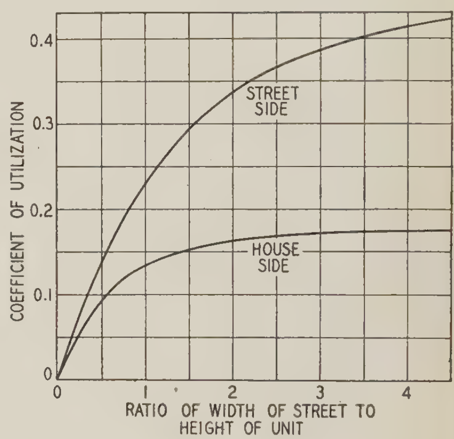


Fig. 10. Utilization curves in terms of pavement widths when various mounting heights are assumed

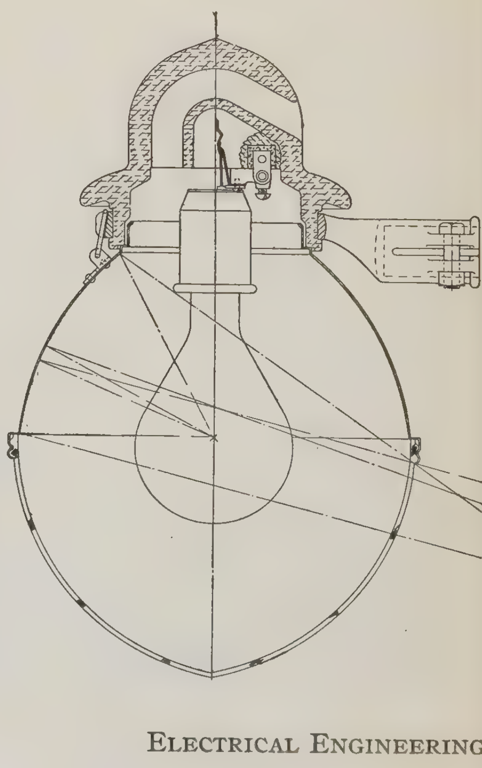


Fig. 11. Optical system showing conical reflecting surfaces and their redistribution of upward light from filament

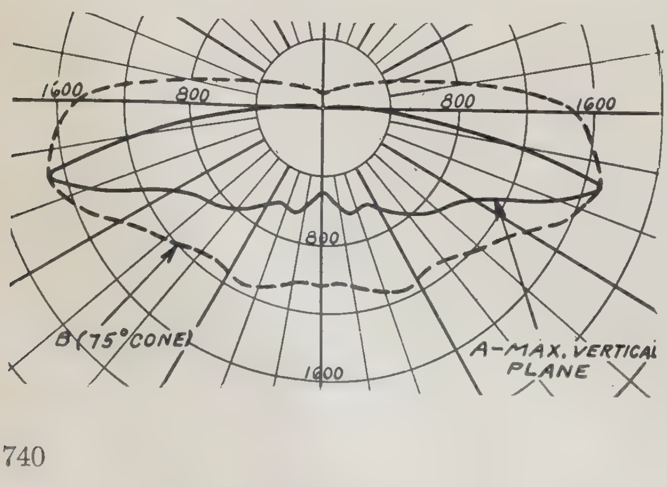


Fig. 9 (left). Isocandle distribution curve of light from luminaire having specially treated aluminum reflector

point source with the light radiating uniformly in all directions. Such a source placed at the center of a sphere would illuminate its surface uniformly throughout as illustrated graphically in figure 7. However, in the illumination of the highway pictured within the spherical globe only a relatively small surface really receives light. In the hypothetical case shown the actual value of the light falling naturally on the highway is only 12.5 per cent of the total light flux generated by the lamp. Thus the luminaire designer has the problem of collecting the wasted light (87.5 per cent) and redirecting it in the most efficient and effective manner to enhance the pavement brightness. Theoretically this would result in 8 times the illumination provided by the bare lamp.

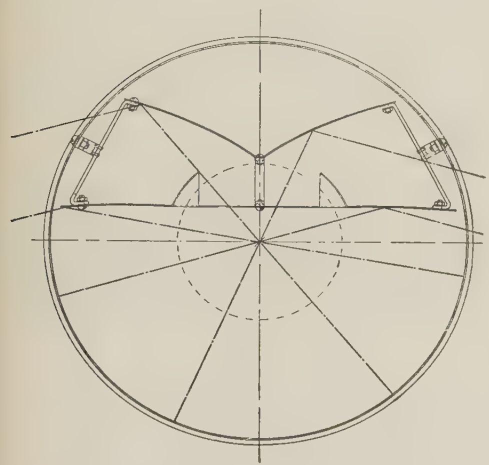


Fig. 12. Redirection of downward light from filament in horizontal planes

Generally the effectiveness of the design of the lighting device depends upon the following factors:

1. The over-all lumen output of the luminaire in per cent of total bare lamp lumens.
2. The efficiency of the luminaire in delivering its lumen output to the pavement in any given case calculated in per cent of total bare lamp lumens (utilization).
3. The candle power intensity of the light from the luminaire in vertical planes around its axis.
4. The intrinsic brilliancy of the luminaire surfaces emitting light toward the eye at angles near the horizontal.

A distribution curve of a conventional type *C* mazda lamp is shown in figure 8. The distribution curves of the light of this lamp modified by a modern reflector design are shown in figure 9. In the process of collecting and redistributing the generated light of the filament 33 per cent of the initial light has been absorbed. While this is important, it is of even greater importance to know how much of the remaining light can be usefully employed. Figure 10 shows the utilization efficiency curve in terms of pavement width (street side) when various mounting heights are assumed. From these it is seen that this particular design is most efficient at approximately 3 to 4 times the mounting height, delivering from 38 to 42 per cent of the light of the bare lamp to the pavement. Everything considered, this is a creditable performance and the luminaire may be said to have increased the efficiency of the lamp threefold so far as light delivered to the pavement is concerned. In the problem just considered it was assumed that the luminaire was placed directly above the edge of the pavement. In practice it probably would overhang the roadway a considerable distance, also there might be a berm or wide shoulder to be lighted in which case the light on the "house side" would play as important a part as the light on the "street side," therefore the utilization efficiency under maximum conditions could be equal to 67 per cent, the full lumen output of the luminaire.

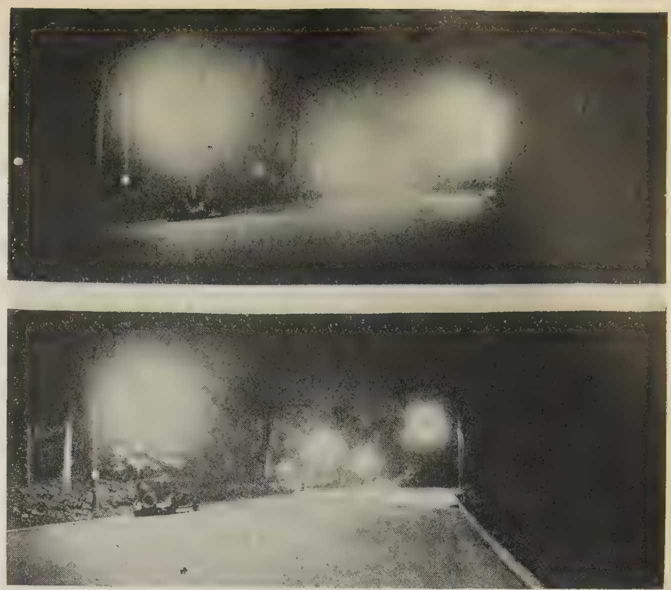


Fig. 13. Lighting of Prospect Park, Brooklyn, N. Y., before and after the installation of luminaires employing the optical system shown in figure 11

Camera exposure and other conditions same in both views

A further analysis of the photometric curves shows that the maximum candle power is at 75 degrees from nadir and is approximately 4 times the maximum candle power of the bare lamp (figure 8). From this luminaire there is practically no direct light, that is, the light source is shaded when viewed along the highway at a distance of 10 times the mounting height. Even at the lower angles, glare is minimized because of the moderate intensities and the diffusion of the light laterally by means of the prismatic devices on the outer surface of the glass bowl. Figure 11 shows the optical design of the gathering reflector surfaces and their effect on the upward light from the filament. Figure 12 shows the downward light from the filament redirected in horizontal planes from deflectors designed to give an asymmetric distribution generally parallel to the roadway.

The development of this particular luminaire resulted from studies of light distribution and glare over a period of years on the miniature highway at Lynn, Mass. These were followed by extensive field tests of an installation in Brooklyn, N. Y., by engineers of the Consolidated Edison group and the Bureau of Gas and Electricity of the City of New York by which the details of design and illumination performance were gradually perfected[14] (figure 13).

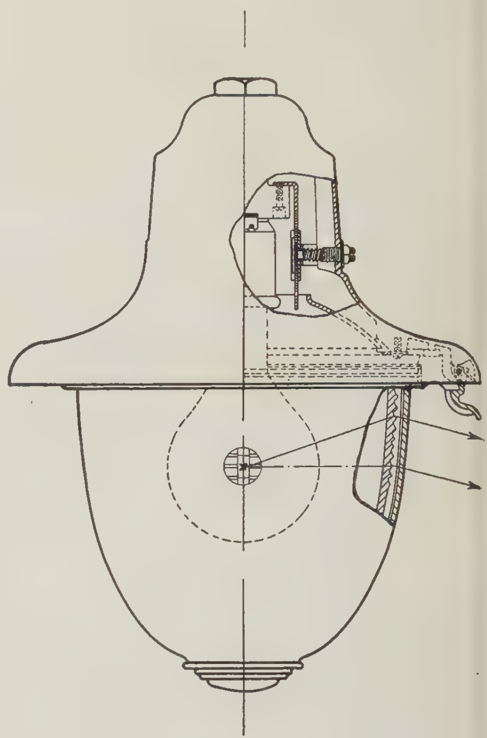
In the foregoing description of a single case of the

design problem many details have been omitted for lack of space; moreover, much could be written about the relationships of various filament forms and the contours of the reflecting and refracting surfaces of the luminaire. Generally, the vertical dimension of a filament positioned at a given distance from a given reflector or refractor surface determines the vertical spread of the redirected beams and the sharpness of the cutoff at the angles near the horizontal which at a distance are the glare producing angles. In other words, the smaller the vertical dimension and the greater the precision of the setting of the filament at the designed focal region, the more nearly will the practical results approach the theoretical. This, of course, is true of all precision projection devices where sharp cutoffs are required as in automobile headlights, airport landing lights, etc. For illustration the lever arm may be considered in its analogy to a light beam, so proportioned that on one side of the fulcrum (the reflecting or refracting surface of the luminaire) it is hundreds of feet in length. At this end of the lever and a few feet above it is the human eye (figure 14). At the end of the short arm of the lever, only a few inches long, is the light source; in the case of headlight beams there is a ratio of thousands to one. If a slight variation occurs in the setting of the filament, even a fraction of an inch, this variation will be multiplied tremendously. In a headlight even $^1/_{32}$ inch may be magnified to a yard at a distance of 100 feet, or if the bracket or support of the device varies angularly from the design position there will be a similar magnification. However, in the matter of light control so far as practical considerations of glare on the highways are concerned, the highway luminaire of precision design has all the advantage since it can be fixed in position permanently while the headlight is constantly in angular motion. As figure 14 shows, the relatively high mounting height of the highway luminaire still further reduces the possibility of glare from variations in beam position because the vertical movement for a given angular displacement is greatly reduced.

Precision is the keynote of modern luminaire design—precision in the design and forming of the surfaces that redirect the light; precision in the design, construction, and location of the filament source (see figure 15); and finally precision in the placement and mounting of the luminaire to secure the designed results (see figure 16). Recently, the lighting engineer has been provided with additional tools which made possible the luminaire shown in figure 11.[15] A specially treated aluminum with a coefficient of reflection of approximately 82 per cent makes possible many novel designs of high efficiency in light control while improvements in lamp making, filament design, and fittings with prefocused attachments all contribute to improved results in highway lighting.

As another outstanding example of the new order of things in highway illumination, attention is directed to a development recently described.[11] As a result of studies on a miniature street of relatively large size, design data were accumulated for a novel luminaire suitable for use in the illumination of 2 and 3 lane highways (figure 17). The reflector is of specially treated aluminum and comprises in part a vertically arranged series of truncated conical surfaces so designed that practically all the upward light of the filament is redirected in planes substantially parallel with the roadway. Since no diffusing glassware is employed, the cutoff at 80 degrees is almost complete. This design lends itself nicely to a special filament form and in fact to any source having small vertical dimensions. Figure 18 shows the trial installation of this new type of luminaire employing 4,000 lumen lamps. The efficiency of the utilization of the light on 3 lane highways is of the order of 45 per cent.

Fig. 15. Cross section of highway refractor showing sighting windows and socket adjustments to position filament accurately



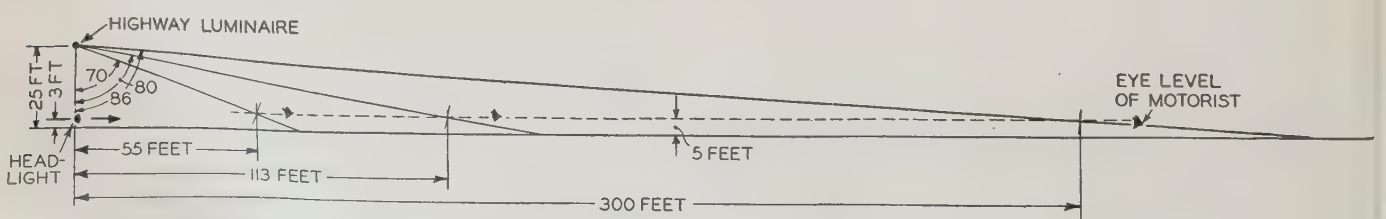


Fig. 14. Diagram showing that glare from headlights is greater than that from high mounted units because of angular relations

Fig. 16. Typical highway installation showing luminaires in staggered arrangement placed over alternate traffic lanes with telescopic brackets at 25 foot mounting height

Sodium Vapor Lamp

In addition to improvements in reflecting materials and incandescent filaments, new and more efficient sources of light are now available for highway lighting in the sodium and high intensity mercury vapor lamps. The luminous efficiency of the sodium lamps, in the 6,000 and 10,000 lumen sizes, is 2 to 3 times that of incandescent street lighting lamps.

This linear source of relatively great physical size presents many interesting problems to the illuminating engineer. Figure 19 shows the isocandle curves of light distribution from the 10,000 lumen lamp. The striking thing about them is that the bulk of the light flux falls in the central zones normal to the axis of the lamp, there being practically no light at the end zones. For the illumination of large circular areas such sources vertically arranged and fitted with large circular reflectors would be ideal. In the lighting of the modern traffic circle or clover-leaf intersection such applications may develop. Where the highway itself is concerned and conventional mounting heights are employed, the best arrangement of the sodium lamp seems to be horizontal and transverse. In this case the luminaire designer starts with a natural distribution of light in which the maximum candle power intensities lie in vertical planes parallel to the roadway. Therefore, instead of reflecting surfaces of revolution around a vertical axis as in the earlier examples where the "point source" of light was described, the procedure in this case is to employ cylindrical surfaces of parabolic cross section to intercept the upward light and reflect it in useful directions along the highway (figure 20). Because of the large vertical dimension of the 10,000 lumen sodium vapor lamp (3 inches), a multiple set of reflectors is required to subtend the light source and gather the upward light. At the rear of the lamp a vertical plane surface is employed to impart a lateral shift to the axes of the light beams toward the middle of the road. Examination of the light distribution curves of the sodium luminaire shows that it has a high over-all efficiency and is especially well adapted to the lighting of the 3 and 4 lane and divided highways (figure 21). Highway lighting by sodium vapor lamps during the past 2 years has met with general public acceptance. The advantages claimed are:

1. Higher efficiency
2. Lower intrinsic brilliancy, resulting in less glare
3. Broad source giving wide band on wet or specular pavement
4. Attractive and distinctive effect
5. Easier vision[16]

In the matter of glare there is very little information of a practical kind available. However some pedestrian visibility tests have been reported which are interesting and important.[17] The tests were made on the Boston-Worcester turnpike to determine the distance at which a pedestrian, in dark clothes, could be seen under similar conditions with sodium lighting and substantially equivalent incandescent illumination. The distances were 1,283 feet and 727 feet, respectively.

High Intensity Mercury Vapor Lamps

The performance of the sodium vapor lamp has been described primarily because of the greater experience in its use. As yet the 400 watt high intensity mercury lamp has not been standardized for horizontal operation. However, several notable street lighting installations of the 400-watt 16,000-lumen lamps have been made in which a 200 watt incandescent lamp is used in a specially designed luminaire to provide color correction, heat for starting in cold weather, and a stand-by in the event of mercury lamp outage.[18] This luminaire provides a symmetrical light distribution.

There is a 250-watt 7,500-lumen high intensity mercury vapor lamp available for universal burning

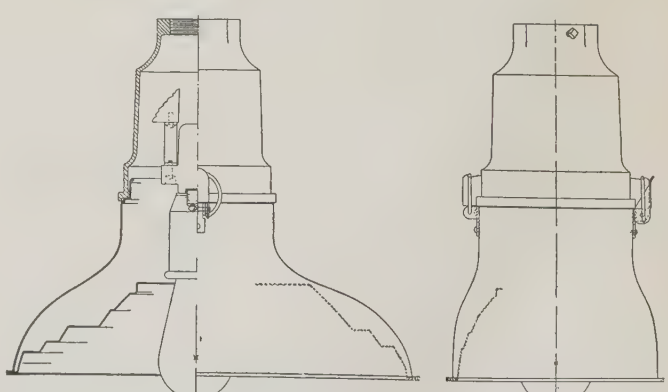


Fig. 17. Highway luminaire with specially treated aluminum reflector having a vertically arranged series of truncated conical surfaces; designed to be placed in transverse position over roadway

Fig. 18. Experimental installation of truncated luminaires on highway near Cleveland, Ohio

in any position. Its characteristics indicate the need of supplementary incandescent lamps to facilitate cold weather starting, with the result that the over-all efficiency is lowered and it becomes less attractive for general street and highway lighting.

In addition to these types of mercury lamps there has been announced as a laboratory development a radically different form, not yet commercial. This employs a small quartz tube about the size of a pencil in diameter and an inch or so long. The arc stream is about 1 millimeter thick. Such a small and concentrated line source may have great potentialities for street lighting in luminaires designed for accurate light control.

The Series Circuit Versus Multiple Distribution

Vapor lamps have a negative resistance characteristic, for their resistance decreases with increasing current. This means that the current flowing in the lamp must be limited to prevent a short-circuit condition and damage to the tube. Limitation of the current may be obtained by suitable resistance or reactance ballast. Resistance ballast is not economical because of the loss in power with consequent decrease in over-all efficiency. Therefore, reactance is the commonly used ballast, although it introduces the problem of power factor, which in most street lighting applications is quite important.

Each vapor lamp to be operated from a multiple source of supply must have an individual series reactance between it and the supply, or an individual transformer may be employed that has sufficient inherent reactance in its design to limit or stabilize the current to the lamp. In a series constant current circuit the stabilizing means is concentrated or lumped in the main constant current transformer that supplies a series or group of lamps. The well known types of constant current transformers used with incandescent lamp circuits will regulate the current for a series of vapor lamps with equal facility. It is quite likely to be more economical to lump the stabilizing reactance for a group of lamps in one unit rather than to distribute it over a number of individual lamp transformers, such as are necessary in multiple distribution.

On most vapor lamps, the voltage necessary to start the lamp is higher than the normal running lamp voltage. Consequently, excess voltage must be provided for starting purposes which the transformer reactance must absorb incidental to providing proper voltage and current for stable operation. This is more particularly true of sodium than of high intensity mercury vapor lamps. A timing relay is used with the sodium vapor lamp which does not permit the arc in the lamp to start until the cathodes have been preheated for approximately 20 seconds. This means that the lamps are ignited at different intervals of time as the result of small irregularities in timing of the individual timing devices, and in effect the load on a constant current circuit is imposed in steps as the individual lamps are ignited. This so-called "staggering" of the load provides the required starting voltage for the individual lamps as they come on the circuit. Were it not for the staggering the simultaneous addition of the starting voltages of all the lamps in the circuit would be much greater than is actually required, and necessitate the use of a much larger constant current transformer. The selection of a transformer that will start a series of sodium vapor lamps in this fashion and operate them at normal current will automatically furnish sufficient ballast for stable operation.

A detailed study will reveal that in any highway system of sodium lighting the series distribution is more satisfactory than the multiple distribution if the system is set up exclusively for the highway lighting.

With the advent of the 20 ampere type *C* mazda lamp in 1914, it was suggested that a 2,300 volt line be carried along the highway and the individual street lamps connected thereto through suitable transformer equipment. However, cost data at that time indicated that it could not compare favorably with

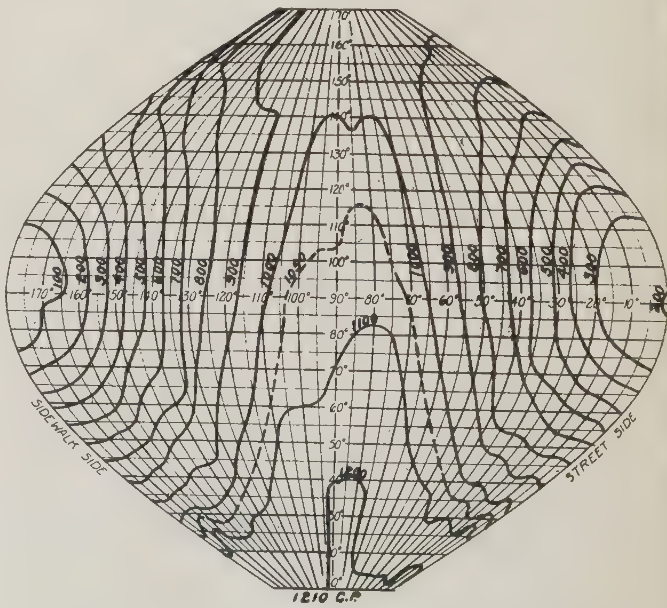


Fig. 19. Characteristic isocandle curves from photometric tests of a 10,000 lumen sodium vapor lamp

the series system because of the high voltage transformers, protective gear, and hardware which necessarily accompanies 2,300 volt distribution apparatus. This method of operation is now being reconsidered, however, for the high intensity mercury lamp, which is better suited for multiple operation. The high intensity mercury vapor lamp is not so easily adapted to series operation as is the sodium vapor lamp because of its inability to restart when hot after a temporary power failure. This property of the mercury lamp produces, in effect, an open circuit, and conventional film cutout protection schemes are not readily applicable. However, a scheme has been proposed for series operation of mercury lamps wherein a time delay device is installed adjacent to the constant current transformer so that after a power interruption, the energy will not be restored to the series circuit for 15 minutes or, in other words, until the lamps have had a chance to reduce their internal vapor pressure by cooling to the point where they will start promptly at a reasonable voltage. There has also been proposed a system using the so-called "air gap transformer." In this type of transformer an air gap is provided in the core which limits the degree of saturation when a lamp becomes open-circuited, and no destructive voltage is generated at the lamp. However, this system introduces an excess of reactance in the circuit which greatly reduces the carrying capacity of the constant current apparatus. In general, this scheme is not an economical means of distribution for operating mercury vapor lamps.

Present knowledge indicates that this mercury characteristic is inherent and there appears no immediate solution for avoiding it. Therefore, multiple distribution systems, which provide only low voltages, are generally simpler and more economical. In the installations made thus far, multiple distribution has been employed and in some cases where combination luminaires are employed, the mercury lamp is shut down at midnight, leaving only a 200 watt incandescent lamp operating on the all night schedule.

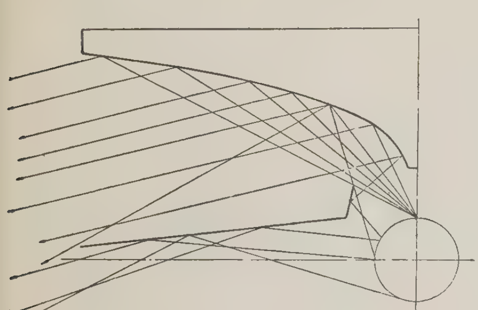


Fig. 20. Optical system of luminaire with horizontal sodium vapor lamp; arrows show redirection of half of light output

Mechanical Design of the Lighting System

Primary consideration should be given to the best practical placement, mounting height, spacing, and maintenance of the complete lighting unit, since from the foregoing it may be seen clearly that to secure optimum conditions for visibility, each type of luminaire must be considered separately. In the opinion of the author, the following suggestions should be helpful.

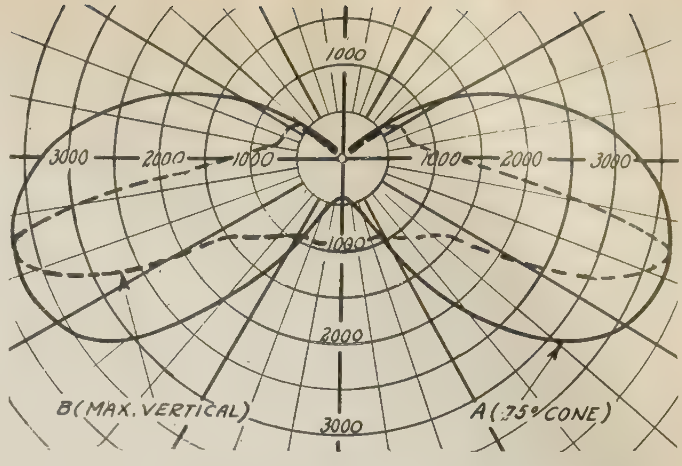


Fig. 21. Curves of distribution in 2 planes of light from sodium vapor luminaire

In general with existing highway conditions, it may be stated that the luminaires should be mounted not less than 25 feet above the pavement. In the case of 2 or 3 lane roadways, they should be placed over the travel lanes and staggered for the best results. If the design of the optical system is such that the cutoff is complete at approximately 80 degrees from nadir, spacings should not exceed 125 feet; that is, the ratio of mounting height to spacing should not exceed 5. This prescription holds good regardless of the type of pavement. However, with light colored pavements 2,500 lumen lamps are the minimum size permissible, and with dark pavements more difficult to light 4,000 lumen or even larger lamps should be used. If the luminaire is of a type that closely confines the light to 2 or 3 traffic lanes, adjustable mast arm supports from one side probably would satisfactorily accomplish the desired result in placement over the traffic lanes. Where highways are divided, as a double highway with a dividing plot between the roadways, each side should be considered as a separate highway and a double system of luminaires should be installed. Whether it would be better to have a single row of twin standards along the middle plot or set up 2 rows of poles along the outer edges of the highway right of way depends upon local conditions of electrical distribution, cost of installation and methods of maintenance, etc.

For 4 lane highways incandescent lamp luminaires giving a wider transverse spread should be employed. If the cutoff is complete at 80 degrees the staggered 125 foot spacing (ratio mounting height to spacing of 5) must be employed but for light colored pavements 4,000 lumen lamps are the permissible minimum size. If the cutoff is not complete at 80 degrees, that is, if there is some light at the higher angles, the spacing-mounting ratio may be increased to 8 with good results, which means that with a mounting height of 25 feet the spacing may be increased to 200 feet. With dark pavements of the diffuse type the spacing should be reduced to 125

feet and the 6,000 lumen lamp should be the minimum employed.

With the sodium vapor luminaire (figure 20) using the 10,000 lumen lamp, excellent results have been secured on all types of highways with maximum spacings of 250 feet on light pavements and minimum spacings of 125 feet on dark pavements. Figure 21 shows that so far as the efficiency of utilization is concerned, the sodium vapor luminaire is at its best on the wider highways; however, this may not be the only consideration because as a rule there are many other factors to be weighed in arriving at a decision in any given case. For example, in the lighting of the General Edwards Bridge which has a 6 lane roadway paved with black diffusing crushed stone, the sodium luminaires are spaced 100 feet apart and opposite, yet the illumination is not excessive (figure 22).

In many highway installations the omission of seemingly unimportant details at the time of installation has given rise later on to severe criticisms of the lighting results. It is important therefore to arrange in advance by a visual survey the placement of luminaires at the crests of hills and grades. Moreover, the axis of the luminaire should be normal to the surface of the roadway, thus obviating glare and conserving the light (figure 23). At clover-leaf intersections, circles, and underpasses even greater care should be exercised in the selection and placement of the lighting units since the difference in roadway levels may result in glaring sources at levels near the eye. For such cases shading luminaires are available and must be employed if confusion at these danger spots is to be avoided.

Future of Highway Lighting

In the foregoing text the author has endeavored to present a highly technical problem in a practical manner, for it is his belief that greater progress in satisfactory highway lighting can be made in the future if the underlying principles are generally known and avoidable mistakes thereby prevented. Unfortunately in some respects, the eye is the sole judge of the results secured in any given case, but all eyes are not alike. Moreover, a type of installation that is satisfactory in one set of conditions may be quite unsatisfactory in another location where the general conditions may be the same, but quite different in important details. However, it is clear that important advances in both the art and in the tools of the lighting engineer have occurred recently and that great strides in the application of highway lighting may be expected as a result in the immediate future. No one questions that there is a need for highway lighting. Statistics prove that it is justifiable from the broad economic viewpoint. Even the faulty installations of the past have proved the worth of illumination on the highway. Therefore is it not reasonable to expect even greater results from the modern systems of highway lighting now available?

Fig. 22. Sodium vapor lighting of the General Edwards Bridge between Lynn and Revere, Mass., with luminaire shown in figure 20

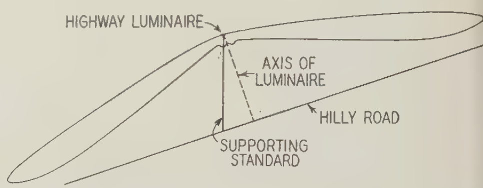


Fig. 23. Diagram illustrating mounting of highway luminaire with vertical axis normal to roadway

References

1. The I.E.S. Street Lighting Code in Pictures. Municipal Index, 1932, p. 411.

2. Report of Committee on the Lighting of Interurban Highways. *Proc.*, Commercial Sessions, National Electric Light Association 37th Convention, p. 168–86.

3. An Unrecognized Aspect of Street Illumination, P. S. Millar. I.E.S. *Trans.*, v. 5, Oct. 1910, p. 546–51. Also, Some Neglected Considerations Pertaining to Street Illumination, p. 653–74.

4. The Design of Luminous Arc Lamps, C. A. B. Halvorson. *Gen. Elec. Rev.*, v. 14, Dec. 1911, p. 578–89.

5. World Almanac, 1935.

6. Research on Driving Skill, Dr. Harry R. De Silva. Federal Emergency Relief Administration project, Massachusetts State College, 1935.

7. Street Lighting in Its Relation to Highway Safety, C. A. B. Halvorson and S. C. Rogers. Third Annual State Conference of Massachusetts Safety Council, Worcester, May 1, 1924.

8. Progress in Outdoor Lighting With Sodium Vapor Lamps, G. A. Eddy. *Gen. Elec. Rev.*, v. 38, Oct. 1935, p. 458–63. Also, Sodium Vapor Highway Lighting on Balltown Road, Schenectady, N. Y., G. A. Eddy. *Gen. Elec. Rev.*, v. 37, Aug. 1934, p. 372–7.

9. Studies in Street and Highway Illumination—Part I, K. M. Reid and H. J. Chanon. *Gen. Elec. Rev.*, v. 38, Nov. 1935, p. 522–4.

10. Headlight Specifications, Illuminating Engineering Society.

11. Studies in Street and Highway Illumination—Part III, K. M. Reid and H. J. Chanon. *Gen. Elec. Rev.*, v. 39, Mar. 1936, p. 150–4.

12. Illuminating Engineering (a book), F. E. Cady and H. B. Dates. John Wiley and Sons, New York, N. Y., 1925.

13. *Proceedings*, Highway Research Board, Washington, D. C., 1932, part I, p. 399. (C. A. B. Halvorson).

14. Lighter Streets at Lighter Costs in New York, Maurice P. Davidson. *American City Mag.*, Feb. 1936.

15. Tests on Alzak Aluminum Reflectors, A. F. Dickerson. I.E.S. *Trans.*, v. 29, 1934, p. 358–63.

16. Notes on Light and Vision, Frank Benford. *Gen. Elec. Rev.*, v. 28, 1925, p. 707–13.

17. Some Visibility Tests on Lighted and Unlighted Highways, Parry Moon and R. C. Waring. Franklin Institute *Jl.*, v. 219, Mar. 1935, p. 285–314.

18. Belmont (Mass.) Lights Concord Avenue, E. P. Taylor. *American City Mag.*, Jan. 1936.

News
Of Institute and Related Activities

The A.I.E.E. Summer Convention This Month at Pasadena, California

ALL is in readiness for the 52d annual A.I.E.E. summer convention to be held at Pasadena, Calif., June 22–26, 1936, with headquarters in the Huntington Hotel. The summer convention committee with R. W. Sorensen, chairman, has arranged an excellent program. In addition to the business features, sports, trips, and entertainment have not been overlooked. Pasadena, by virtue of its location, provides an ideal setting with the most favorable of climatic conditions and places of interest which may be visited. For a more complete account of the convention features, including both preconvention and postconvention trips, as well as transportation and hotel facilities, see ELECTRICAL ENGINEERING for May, pages 554–8.

Technical Sessions

Many interesting and timely papers will be presented and discussed at the 9 technical sessions which have to do with a variety of subject matter—electrophysics, conductor vibration, power transmission, electrical machinery, electrical measurements and selected subjects, transformers, illumination, protective devices, and current aspects of engineering education. The consolidation of views brought about through the discussions by well-known eastern and western engineers is expected to provide valuable engineering information.

The technical program, as given in ELECTRICAL ENGINEERING, for May, page 555, will be the final technical program. The name of C. P. Garman, Bureau of Power and Light, City of Los Angeles, should have been given as co-author with A. J. Bowie of the paper "287 Kv Boulder Dam Disconnecting Switches," and O. W. Eshbach, American Telephone and Telegraph Company, has become co-author with Morland King of the paper "Engineering Education—Opinions and Influencing Factors."

Technical Conferences

Arrangements are being made to schedule 6 or 7 technical conferences or informal round table meetings during some of the afternoons. Neither the papers and the talks nor the conclusions reached at these conferences are scheduled for publication, but those who attend will profit by the exchange of views. The objectives or agenda for several of these conferences available to date are given in the following paragraphs.

High Voltage X Ray Tubes and Allied Apparatus. This conference, with J. P. Youtz presiding, will probably consider a multiplicity of electrical problems that have risen in connection with the operation of high voltage X ray tubes. Dr. C. C. Lauritsen of the California Institute of Technology, who has developed and used the million volt X ray tube, probably will lead the discussion.

Mechanical Properties of Electrical Conductors. This conference, with probably F. C. Lindvall presiding, will be of great benefit to all engineers interested in conductors, conductor failures, and the expected life. The discussion at this conference may be started by Alex Goetz of the physics department of California Institute of Technology, who has done work with regard to the crystal structure of materials that probably has not been done anywhere else in the United States. An outline of some of his experiments on materials and a discussion of the meaning of fatigue will be most profitable.

The Use of Electronic Tubes in Industry. Electronic developments in general will probably be considered at this conference with W. C. White of the General Electric Company presiding. The scope of the conference will include also the timing of short intervals by electronic means, which is of particular interest to the airplane industry in California in connection with the quick welding and the fabrication of airplanes.

Synchronous Machines. This conference, with Prof. B. L. Robertson presiding, in all probability will be limited to synchronous machines, but may include other types of rotating machines and especially induction machines. The conference probably will be held after a session on electrical machinery so that the authors may meet informally with others interested and talk more in detail about the general subject of their papers. The general subject of the present performance of synchronous machines, both from the theory and testing method points of view, also has been suggested.

General Transformer Problems. This conference will be presided over by J. E. Clem, chairman of the transformer subcommittee of the committee on electrical machinery. Among the subjects that will be discussed are the following:

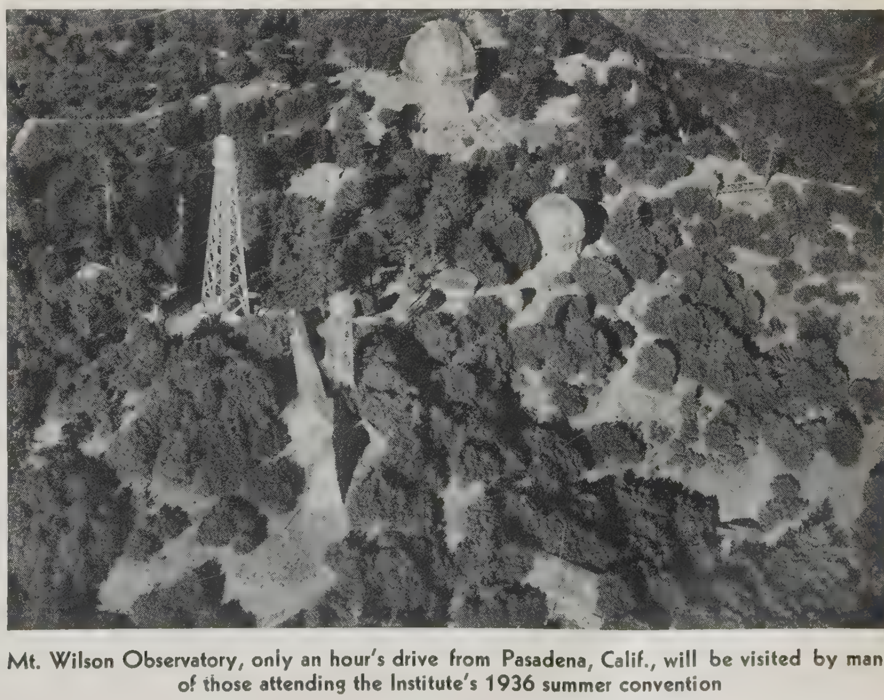
1. Insulation level for transformers.
2. Bushings and bushing troubles.
3. Drying out transformers.
4. Use of reclaimed or new oil after transformer has been reconditioned.
5. What transformer might be subject to further standardization.

Any other subjects that those attending may propose will be discussed as fully as time permits.

Student Sessions

The following papers by students are scheduled for presentation at the student sessions:

A 30 KW HIGH VOLTAGE D-C SUPPLY FOR X RAY USE, R. L. Hand, California Institute of Technology, Pasadena.



Mt. Wilson Observatory, only an hour's drive from Pasadena, Calif., will be visited by many of those attending the Institute's 1936 summer convention

Core Materials for Audio Frequency Transformers, J. B. Powers, University of California, Berkeley.

The Control of the Regulation and Stability of Long Distance A-C Power Lines, James Miller, University of Idaho, Moscow.

Effect of High Frequency Radiation on Living Cells, R. M. Hanson, Montana State College, Bozeman.

Heat Transfer Efficiency of Electric Range Surface Units, W. James Walsh, Oregon State Agricultural College, Corvallis.

Photoelectric Color Sorting, H. W. Bartlett, Jr., University of Santa Clara, Calif.

The Effect of Disk Position on the Accuracy of Watt-Hour Meters, J. A. Harkness, University of Southern California, Los Angeles.

A System of High Efficiency Grid Modulation, F. A. Everest, Stanford University, Calif.

Power Measurements at High Frequency, Richard Borton, State College of Washington, Pullman.

The Vacuum Tube Distortion Compensator, J. S. Campbell, University of Washington, Seattle.

Because of the large number of papers submitted for these sessions, each school has been limited to one paper.

Victorville Inspection Trip

An additional inspection trip is planned to Victorville, which is about 100 miles east of Los Angeles, to inspect one of 2 automatic switching stations on the Boulder Canyon 287 kv transmission line. At this station are installed the highest voltage high speed switches in the world. Of particular interest will be supervisory control, telephone, and relays, operating by carrier current, and also 287 kv coupling capacitors.

Hotel Reservations

The Huntington Hotel, convention headquarters, has made a special rate for the convention of $6 per person per day, American plan, for 2 in a room; $7 per day for one in a room. For the price of meals and the rates of other hotels in Pasadena see Electrical Engineering for May, page 558.

Members should make their own hotel reservations by writing directly to the hotel of their preference. Reservations for students only who wish accommodations at the "Athenaeum," California Institute of Technology, should be made with Edward Simmons, chairman, A.I.E.E. California Institute of Technology Branch, Pasadena, Calif.

Advance Registration

Members who will attend the convention and who have received a return addressed advance registration card should fill in and post the card promptly, if they have not already done so.

Nonmembers will be charged a registration fee of $3; members $2; immediate families of members and Enrolled Students no charge.

North Eastern District Meeting and Student Convention Held at New Haven

More than 300 members, guests, and students attended a very successful 4 day meeting of the A.I.E.E. North Eastern District held at New Haven, Conn., May 6–9, 1936. The student convention, held in conjunction with the meeting, was represented by counselors and students from the various Branches within the District. The campus of Yale University and the excellent facilities of Strathcona Hall, meeting headquarters, lent added interest and color to a program that was well arranged by the local committee.

The opening session on Wednesday morning was called to order by A. C. Stevens, the presiding officer, who introduced Vice President W. H. Timbie, who in turn welcomed the members of the North Eastern District, as well as those from outside the District. He commented on several features of the program and especially urged the engineers present to attend the student session and to come and hear the address to be given by Samuel Ferguson, chairman of the board of the Hartford Electric Light Company, on Thursday morning. President E. B. Meyer was introduced, and he addressed the assemblage with emphasis on the importance of association with other engineers and the value of friendship in business. At the conclusion of this address H. H. Henline, national secretary, referred to the excellent spirit of co-operation and work being done in the other Sections and Branches of the Institute, many of which he had visited recently. Following his remarks the remainder of the session was devoted to a symposium on highway lighting.

Technical Sessions

In addition to the symposium on highway lighting, 3 other general sessions were held at which technical papers having to do with a variety of subjects were presented, mostly by authors residing within the District. In general, the presentations were closely followed by those in attendance, some of whom later discussed the papers or asked questions which were answered by the authors.

At the symposium on highway lighting 2 addresses were given: one by C. A. B. Halvorson of the General Electric Company, and the other by L. A. S. Wood of the Westinghouse Electric and Manufacturing Company and president of the Illuminating Engineering Society. Mr. Halvorson described several of the modern light sources including the sodium vapor, mercury vapor, and capillary lamps. Mr. Wood gave some startling statistics on the number of night accidents and commented that, while safety campaigns had reduced the number of accidents occurring during daylight hours, they were ineffective in so far as reducing the number of accidents occurring after dark. He illustrated with lantern slides the value of street lighting by showing a man on the road at various distances from the driver. With the headlights on and the street lights off, under wet conditions, the object was not nearly so discernible as when the silhouette effect was produced by the street lighting. Papers based upon both these addresses have been scheduled for presentation at the A.I.E.E. 1936 summer convention to be held June 22–26 at Pasadena, Calif., and both papers are published elsewhere in this issue.

In a discussion of the subject, Robert Sparks advocated the use of polarized light. Other discussers thought that this might possibly be one solution of the problem but it was thought that the analyzer might block out too much light. Among the discussers of this subject was Dr. Clayton H. Sharp, who had served a number of years on the original committee on motor vehicle lighting of the Illuminating Engineering Society with Mr. Halvorson, who gave the first address.

One of the features of the technical sessions took place on Thursday morning in a general session with Past-President Chas. F. Scott presiding. Samuel Ferguson, chairman of the board of the Hartford Electric Light Company, showed pictures and described conditions that existed during the flood this spring. Among the many pictures of noteworthy interest was the setting of a pole in 15 feet of water, which was accomplished by means of lashing 4 oil barrels filled with rock to the butt of the pole; this may have been the first time that setting a pole under such adverse conditions has been accomplished. Following the description of the flood conditions, Mr. Ferguson gave an address "Do Rate Reductions Per Se Stimulate Use—An Argument in the Negative."

The chairman of the 2 other general sessions were R. S. Judd and C. T. Hughes. The program and a list of the papers presented are given on page 417 of the April issue of Electrical Engineering, with the exception of 2 additional papers presented at the general session on Wednesday afternoon; these were "Aging in Copper Oxide Rectifiers" by E. A. Harty of the General Electric Company, and "Frequency Tripling Transformers" by J. L. Cantwell of the General Electric Company.

District Executive Committee Luncheon

The executive committee of the North Eastern District met at luncheon on Thursday, and held a brief business session afterward, with Vice President W. H. Timbie presiding.

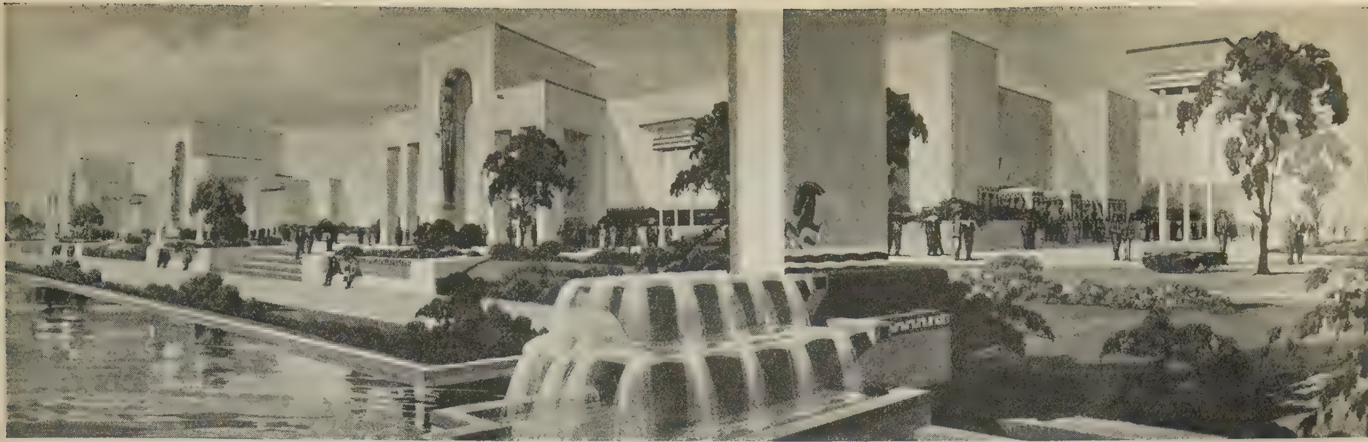
In connection with a discussion of general plans for a District meeting in May 1937, for which a request had been submitted, it was decided that the main headquarters should be in Buffalo, N. Y., but that it may

Future AIEE Meetings

Summer Convention,
Huntington Hotel, Pasadena, Calif., June 22–26, 1936

South West District Meeting,
Dallas, Texas, Oct. 26–28, 1936

Southern District Meeting,
Birmingham, Ala., Dec. 1936



The Texas Centennial Exposition to be held in Dallas from June 6 to November 29, 1936, offers an interesting side trip to many members on their way to or from the Institute's forthcoming summer convention at Pasadena, Calif.

be desirable to have headquarters at Niagara Falls for one day.

The committee decided to recommend to the A.I.E.E. publication committee that the address delivered at the Thursday morning session by Samuel Ferguson be published in ELECTRICAL ENGINEERING.

District Secretary A. C. Stevens reported upon the finances for the year and mentioned plans for a special car to Pasadena, Calif., with stops at certain points of special interest, for those attending the Institute's summer convention. He expressed the hope that some of the delegates of the Sections in the North Eastern District might wish to join the party.

ENTERTAINMENT

The main social event, which was held on Thursday and which was well attended, consisted of an informal dinner and meeting at the New Haven Lawn Club, followed by dancing. After brief remarks by Vice President Timbie, the toastmaster, and President E. B. Meyer, A. C. Stevens, secretary-treasurer of the North Eastern District, presented the District prizes for papers presented during the calendar year 1935. The prize for the best paper was presented to H. D. Braley and J. L. Harvey for their paper entitled "Fault and Out-of-Step Protection of Lines." The prize for initial paper was awarded to H. M. Cushing for his paper entitled "Design and Operation of Huntley Station No. 2." The prize for Branch paper was presented to W. R. Harry of Cornell University for his paper entitled "A High Sensitivity Vacuum Tube Voltmeter."

After the prizes were presented the toastmaster introduced Prof. H. E. Edgerton, of Massachusetts Institute of Technology, Cambridge, who gave a very interesting demonstration showing motion pictures of various slow motion effects produced by means of the high speed motion picture camera employing stroboscopic light. Some of the slow motion pictures, such as a carrier pigeon taking off on flight, the analysis of golf and tennis strokes, and the dropping of a ball bearing into a pail of milk, produced striking effects of extreme grace and beauty. This development and investigation which is being conducted at M.I.T. may prove to be of considerable scientific value, particularly in the field of aeronautics.

During the dinner the orchestra rendered classical selections, and after the demonstration dancing was enjoyed.

On Wednesday, the preceding evening, many members and guests enjoyed a play "The Mercenary Match," presented by the Yale department of drama, which was very well done. The play, with its setting in Boston, Mass., in the year 1786, possessed considerable historical interest, as it was the first play written by a student of Yale University, Barnabas Bidwell, class of 1786. "The Mercenary Match," originally described as a domestic tragedy, is thought by some to have been intended deliberately as a satire on contemporary English dramas.

In addition to the foregoing entertainment, the ladies' entertainment committee arranged a busy program, consisting of a tea, a drive through New Haven parks, a visit to the gallery of the Yale School of Fine Arts, inspection of the University buildings, and a luncheon-bridge at the Faculty Club.

STUDENT TECHNICAL SESSION

On Friday morning a student technical session was held, which was well attended by a large delegation of students and interested engineers. Nine technical papers were presented by the students, and each paper was from a different Student Branch representing more than half of the 16 Branches within the District. Most of the papers had been given considerable care in preparation and attention was given also to delivery and presentation. Richard Porter of Yale University acted as chairman. The following papers were presented:

THE CATHODE RAY TUBE AS A VOLTMETER AT ULTRA-HIGH RADIO FREQUENCIES, L. L. Libby, Worcester (Mass.) Polytechnic Institute.

ELECTRICITY IN THE FIELD OF MEDICINE, T. N. Wilcox, Massachusetts Institute of Technology, Cambridge.

EARLY EVENTS IN ALTERNATING CURRENT HISTORY, F. B. Hunt, University of Vermont. Burlington.

A HIGH SPEED VIBRATION ANALYZER, W. R. Harry, Cornell University, Ithaca, N. Y.

FREQUENCY MULTIPLIERS, R. F. Huminski and E. M. Williams, Yale University, New Haven, Conn.

EXTENDING THE SCOPE OF THE CATHODE RAY OSCILLOGRAPH FOR LABORATORY USE, R. K. Saxe and R. B. Shimer, Northeastern University, Boston, Mass.

THE EFFECTS OF OXIDATION AND MOISTURE ON INSULATING OILS, J. A. Kaup, Tufts College, Mass.

INDUCTION HEATING AT COMMERCIAL FREQUENCY, E. G. Ball, Rensselaer Polytechnic Institute, Troy, N. Y.

RANDOM OBSERVATIONS OF THE BRUSH-SHIFT ADJUSTABLE SPEED MOTOR, R. P. Richard, III, Clarkson College of Technology, Potsdam, N. Y.

President Meyer addressed the students, remarking that good men are needed now as they always will be. He emphasized the importance of character development and explained that one of the failings of engineers in general was the lack of ability to express themselves clearly, but that the Institute through its work in the Sections and Branches offers engineers and students a means of development along these lines.

At the conclusion of the session the judges considered the 9 presentations and awarded the first cash prize of $10 to L. L. Libby, Worcester Polytechnic Institute; second prize of $5 to T. N. Wilcox, Massachusetts Institute of Technology, and third prize of $3 to E. G. Ball, Rensselaer Polytechnic Institute. The awards were based only on the presentations of the papers and not on their contents.

LUNCHEON CONFERENCE OF COUNSELORS AND BRANCH CHAIRMEN

The Branch counselors and chairmen, District officers, and a few guests attended a luncheon meeting on Friday, which was held as the annual District conference on student activities. Prof. W. B. Hall, chairman of the District committee on student activities, presided. Announcement of the award of prizes for papers presented during the student technical session, as reported in another paragraph, was made.

There was an extensive discussion of the importance of the national and District prizes offered by the Institute, and Branch counselors and chairmen were urged to encourage students to submit more papers for consideration. The desirability of having suitable prizes available for graduate students rather than having them compete with practicing engineers for the initial paper prizes, or having undergraduates in competition with graduates, was discussed

in considerable detail. The conference voted to recommend that an additional prize be established for graduate students who are enrolled in the Institute.

The difficulties encountered in the grading of student papers were discussed, and the chairman was requested to appoint a committee to make recommendations.

There was some discussion of the desirability of including in ELECTRICAL ENGINEERING more papers of special interest to students. However, it was thought by several that the seniors and recent graduates are in better position to understand many of the technical papers than are the practicing engineers whose work has taken them away from the more theoretical aspects of electrical engineering.

Prof. F. N. Tompkins, counselor of the Brown University Branch, was elected chairman of the District committee on student activities for the year beginning August 1, 1936, and Profs. E. M. Strong and E. A. Walker, counselors of the Cornell University and Tufts College Branches, respectively, were elected to serve with him on the counselor's executive committee.

INSPECTION TRIPS

A number of interesting inspection trips to places of interest and to industries of New Haven and vicinity were offered by the inspection trips committee under the chairmanship of E. D. Lynch. On Thursday afternoon a large number of members and students inspected transportation equipment of the New York, New Haven and Hartford Railroad at the New Haven Station and the plants of the Wallingford Steel Company and the R. Wallace and Sons Manufacturing Company.

At the New Haven station modern electric freight and passenger locomotives and a mountain type steam locomotive were inspected, as well as the Ansonia, Derby, and Birmingham locomotive which was operated in 1888 and was probably the first electric freight locomotive in existence. This locomotive, which was on the same track with the modern locomotives, and which was similar to an early horse-drawn car but propelled by a 600 volt d-c motor and a chain drive, strikingly depicted the progress in motive power for transportation during the past 50 years.

Later, a special air-conditioned train was taken to Wallingford where the various sizes of mills for converting flat hot-rolled strip steel into flat cold-rolled strip steel of different degrees of hardness were inspected during several stages of operation. After inspection of the steel mill, the adjacent plant of R. Wallace and Sons Manufacturing Company, where, after pickling, the steel is punched out into a variety of kitchen utensils, knives, forks, and spoons, was visited. Courteous guides explained the various operations of tinning and plating and most of the machinery employed was designed by the engineers of this company. The production of silverware from the ingot to the finished product were seen in the various stages of process. Silver shod spoons were given as souvenirs to those in the party, and the ladies on the trip found the finished products of silverware on display of considerable interest.

On Friday evening the equipment building of Southern New England Telephone Company was visited by a number of members and students. This telephone exchange serves approximately 36,000 terminals. The inspection offered those not directly connected with the communication field an opportunity to see the equipment for handling a large number of messages expediently, which so frequently are put forth without appreciating the intricacies involved.

On Saturday morning a number visited the plant of the Sikorsky Aircraft Company in Bridgeport, where the detailed construction of airplanes was seen first hand in all stages of production. It was at this plant where large transport planes, such as the type of the Pan American Clipper, were fabricated. The trip was evidence of the increasingly popular interest in aeronautics.

Analysis of Registration at North Eastern District Meeting

Classification	Connecticut Section	District No. 1*	Other Districts	Total
	Location			
Members	54	62	22	138
Students	17	103	9	129
Men Guests	3	8	4	15
Women Guests	13	12	3	28
Totals	87	185	38	310

*Exclusive of Connecticut Section Territory.

Student Conference Held by Southern District

A conference of Student Branches in the Institute's Southern District (Number 4) was held April 16–18, 1936, at Clemson College, S. C. The Branches in the District were well represented, 142 counselors and Enrolled Students being present.

The program included several technical papers by Students, social and entertainment features, and inspection trips to Saluda Dam and Tallulah Falls hydroelectric plants. The following student papers were presented:

A VACUUM TUBE VOLTMETER, C. T. Coffey, Georgia School of Technology.

A HISTORY OF ALTERNATING CURRENT IN THE UNITED STATES, H. W. Jeffcock, University of Alabama, University.

A SELF-CONTAINED A-C IMPEDANCE BRIDGE, Ben Ragland, University of Kentucky, Lexington.

THE LOCATION OF TROPICAL STORMS BY MEANS OF THE CATHODE RAY OSCILLOGRAPH, Wayne Mason, University of Florida, Gainesville.

ENGINEERING EDUCATION AT THE UNIVERSITY OF FLORIDA, A. C. Ewert, University of Florida, Gainesville.

BRINGING RADIO TO THE RURAL HOME, G. F. Rogers, The Clemson Agricultural College, S. C.

The papers presented by the students who had won places on the program were of high order. Prizes were awarded as follows: G. F. Rogers, first; Wayne Mason, second; and C. T. Coffey, third.

The next conference will be held in Birmingham, Ala., in connection with the Institute's forthcoming Southern District meeting which is scheduled to be held early in December 1936. The Alabama Polytechnic Institute and University of Alabama Branches will be joint hosts.

E.C.P.D Seeks Uniformity in Engineering Degrees

On several occasions, educators in the field of engineering and members of engineering societies have concerned themselves with the lack of uniformity in engineering degrees and the consequent confusion arising from the various practices of engineering schools. The Society for the Promotion of Engineering Education has had 3 different committees and reports on the subject, and the Engineers' Council for Professional Development has recognized the problem in the deliberations of its committee on professional recognition.

The latest reports of the S.P.E.E. committee on degrees in engineering, presented at the Ithaca, N. Y., meeting in 1934, contained provisions for unifying the names of, and requirements for, engineering degrees, including honorary degrees. The committee on professional recognition of E.C.P.D. heartily concurred in this report, with one exception, and urged its adoption as a step in clarifying the indefiniteness of the present situation. The single exception was taken to the proposal to abandon completely the professional degree, and E.C.P.D. expressed the hope that this proposal would receive further consideration by S.P.E.E. before adoption.

Using civil engineering as an example typifying the lack of uniformity and confusion in all engineering degrees, S.P.E.E. found the following forms of the first, or bachelor's, degree:

Bachelor of Arts
Bachelor of Science
Bachelor of Science in Engineering
Bachelor of Science in Civil Engineering
Bachelor of Engineering
Bachelor of Civil Engineering
Civil Engineer

In contrast to this diversity, the S.P.E.E. committee recommended the following plan for engineering degrees, again using civil engineers as an example:

Type	Degree
Bachelor's	Bachelor of Civil Engineering (B.C.E.)
Master's	Master of Civil Engineering (M.C.E.)
Doctor's	Doctor of Civil Engineering (D.C.E.)
Honorary	Master of Engineering (M.Eng.)
Honorary	Doctor of Engineering (D.Eng.)
Professional	Civil Engineer (abandon as earned or honorary)

In order to get a consensus of opinion on this plan, S.P.E.E. submitted the recommendations to letter ballot, with the following results:

Degrees Earned in Course	Votes in Favor	Per Cent	Votes Opposed
B.C.E.	657	81	152
M.C.E.	650	81	150
D.C.E.	639	80	156
Honorary Degrees			
M.Eng.	666	85	116
D.Eng.	702	90	78
Discontinue Professional			
C.E.	474	61	304

Total ballots numbered 813 out of a membership of 2,325, representing 160 institutions, with 50 voters outside of faculties. The significance of the ballot is outstanding, and the results more than ordinarily conclusive. With the exception of the vote favorable to abandonment of the professional degree (C.E.) the results are also in accord with the preference of the committee on professional recognition of E.C.P.D. The latter organization had proposed that the professional degree might be granted only to experienced engineers who otherwise had met the requirements for certification into the profession.

1936 Lamme Medal Nominations Due Nov. 1

Special attention is directed to the fact that the names of Institute members who are considered eligible for the Lamme Medal, to be awarded in the fall of 1936, may be submitted by any member in accordance with Section 1 of Article VI of the by-laws of the Lamme Medal committee, as quoted in the following:

> The committee shall cause to be published in one or more issues of Electrical Engineering, or of its successors, each year, preferably including the June issue, a statement regarding the "Lamme Medal" and an invitation for any member to present to the national secretary of the Institute by November 1, the name of a member as a nominee for the medal, accompanied by a statement of his "meritorious achievement" and the names of at least 3 engineers of standing who are familiar with the achievement.

Each nomination should give concisely the specific grounds upon which the award is proposed, and also a complete detailed statement of the achievements of the nominee to enable the committee to determine its significance as compared with the achievements of other nominees. If the work of the nominee has been of a somewhat general character in co-operation with others, specified specific information should be given regarding his individual contributions. Names of endorsers should be given as specified above.

The Lamme Medal, founded as a result of a bequest of the late Benjamin Garver Lamme, chief engineer of the Westinghouse Electric and Manufacturing Company (deceased July 8, 1924), provides for the annual award by the Institute of a gold medal —together with bronze replica thereof— to a member of the A.I.E.E. "who has shown meritorious achievement in the development of electrical apparatus or machinery"; and for the award of 2 such medals in some years if the accumulation of funds warrants.

The eighth (1935) Lamme Medal has been awarded to Dr. Vannevar Bush (A'15, M'19, F'24), vice president of the Massachusetts Institute of Technology, and dean of the school of engineering, "for his development of methods and devices for application of mathematical analysis to problems of electrical engineering." Presentation will be made during the A.I.E.E. summer convention at Pasadena, Calif., June 22–26, 1936. A brief biographical sketch of Dr. Bush appeared in Electrical Engineering for March 1936, page 313.

Student Conference Held by North Central District

On Saturday, April 18, 1936, the ninth annual conference of Student Branches in the A.I.E.E. North Central District (Number 6) was held at the University of Colorado, Boulder. In conjunction with the Student Conference, a technical conference on electric power and communication problems was held on Friday, April 17, under sponsorship of the university. This was attended by about 190, which included most of those attending the Student Branch conference as well as many members of the Institute's Denver Section.

About 40 delegates and guests attended the Student conference on Saturday morning. R. H. Fair, vice president of the A.I.E.E. North Central District, presided. The following topics were discussed:

1. How to sell A.I.E.E. to the students.
2. Should A.I.E.E. meetings be given credit for seminar?
3. Should A.I.E.E. meetings assume a more social status, and should a larger fee be charged for local programs?
4. How frequently should meetings be held?
5. Should underclassmen be encouraged to attend meetings, or should the meetings be limited to upperclassmen?

In the discussion of topic 1, various means of making the student cognizant of the many advantages of Institute membership were advanced. The value of interesting meeting programs was stressed. In discussing topic 2, the question involved was unanimously answered in the negative. In regard to topics 3 and 4, it was generally agreed that meetings should involve a fellowship among the engineers not of a purely social nature, but rather in the form of a men's club for the discussion of professional progress; it was generally agreed that meetings should be held about every 2 weeks. A general conclusion concerning topic 5 was that a compromise program for upperclassmen and underclassmen should be arranged.

Prof. W. H. Gamble of the South Dakota State College, Brookings, was elected chairman of the committee on Student activities for the coming year, and it was decided that the conference of 1937 should be held at that institution.

The technical conference held on Friday included the following papers:

Methods of Suppressing Faults on Power Systems, W. W. Lewis (A'09, M'13) General Electric Company, Schenectady, N. Y.

Electrical Features of the Boulder Canyon Power Plant, L. N. McClellan (A'14, M'26) U.S. Bureau of Reclamation, Denver, Colo.

Wood Poles, Pests, and Preservatives, R. W. Linsday (A'35) Mountain States Telephone and Telegraph Company, Denver, Colo.

Earth Characteristics in Regard to Electric Currents, L. M. Robertson (A'27) Public Service Company of Colorado, Denver.

Papers by electrical engineering faculty members of the University of Colorado included:

Telemetering Systems, C. M. McCormick (M'23).

Electric and Magnetic Field Maps, H. B. Palmer (A'25, M'35).

Noise Silencing in Radio Receiving Systems, W. L. Cassell (A'25).

A New Type of Oil-Less Circuit Breaker, Frank Eastom (A'21, M'35).

The conference program also included an inspection of the electrical engineering laboratories at the university, a demonstration of a new circuit breaker, and operation of a lightning impulse generator, grid controlled rectifier tubes, and a high voltage d-c transmission system. The conference was under the general direction of Dean H. S. Evans and Prof. W. C. Du Vall of the department of electrical engineering of the University of Colorado. Donald M. Nicholson, chairman, University of Colorado Branch, acted as secretary of the conference in the absence of the District secretary.

Membership—

Mr. Institute Member:

Your participation in the work of the membership committee by sending in names, together with the splendid co-ordinated activity of the Section membership committees, has resulted to give a net increase in membership for the fiscal year ending April 30, 1936, which is the first yearly increase recorded since 1931. The figures are:

As of April 30, 1935	As of April 30, 1936
14,253	14,600

We ask for your continued participation in sending in names as requested in our letter of April 8.

Chairman, National Membership Committee

Section and Branch Activities

Annual Report for 1935–36

THE following constitutes the annual report on Institute Section and Branch activities for the fiscal year which ended April 30, 1936. Similar information for 3 preceding fiscal years appeared in ELECTRICAL ENGINEERING for June 1935, pages 674–75; June 1934, pages 1027–29; and June 1933, pages 726–28.

The names of present members of the Sections committee and the committee on Student Branches, which supervise the 2 important divisions of Institute activities covered by this report, are: Sections—I. M. Stein, *chairman*, Mark Eldredge, O. W. Eshbach, F. A. Hamilton, Jr., A. P. Hill, W. H. Timbie, and, *ex-officio*, the chairmen of all Sections of the Institute. Student Branches—O. W. Eshbach, *chairman*, R. B. Bonney, L. A. Doggett, F. O. McMillan, C. F. Scott, G. C. Shaad, W. H. Timbie, and, *ex-officio*, all Student Branch counselors.

SECTION ACTIVITIES

Nearly all the Sections carried on a normal amount of activity during the year, and, although no new Sections were organized, the total number of meetings held was the largest ever reported—540. Several prospective new Sections are being discussed.

Interest in technical groups and special technical meetings as means of supplying types of activities desired by members of Sections has materially increased. The New York and Chicago Sections attained the usual success in the conduct of their group activities. The special technical meetings of the San Francisco Section have successfully met a real need. The Portland Section organized 2 additional technical committees, (1) electrochemistry and electrometallurgy and (2) illumination, making a total of 5. The Cleveland Section organized a technical division dealing with motors and control, and expects to form other divisions later. The Niagara Frontier Section decided to organize 2 technical discussion groups, (1) generation, transmission, and distribution of electricity, and (2) electrical communication. The Schenectady Section held a series of 5 technical discussion meetings, each preceded by a dinner, with an average attendance of about 70. The Sharon Section held 2 technical discussion meetings.

The Boston and Lynn Sections held their annual joint meeting for competition among their members for prizes of $25, $20, $15, and $10, with 4 selected papers from each Section presented by their authors. The Pittsfield and Schenectady Sections held their seventh annual competition among members not over 30 years of age presenting their initial papers. Two prizes of $15 and $10 were presented at each of 2 meetings, one held in each city. The winning Section holds the F. F. Brand cup during the succeeding year. The Schenectady Section held its third annual test men's competition, with 6 papers presented and prizes of $10 and $5 awarded. The Seattle Section held its annual meeting for competition among its members for a $25 prize for best paper.

The power group of the New York Section has been conducting courses in structures, effective speaking, review of electrical engineering, and electronics. Courses in economics and business law are under consideration. A comprehensive report on the activities of the power group was published on pages 421–22 of ELECTRICAL ENGINEERING for April 1936.

The Philadelphia Section conducted a course in electronics limited to members of the Institute, during a period of 18 weeks, with a fee of $12 paid in advance. The enrollment was 53, more than twice the estimated number, and it was necessary to conduct the work in 2 classes. The enthusiasm was so great that the Section was requested to repeat the course next year, provide a continuation course, and offer courses covering mathematics review, electrical theory review, etc.

The St. Louis Section planned a joint meeting with the Student Branches at Washington University, Missouri School of Mines and Metallurgy, and University of Missouri, to be held on May 15, with prizes offered for the best papers. The Washington Section held its second annual "college night" meeting on April 16, with a program composed of talks on social and economic problems of interest to engineers by representatives of the 3 local Student Branches, including the Branch recently organized at the University of Maryland. A considerable number of Sections continued their well established practice of holding meetings with programs supplied by neighboring Student Branches.

The Urbana Section and the University of Illinois Branch held an unusually interesting joint meeting, combining a lecture on the electric organ with a recital on that instrument by a well known organist. The attendance was about 2,000.

Detailed information on Section meetings during the past year appears in table I, and table II contains a brief summary of such meetings held during the past 3 years.

BRANCH ACTIVITIES

In March 1936, a Student Branch was organized at the University of Maryland, bringing the total number to 118.

Many of the Branches held large numbers of meetings, and the total number—1,045—reported for the year exceeds the total of each of the 3 preceding fiscal years. Eleven Branches held more than 15 meetings each, 59 held from 8 to 15, 30 held from 4 to 7, and 14 held from 1 to 3, leaving among the total number of 118 Branches only 4 which reported no meetings.

Students presented 10 papers in 2 regular sessions of the Pacific Coast convention, and 15 papers during one of the sessions of the Great Lakes District meeting. Some of the Branches have shown increased interest in papers by students. Students have been keenly interested in opportunities to present

Table I—Section Meetings Held During Year Ending April 30, 1936

Section	A.I.E.E. Members: August 1934	A.I.E.E. Members: August 1935	Meetings During Year: Number	Meetings During Year: Average Attendance	Avg. Attendance as Per Cent of Membership, August 1935
Akron	69	64	8	64	100
Alabama	39	38	5	279	734
Atlanta	76	71	—		—
Baltimore	157	167	9	127	76
Boston	370	378	8	237	63
Central Indiana	85	94	7	135	144
Chicago	591	614	6	234	38
Power Group			5	114	
Cincinnati	140	143	9	397	278
Cleveland	200	220	8	130	59
Motor and Control Technical Division			2	61	
Columbus	58	62	6	75	121
Connecticut	237	225	7	133	59
Dallas	78	87	7	89	102
Denver	130	138	9	52	38
Detroit-Ann Arbor	246	268	10	155	58
Erie	49	48	6	51	106
Florida	40	47	3	127	270
Fort Wayne	52	55	9	75	136
Houston	49	65	10	58	89
Iowa	52	50	4	75	150
Ithaca	46	56	4	40	71
Kansas City	125	134	9	147	110
Lehigh Valley	176	185	9	178	96
Los Angeles	333	385	11	157	41
Louisville	46	56	9	44	79
Lynn	100	99	17	523	528
Madison	56	51	9	53	104
Memphis	29	31	10	42	135
Mexico	65	66	4	50	76
Milwaukee	153	173	19	134	77
Minnesota	76	74	7	95	128
Montana	29	34	11	21	62
Nebraska	46	51	1	60	118
New Orleans	42	44	3	213	484
New York	2,703	2,773	4	464	17
Communication Group			3	417	
Illumination Group			3	967	
Power Group			8	874	
Transportation Group			4	255	
Niagara Frontier	150	197	9	108	55
North Carolina	68	78	2	91	117
Oklahoma City	90	110	7	131	119
Philadelphia	512	517	9	241	47
Pittsburgh	383	379	7	137	36
Pittsfield	96	107	11	723	676
Portland	86	92	8	103	112
Communication			5	20	
Electrochemistry and Electrometallurgy			4	20	
Illumination			6	20	
Transmission and Distribution			12	20	
Providence	71	85	7	70	82
Rochester	59	71	15	99	139
St. Louis	173	182	8	119	65
San Antonio	30	26	7	42	162
San Francisco	359	373	8	95	25
Special Technical Meetings			5	69	
Saskatchewan	23	24	5	22	92
Schenectady	350	353	12	195	55
Technical Division Series			5	70	
Seattle	124	117	9	107	91
Sharon	50	55	8	92	167
Technical Discussion Meetings			2	32	
Spokane	32	44	8	50	114
Springfield, Mass.	69	65	8	100	154
Syracuse	62	65	3	330	508
Toledo	59	69	10	76	110
Toronto	287	286	13	102	36
Urbana	34	36	6	373	1,036
Utah	38	39	8	57	146
Vancouver	81	88	10	52	59
Virginia	81	86	4	66	77
Washington	173	229	8	223	97
Worcester	54	62	8	70	113
Total...61	10,337	10,881			

Total number of meetings 540
Total attendance 85,501

papers at Section and other meetings including many Institute members in the audiences.

The Purdue University Branch offered 2 prizes for seniors and 2 for juniors in the school of electrical engineering for the best papers on electrical engineering subjects. The first prize in each case was the Student enrollment fee in the A.I.E.E. for one year, and the second was the fee for 1/2 year. All papers to be considered were required to be submitted in advance, in order that judges might select 4 from

Table II—Section Meetings Held During Last 3 Fiscal Years

	Fiscal Year Ending April 30		
	1934	1935	1936
Number of Sections	61	61	61
Number of meetings held	472	521	540
Average number of meetings	7.7	8.5	8.9
Total attendance	73,271	73,381	85,501
Average attendance per meeting	156	141	158

Table III—Branch Meetings Held During Year Ending April 30, 1936

Branch	Meetings During Year: Number	Meetings During Year: Average Attendance	Approx. No. of Talks by Students
Akron, University of	3	11	—
Alabama, Polytechnic Institute	6	68	—
Alabama, University of	14	23	18
Arizona, University of	11	9	7
Arkansas, University of	20	29	30
Armour Institute of Technology	15	44	2
British Columbia, Univ. of	10	16	9
Brooklyn, Polytechnic Inst. of	8	62	4
Brown University	5	29	6
Bucknell University	6	13	2
California Institute of Tech	5	25	2
California, University of	5	58	—
Carnegie Institute of Tech	4	43	1
Case School of Applied Science	23	46	52
Catholic University of America	3	22	1
Cincinnati, University of	9	97	7
Clarkson College of Technology	3	22	—
Clemson Agricultural College	12	25	8
Colorado State Agri. College	7	14	3
Colorado, University of	9	68	—
Cooper Union (Day Division)	12	28	6
(Evening Division)	1	18	—
Cornell University	5	56	1
Denver, University of	11	40	2
Detroit, University of	5	64	—
Drexel Institute	12	16	2
Duke University	9	23	12
Florida, University of	12	48	2
George Washington University	10	34	4
Georgia School of Technology	9	46	—
Harvard University	4	35	—
Idaho, University of	7	34	1
Illinois, University of	12	612	2
Iowa State College	12	73	2
Iowa, University of	28	48	20
Johns Hopkins University	17	43	9
Kansas State College	10	45	8
Kansas, University of	13	37	9
Kentucky, University of	8	56	7
Lafayette College	7	13	12
Lehigh University	8	59	1
Lewis Institute	3	66	—
Louisiana State University	9	28	9
Louisville, University of	8	11	8
Maine, University of	3	21	4
Marquette University	12	37	6
Maryland, University of*	1	40	—
Massachusetts Inst. of Tech	5	106	6
Michigan College of Min. & Tech	8	43	1
Michigan State College	5	37	3
Michigan, University of	6	47	4
Milwaukee School of Engineering	11	64	4
Minnesota, University of	9	35	1
Mississippi State College	13	15	19
Missouri School of Mines & Met	9	76	—
Missouri, University of	5	33	2
Montana State College	27	25	70
Nebraska, University of	11	30	6
Nevada, University of	7	17	8
Newark College of Engineering	9	38	6
Hew Hampshire, University of	23	19	39
New Mexico, University of	5	11	2
New York, College of the City of (Day Division)	4	39	2
(Evening Division)	6	19	—
New York University (Day Division)	9	23	19
(Evening Division)	2	23	8
North Carolina State College	13	42	4
North Carolina, University of	8	22	9
North Dakota State College	3	11	—
North Dakota, University of	14	11	10
Northeastern University	8	40	3
Notre Dame, University of	9	63	12
Ohio Northern University	16	14	11
Ohio State University	9	58	1
Ohio University	9	24	—
Oklahoma A. & M. College	13	60	10
Oklahoma, University of	8	31	—
Oregon State College	8	62	—
Pennsylvania State College	7	48	4
Pennsylvania, University of	—	—	—
Pittsburgh, University of	12	70	5
Porto Rico, University of	7	31	1
Pratt Institute	10	48	10
Princeton University	3	6	—
Purdue University	7	137	—
Rensselaer Polytechnic Institute	7	94	4
Rhode Island State College	7	23	6
Rice Institute	10	18	4
Rose Polytechnic Institute	2	11	—
Rutgers University	4	—	—
Santa Clara, University of	8	36	3
South Carolina, University of	12	18	—
South Dakota State College	5	24	1
So. Dakota State School of Mines	2	21	—
South Dakota, University of	—	—	—
Southern California, Univ. of	16	20	4
Southern Methodist University	4	52	3
Stanford University	10	21	—
Stevens Institute of Technology	1	19	—
Swarthmore College	—	—	—
Syracuse University	20	43	18
Tennessee, University of	2	18	—
Texas A. & M. College	8	73	7
Texas Technological College	9	19	2
Texas, University of	—	—	—
Tufts College	7	36	—
Union College	5	30	—
Utah, University of	10	34	9
Vermont, University of	8	15	3
Villanova College	15	19	4
Virginia Military Institute	4	24	6
Virginia Polytechnic Institute	20	36	24
Virginia, University of	9	35	5
Washington, State College of	13	38	4
Washington, University of	14	50	10
Washington University	13	30	—
West Virginia University	16	26	90
Wisconsin, University of	5	80	4
Worcester Polytechnic Institute	3	23	—
Wyoming, University of	9	9	—
Yale University	3	24	3
Total	118		772

Total number of meetings 1,045
Total attendance 45,304

* Authorized by executive committee, March 9, 1936.

Table IV—Branch Meetings Held During Last 3 Fiscal Years

	Fiscal Year Ending April 30		
	1934	1935	1936
Number of Branches	113	117	118
Number of meetings held	1,015	986	1,045
Average number of meetings	9.0	8.4	8.9
Total attendance	41,772	36,629	45,304
Average attendance per meeting	41	37	43
Number of student talks	1,004	708	772

Table V—Comparison of Branch Activities by Districts

District	No. of Branches Jan. 1	Avg. No. Meetings Per Branch	Avg. Attendance Per Meeting	Approx. Avg. No. Student Talks per Branch	No. Branches Reporting 8 or More Student Talks
1	16	7.3	37	5.8	2
2	21	9.0	37	9.8	5
3	9	8.1	33	6.2	2
4	17	9.9	32	7.7	8
5	16	9.3	100	3.8	2
6	10	7.1	27	2.2	1
7	14	9.1	38	5.5	4
8	7	8.9	23	3.4	1
9	6	13.2	37	15.7	3
10	1	10.0	16	9.0	1

Table VI—Conferences on Student Activities

District	Location	Date
1	Rensselaer Polytechnic Institute, Troy, N. Y.	5/3/35
8 & 9	Seattle, Wash. (Pacific Coast Convention)	8/28/35
5	Purdue Univ., Lafayette, Ind. (Great Lakes District Meeting)	10/25/35
2	Pennsylvania State College, State College, Pa.	10/25/35
Pittsburgh Section	Pittsburgh, Pa.	1/14/36
4	Clemson College, Clemson, So. Carolina	4/16–18/36
6	University of Colorado, Boulder, Colo.	4/17–18/36

Table VII—Student Conventions

Sponsored by District	Location	Date	No. of Student Papers
1	Rensselaer Polytechnic Institute, Troy, N. Y.	5/4/35	12
8 & 9	Seattle, Wash. (Pacific Coast Convention)	8/28–30/35	10
5	Purdue Univ., Lafayette, Ind. (Great Lakes District Meeting)	10/25/35	15
2	Pennsylvania State College, State College, Pa.	10/26/35	6
Brown University	Providence, R. I.	12/14/35	...
Pittsburgh Section	Pittsburgh, Pa.	1/14/36	6
4	Clemson College, Clemson, So. Carolina	4/16–18/36	....
New York Section	New York	4/22/36	2

Table VIII—Section or Joint Section and Branch Meetings With Active Student Participation

Sections	Schools	Date	Student Talks	Attendance
St. Louis	Missouri School of Mines and Metallurgy, Univ. of Missouri, Washington Univ.	5/10/35	6	145
Cincinnati	Univ. of Cincinnati	5/14/35	6	97
Baltimore	Johns Hopkins Univ.	5/17/35	3	71
Utah	Univ. of Utah	5/20/35	4	40
Portland	Oregon State College	5/25/35	4	62
Vancouver	Univ. of British Columbia	12/2/35	1	31
Kansas City	Kansas State College, Univ. of Kansas	12/5/35	5	93
Houston	Rice Institute, A. & M. College of Texas	3/20/36	2	46
Madison	Univ. of Wisconsin	3/25/36	2	73
Spokane	Univ. of Idaho, State College of Washington	3/27/36	2	80
Los Angeles	Calif. Inst. of Technology, Univ. of Southern Calif.	4/14/36	5	127
Cleveland	Case School of Applied Science	4/16/36	2	140
Washington, D. C.	Catholic Univ. of America, George Washington Univ., Univ. of Maryland	4/16/36	3	150
Seattle	Univ. of Washington	4/21/36	3	79
Dallas	Southern Methodist Univ.	4/24/36	2	40
San Francisco	Univ. of California, Univ. of Santa Clara, Stanford Univ.	4/24/36	3	69
Totals—16 Sections, 26 Branches			53	1,343

each class for presentation at a Branch meeting in competition for the prizes.

During the past fiscal year, 1,991 students applied for enrollment, and the total enrollment on April 30 was 4,049. About 50 per cent of the 1,226 whose periods of enrollment expired at that time applied for admission as Associates.

Comprehensive information on Branch activities during the year is given in tables III to VII.

Sections and Branch Joint Meetings

As indicated hereinbefore, the keen interest in co-operation between Sections and Branches has continued to receive much emphasis, and the opportunities thus afforded the students are deeply appreciated by them. Table VIII contains brief information on some of the outstanding examples.

Institute Prize Awards Announced for 1935 Papers

Five national prizes for papers presented during the calendar year 1935 have been announced by the committee on award of Institute prizes, which consists of W. R. Smith (M'18, F'30), *chairman*, C. O. Bickelhaupt (M'22, F'28), R. N. Conwell (A'15, F'31), and F. M. Farmer (A'02, F'13). Each prize consists of a suitable certificate. Personal presentation of the prizes will take place at the opening session of the Institute's summer convention to be held at The Huntington Hotel, Pasadena, Calif., June 22–26, 1936.

Authors presenting papers which are eligible for future prizes should bear in mind the following: All papers approved by the technical program committee and presented at national conventions or District meetings will be considered for the national best paper prizes and the national initial paper prize automatically. All other papers which were presented at Section, Branch, or student meetings in order to receive consideration for national prizes must be submitted in triplicate with written communications to the national secretary on or before February 15 of the following year. Papers which are to receive consideration for District prize awards must be submitted in duplicate by the authors or by the officers of the Branch, Section, or District concerned to the District secretary on or before February 15 of the following year.

National Prizes

After due consideration of all highly recommended papers, the committee on award of Institute prizes made the following awards of national prizes for papers presented in 1935:

Best Papers

Best Paper in Engineering Practice. This prize was awarded to E. F. Scattergood (A'08, F'13) for his paper "Engineering Features of the Boulder Dam-Los Angeles Lines," published in Electrical Engineering for May 1935, pages 494–512, and discussed at the summer convention, Ithaca, N. Y., June 24–28, 1935.

Best Paper in Theory and Research. This prize was awarded to Lloyd Espenchied (A'18, F'30) and M. E. Strieby (M'22) for their paper "Wide Band Transmission over Coaxial Lines," published in Electrical Engineering for October 1934, pages 1371–80, and discussed at the winter convention, New York, N. Y., January 28–31, 1935.

Honorable mention was made of "Sparking Under Brushes of Commutator Machines" by R. E. Hellmund (A'05, F'13) and L. R. Ludwig (A'28) published in Electrical Engineering for March 1935, pages 315–21, and discussed at the summer convention, Ithaca, N. Y., June 24–28, 1935.

Best Paper in Public Relations and Education. This prize was awarded to Alex Dow (A'93, F'13) for his paper "On the Schooling of Engineers," published in Electrical Engineering for December 1934, pages 1589–91, and discussed at the winter convention, New York, N. Y., January 28–31, 1935.

Initial Paper

Prize for initial paper was awarded to R. N. Stoddard (M'34) for his paper "A New Timer for Resistance Welding," published in Electrical Engineering for October 1934, pages 1366–70, and discussed at the winter convention, New York, N. Y., January 28–31, 1935.

Honorable mention was made of "D-C Braking of Induction Motors" by F. E. Harrell (A'26, M'35) and W. R. Hough (A'35) published in Electrical Engineering for October 1934, pages 1366–70, and discussed at the winter convention, New York, N. Y., January 28–31, 1935.

Branch Paper

Prize for branch paper was awarded to P. H. Odessey for his paper "A Direct Current Controlled Transformer Regulator," presented at the annual student convention, District 3, New York, N. Y., April 22, 1935.

The committee on award of Institute prizes recognizes the excellent quality of the large number of papers presented during the calendar year. The committee wishes to commend all authors for the high standard of their contributions and also the technical committees for having sponsored a number of the papers and having graded them at the time they were initially reviewed. This has been of valuable assistance to the committee in determining the most worthy papers for the awards.

District Prizes

District prizes as announced by 5 Districts to date include 3 awards of $25 each, together with appropriate certificates. Other District awards will be announced later, as the information becomes available.

District 1

Prize for best paper was awarded to H. D. Braley (A'18) and J. L. Harvey (A'25) for their paper "Fault and Out-of-Step Protection of Lines," published in Electrical Engineering for February 1935, pages 189–200, and discussed at the summer convention, Ithaca, N. Y., June 24–28, 1935.

Prize for initial paper was awarded to H. M. Cushing (A'06, F'35) for his paper "Design and Operation of Huntley Station No. 2," published in Electrical Engineering for June 1935, pages 632–45, and discussed at the summer convention, Ithaca, N. Y., June 24–28, 1935.

Prize for Branch paper was awarded to W. R. Harry for his paper "A High Sensitivity Vacuum Tube Voltmeter," presented at the Rensselaer Polytechnic Institute student convention, May 4, 1935.

District 2

Prize for best paper was awarded to L. M. Olmsted (A'30) for his paper "Improving Voltage Regulation on Distribution Feeders," presented at a meeting of Pittsburgh Section, February 12, 1935.

District 5

Prize for Branch paper was awarded to W. H. Budd for his paper "Design of Cornices for Built-in Lighting," presented at the Great Lakes District meeting, West Lafayette, Ind., October 24–25, 1935.

District 7

Prize for Branch paper was awarded to W. H. Mayne for his paper "An Alternator Voltage Regulator Utilizing a Nonlinear Circuit," presented at a joint meeting of the San Antonio Section and the University of Texas Branch, April 15, 1935.

DISTRICT 10

Prize for best paper awarded to D. G. Geiger (A'25, M'36) for his paper "The Transmission Design of Telephone Systems," presented at a meeting of the Toronto Section, January 11, 1935.

Prize for initial paper awarded to R. M. Morton (A'35) for his paper "Torque in a Bipolar Induction Meter," published in ELECTRICAL ENGINEERING for April 1936, pages 354–8, and presented at a meeting of the Vancouver Section, December 2, 1935.

A.I.E.E. Directors Meet at Institute Headquarters

The regular meeting of the board of directors of the American Institute of Electrical Engineers was held at Institute headquarters, New York, on May 25, 1936.

There were present: *President*—E. B. Meyer, Newark, N. J. *Past-President*—J. B. Whitehead, Baltimore, Md. *Vice-Presidents*—Mark Eldredge, Memphis, Tenn.; W. H. Harrison, Philadelphia, Pa.; F. O. McMillan, Corvallis, Ore.; R. H. Tapscott, New York, N.Y.; W. H. Timbie, Cambridge, Mass. *Director*—F. M. Farmer, New York, N. Y.; H. B. Gear, Chicago, Ill.; C. R. Jones, New York, N. Y.; W. B. Kouwenhoven, Baltimore, Md.; Everett S. Lee, Schenectady, N. Y.; L. W. W. Morrow, New York, N. Y.; G. C. Shaad, Lawrence, Kans.; A. C. Stevens, Schenectady, N. Y. *National Treasurer*—W. I. Slichter, New York; N. Y. *National Secretary*—H. H. Henline, New York, N. Y. Present by invitation—A. M. MacCutcheon, Cleveland, Ohio, nominee for president.

Minutes were approved of meetings of the board of directors held January 27, and the executive committee on March 9, 1936.

Report was made of the actions of the executive committee, under date of April 14, 1936, on applications, as follows: transferred 2 applicants to grade of Fellow; elected 14 and transferred 14 applicants to grade of Member; elected 468 Associates; enrolled 89 Students.

Reports were presented and approved of meetings of the board of examiners held March 25, April 23, and May 22, 1936. Upon the recommendation of the board of examiners, the following actions were taken: 2 applicants were elected and 4 were transferred to the grade of Fellow; 21 applicants were elected and 24 were transferred to the grade of Member; 292 applicants were elected to the grade of Associate; 184 Students were enrolled.

Disbursements as follows were reported by the finance committee and approved: March, $16,602.76; April, $21,208.18; May, $25,362.34.

Upon the recommendation of the committee on co-ordination of Institute activities, the following schedule of meetings for 1937 was adopted:

Winter convention—New York, N. Y.—January 25–29

Summer convention—Milwaukee, Wis.—June 21–25

Pacific Coast convention—Spokane, Wash.—date to be determined later

North Eastern District meeting—Buffalo, N. Y.—in May

Middle Eastern District meeting—Akron, Ohio—in the fall

The board approved a recommendation of the standards committee that the publication and distribution of A.I.E.E. Standards Numbers 5, 7, 8, 9, and 10 be discontinued, as they have been superseded by Number C-50, American Standard for Rotating Electrical Machinery.

Upon the recommendation of the standards committee, approval was given to new 60 cycle values to supersede material in the present A.I.E.E. Standard Number 4, "Measurement of Test Voltage in Dielectric Tests," these values having been revised by the subcommittee on sphere gap calibrations of the committee on instruments and measurements. Approval was given also to the publication, in an informal manner, in ELECTRICAL ENGINEERING of pertinent explanations regarding transients, with the understanding that such material is not standard at the present time.

A petition for authority to organize an East Tennessee Section was presented; and, upon the recommendation of the Sections committee, the formation of such Section was authorized.

A report of the special committee on celebration (in March 1936) of the fiftieth anniversary of the establishment of the alternating current system in America was presented and accepted with appreciation.

The committee on award of Institute prizes presented a report of the awards for papers presented in 1935. (The report is published elsewhere in this issue.)

The appointment by the president of the following committee of tellers to canvass and report upon the ballots for the election of Institute officers was approved: Henry Kurz (chairman), W. E. Coover, G. F. Drum, M. S. Mason, O. F. Olsen, C. H. Thomas, and H. R. Wemple, Jr.

The annual report of the board of directors to the membership, prepared by the national secretary, was approved for presentation at the annual meeting of the Institute to be held in Pasadena, Calif., June 22, 1936. Contained therein were reports of general standing committees.

The annual report of the national treasurer was presented and accepted.

In accordance with Section 37 of the Constitution, the appointment of a national secretary for the administrative year beginning August 1, 1936, was made. National Secretary H. H. Henline was reappointed.

H. R. Woodrow was reappointed a representative of the Institute on the board of

A.I.E.E. Schenectady, N. Y., Section Hears Tenth Steinmetz Memorial Lecture

AT A meeting of the Institute's Schenectady, N. Y., Section on April 20, 1936, the tenth Steinmetz Memorial Lecture was delivered by Gerard Swope (A'99, F'22), president, General Electric Co. Full text of the lecture appears on pages 572–4, of this issue. Those shown above attended a dinner at the Mohawk Club, preceding the lecture. Standing, left to right: B. W. Bullock, C. M. Foust (M'22, F'31), C. F. Garis, E. M. Hunter (A'28), C. W. La Pierre (A'28), J. L. R. Hayden, R. C. Muir (A'08, M'19), E. O. Shreve (A'06), Owen D. Young, D. R. Fox, F. A. Hamilton (A'28, M'36), S. A. Holme, Alan Howard (A'28, M'35), and B. R. Prentice (A'35). Sitting, left to right: L. A. Hawkins (A'03, M'13), E. J. Berg (A'94, F'13), J. W. Butler (A'31), A. J. Nerad, Gerard Swope (A'99, F'22), H. H. Race (A'24, M'31), F. N. Neal, C. E. Kilbourne (A'29), and O. C. Rutledge (A'33).

trustees of the United Engineering Trustees, Inc., for the 4 year term beginning in October 1936, and William B. Jackson was reappointed an Institute representative on the Commission of Washington Award for the 2 year term beginning August 1, 1936.

Directors C. R. Jones and L. W. W. Morrow were appointed the 2 official delegates of the Institute to the Third World Power Conference, Washington, D. C., September 7-12, 1936.

Other matters were discussed, reference to which may be found in this or future issues of ELECTRICAL ENGINEERING.

Additions to Member-for-Life List

Membership for life is granted by the Institute for either of the following 2 reasons: a member has paid annual dues for 35 years; or has reached the age of 70 and has paid dues for 30 years.

The fiftieth anniversary issue of ELECTRICAL ENGINEERING published in May 1934 and the September 1935 number included lists of all members who had been enrolled upon the Institute records as Members for Life during the period up to May 1, 1935.

The list that follows indicates those members who have become Members for Life since the publication of the list in the September 1935 issue:

Amstutz, N. S.
Anderson, D. S.
Bergenthal, V. W.
Blakemore, M. N.
Bliss, L. D.
Clifford, H. E.
Cox, F. P.
Dean, G. C.
Deeds, E. A.
Dodd, S. T.
Edstrom, J. S.
Farnsworth, A. J.
Gille, H. J.
Hassler, C. I. F.
Haubrich, A. M.
Hawks, H. D.
Heitman, E.
Henry, I. W.
Howe, W. K.
Howes, R.
Howland, L. A.
Hoxie, G. L.
Hulse, W. S.
Kirker, H. L.
Leeds, M. E.
Lowther, C. M.
McBerty, F. R.
McLimont, A. W.
Nicholson, S. L.
Northrup, E. F.
Poirier, A. E.
Powell, E. B.
Rice, M. P.
Sanville, H. F.
Snow, J. E.
Sparks, C. P.
Thomas, P. H.
Thompson, E. J.
Thornton, K. B.
Tidd, G. N.
Tingley, E. M.
Twining, W. S.
Van Vleet, R. M.
Watson, A. E.
Whitney, W. R.
Wintner, L.
Woodbridge, J. L.
Woodworth, P. B.
Zabel, M. W.

Membership

Recommended for Transfer

The board of examiners, at its meeting on May 22, 1936, recommended the following members for transfer to the grade of membership indicated. Any objection to these transfers should be filed at once with the national secretary.

To Grade of Fellow

Honaman, R. K., protector methods engr., Bell Tel. Labs. Inc., New York.
Muir, Roy C., vice pres. in charge of engg., Gen. Elec. Co., Schenectady, N. Y.

2 to grade of Fellow

To Grade of Member

Berberich, L. J., research E.E., Socony-Vacuum Oil Co., Paulsboro, N. J.
Black, W. L., member of technical staff, Bell Tel. Labs., Inc., New York.
Bousman, H. W., development engr., Gen. Elec. Co., Schenectady, N. Y.
Brandon, M. M., associate E.E., Underwriters' Labs., New York.
Dowell, J. C., in charge of high voltage engg. lab., Gen. Elec. Co., Pittsfield, Mass.
Harness, G. T., instructor in elec. engg., Columbia Univ., New York.
Jebb, Thomas K., head of dept. of E.E. and Mathematics, Tech. Coll., Launceston, Tasmania, Australia.
Journeaux, D., elec. engr. & patent attorney, Allis Chalmers Mfg. Co., Milwaukee, Wis.
Kaylor, P. P., elec. engr., Olympic Portland Cement Co., Bellingham, Wash.
Kingsley, C., Jr., instructor in E.E., Mass. Inst. of Tech., Cambridge.
Knack, W. L., asst. plant engr., Dunlop Tire & Rubber Co., Buffalo, N. Y.
Koontz, R. M., vice pres., Koontz-Wagner Elec. Co., South Bend, Ind.
Martin, B. H., supt. of operations, Metropolitan Water District of Southern Calif., Banning, Calif.
McConnell, E. S., wire & cable engr., Anaconda Wire & Cable Co., Hastings-on-Hudson, N. Y.
McCreary, H. J., chief engr., Leich Elec. Co., Genoa, Ill.
O'Connell, E. J., telephone engr., Am. Tel. & Tel. Co., New York.
Pike, O. W., E.E., Gen. Elec. Co., Schenectady, N. Y.
Potts, J. A., system engr., The Milwaukee Elec. Ry. & Lt. Co., Milwaukee, Wis.
Reynolds, R. W., chief E.E., Venezuela Gulf Oil Co., Maracaibo, Venezuela, So. America.
Robinson, P. H., power sales engr., Houston Ltg. & Pwr. Co., Texas.
Scobie, J. K., system control engr., Shanghai Pwr. Co., Shanghai, China.
Stark, I. A., engg. asst. Brooklyn Edison Co., Brooklyn, N. Y.
Steinmayer, A. G., vice pres. in charge of engg., Line Material Co., So. Milwaukee, Wis.
Van Horn, R. H., mgr., The United Illuminating Co., Bridgeport, Conn.
Zimmermann, J. E., equipment program supervisor, Wisconsin Tel. Co., Milwaukee.

25 to grade of Member

Applications for Election

Applications have been received at headquarters from the following candidates for election to membership in the Institute. If the applicant has applied for direct admission to a grade higher than Associate, the grade follows immediately after the name. Any member objecting to the election of any of these candidates should so inform the national secretary before June 30, 1936, or Aug. 31, 1936, if the applicant resides outside of the United States or Canada.

Barksdale, G. R. (Member), 230 E. Cambridge St., Greenwood, S. C.
Bergmann, W. F., Byllesby Elec. & Mfg. Corp., Chicago, Ill.
Binford, H. F. (Member), Howard Univ., Washington, D. C.
Bronson, F. M. (Member), Am. Tel. & Tel. Co., New York, N. Y.
Bronstein, J. L., Cutler Hammer Co., Milwaukee, Wis.
Brooks, C. J., Cleveland Elec. Illuminating Co., Ohio.
Broughton, L. I., Monsanto Chem. Co., Anniston, Ala.
Brown, W. B. (Member), Vanderbilt Univ., Nashville, Tenn.
Bushnell, R. J., Commonwealth Edison Co., Chicago, Ill.
Coolidge, F. C., 1779 The Alameda, San Jose, Calif.
Coomes, E. A., Univ. of Notre Dame, Ind.
Denmead, H., West Penn Pwr. & Lt. Co., Pittsburgh, Pa.
Dunkelberger, B. W. Jr., 1529—4th Ave., So., Fargo, N. D.
Eddie, W. Jr., 285 Machray Ave., Winnipeg, Manitoba, Can.
Gault, T. C., 8719 So. Racine Ave., Chicago, Ill.
Greenhut, J., Fred Polster Elec. Co., Cleveland, Ohio.
Gresko, J. S., S. K. Wellman Co., Cleveland, Ohio.
Hagen, A. C., 1314 N St., N. W., Washington, D. C.
Hector, J. R., Gen. Elec. X-Ray Corp., Berkeley, Calif.
Heine, G. M., Cutler Hammer Inc., Milwaukee, Wis.
Hejda, C. J. (Member), Commonwealth Edison Co., Chicago, Ill.
Holleran, J. T., Gen. Elec. Co., Pittsfield, Mass.
Johnson, R. P., Kuhlman Elec. Co., Bay City, Mich.
Kahler, F. C., Bethlehem Steel Co., Baltimore, Md.
Kelso, F., Westinghouse Elec. & Mfg. Co., Philadelphia, Pa.
Kleinsmith, C. W., Am. Dist. Telegraph Co., Cleveland, Ohio.
Krantz, C. H., 329 Lexington Ave., El Monte, Calif.
Kruse, G. A., Hospital Specialty Co., Cleveland, Ohio.
Langfelder, H., Ray Oil Burner Co., San Francisco, Calif.
Luce, D. C., Pub. Serv. Elec. & Gas Co., Kearny, N. J.
Marson, J., 99 Clifton Terrace, Weehawken, N. J.
Martin, A. F., Bausch & Lomb Optical Co., Rochester, N. Y.
McKee, L. G., Moncton Electricity & Gas Co., Ltd., Moncton, N. B., Can.
Mechler, H. B., Intl. Business Machines Corp., Cleveland, Ohio.
Millard, J. A., Pub. Serv. Comm. of West Va., Charleston.
Mitchell, J. D., Consolidated Mining & Smelting Co., Trail B. C., Can.
Morawetz, R. J., Harnischfeger Corp., Milwaukee, Wis.
Nathan, R., Midwest Radio Corp., Cincinnati, Ohio.
Penney, W. M., Union Elec. Lt. & Pwr. Co., St. Louis, Mo.
Ramsey, L. W., Hartford Steam Boiler Insp. & Ins. Co., Baltimore, Md.
Rebholz, J. T. Jr., N. Y. State Employment Serv., New York, N. Y.
Reid, W. E. (Member), Bell Tel. Labs. Inc., New York, N. Y.
Ross, F. (Member), Room 402, Union Station, St. Louis, Mo.
Sachs, S. C., 817 No. 9th St., St. Louis, Mo.
Sherman, J. B., RCA Mfg. Co., Harrison, N. J.
Shirreffs, A., Westinghouse Elec. & Mfg. Co., New York, N. Y.
Singer, F. J., Bell Tel. Labs., New York, N. Y.
Snelson, F. O., Union Elec. Lt. & Pwr. Co., St. Louis, Mo.
Soliday, H., Westinghouse Elec. & Mfg. Co., St. Louis, Mo.
Stewart, H. C. (Member), Gen. Elec. Co., Pittsfield, Mass.
Sunde, A. M., Cornucopia, Ore.
Zornik, A. W., Utah Pwr. & Lt. Co., Salt Lake City.

52 Domestic

Foreign

Bhowmik, D. N., Dacca Elec. Supply Co. Ltd., Bengal, India.
Godinez, S. H., Gen. Hershey Corp., Central Hershey, Prov. Habana, Cuba.
Guimaraes, A. de S. M. (Member), Cia Telephonica Brasileira, Rio de Janeiro, Brazil, S. A.
Menge, W. H., Sao Paulo Elec. Co. Ltd., Sorocaba, Sao Paulo, Brazil, S. A.
Vyas, B. D., Andhra Paper Mills Co. Ltd., Rajahmundry, So. India.

5 Foreign

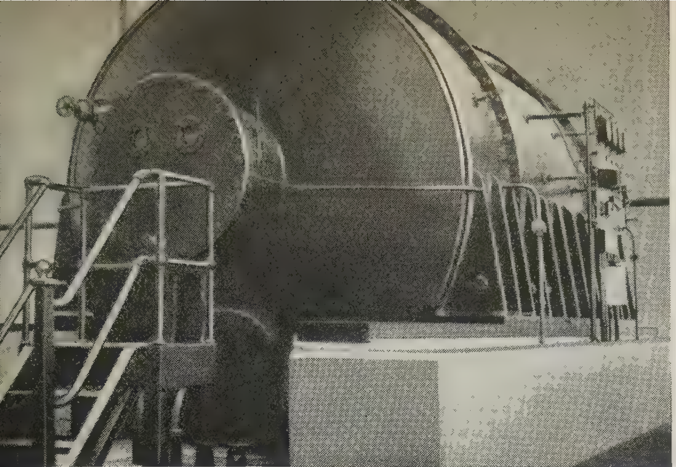
Addresses Wanted

A list of members whose mail has been returned by the postal authorities is given below, with the addresses as they now appear on the Institute record. Any member knowing of corrections to these addresses will kindly communicate them at once to the office of the secretary at 33 West 39th St., New York, N. Y.

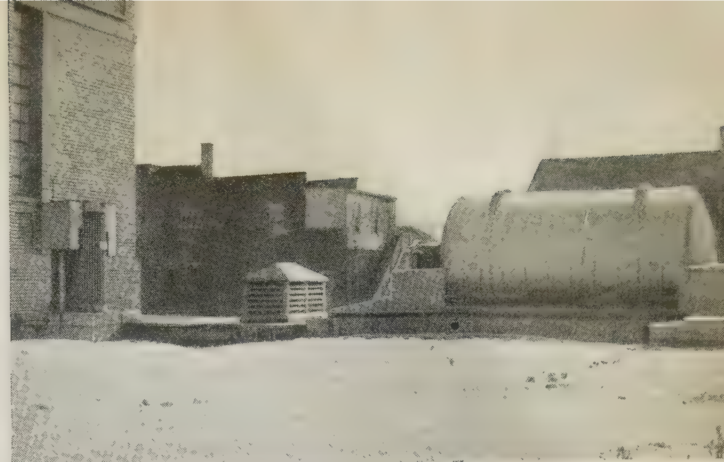
Blocksom, Franklin C., Randle, Wash.
Bukley, E. J., Malaja-Dmitrovka D. 8 Kv. 38, Moscow, U.S.S.R.
Collins, Ogie B., Minimum, Mo.
DeKeyser, Jacques F., 37-53—78th St., Jackson Heights, N. Y.
Gardner, Fred V., 1180 Clayton St., San Francisco, Calif.
Johnson, James W., 3506—16th St., N. W., Washington, D. C.
Jones, Robert W., 565 Thompson Ave., Donora, Pa.
Luther, Herbert A., 50 Atwood Ave., Johnston, R. I.
Megeath, S. A., Jr., 14 North Ave., Elizabeth, N. J.
Meltvedt, Henry, 742 S. Douglas, Springfield, Mo.
Merrill, Warren C., 208 W. 8th St., Los Angeles, Calif.
Miyamoto, Tatsuo Charles, 517 M St., Sacramento, Calif.
Murray, Forrest H., 5530 Dorchester Ave., Chicago, Ill.
Ritter, Edward A., 40 Lexington St., Hamden, Conn.
Williams, Thomas J. C., 827 S. 48th St., Philadelphia, Pa.
Willson, William H., Jr., 1720—2nd Ave., Cedar Rapids, Iowa.

16 Addresses Wanted

BE GUIDED
by facts, not claims
by service records, not initial tests
by experience, not prophecy—
The 11,000 volt generator leads on these Turbo-Alternators of the 59th Street Power Station of the Interborough Rapid Transit Co., New York City, are all single conductor braided
KERITE CABLES
INSTALLED IN 1909, 1910, 1911
They have all given and are still giving continuous and satisfactory service.
THE KERITE COMPANY INC



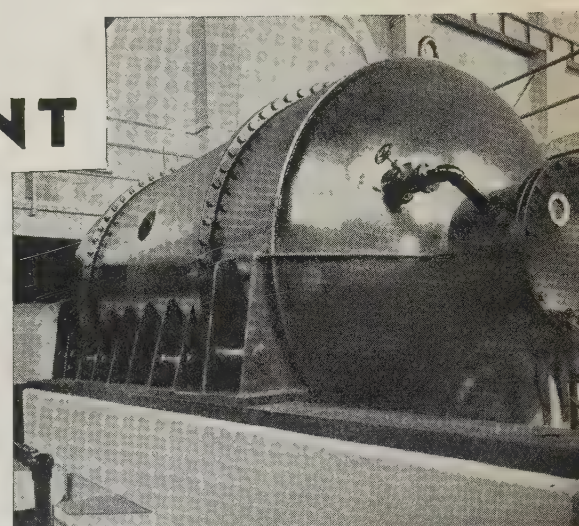
nchronous condenser, Atlantic City Electric Company, Atlantic City. talled 1930



Synchronous condenser, Public Service Company of Northern Illinois, Joliet, Ill. Installed 1930

COOLING

ELECTRIC DEVELOPMENT

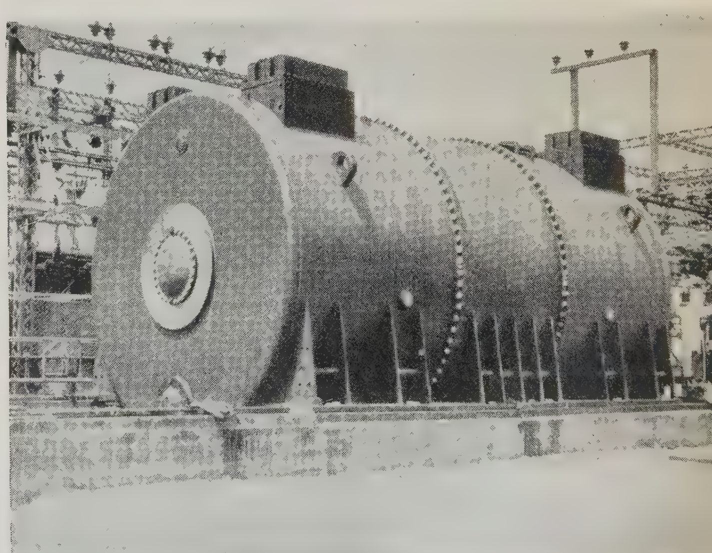


Synchronous condenser, Appalachian Electric Power Co., Kenova, W. Va. Installed 1935

THE inherent design characteristics of a hydrogen-cooled achine permit outdoor installaon and give you the additional enefits of lower over-all operating osts — decreased losses, reduced aintenance and inspection costs, nd longer insulation life.

he first of the hydrogen-cooled nstallations illustrated was made t Pawtucket, R. I., in 1928 (a 12,500-kva G-E condenser), and the most recent is represented by two G-E 60,000-kva condensers now in course of installation for the City of Los Angeles, Department of Water and Power. This wide range of G-E experience is your further assurance of dependable G-E equipment. We invite your inquiries. General Electric Company, Schenectady, N. Y.

Frequency changer, Pennsylvania Power and Light Company, Freemansburg, Pa. Installed 1935



One of two new 60,000-kva synchronous condensers built for City of Los Angeles, Department of Water and Power

600-49

ELECTRIC

The *Big Creek Line* is famous as the first in the world to be operated at 220,000 volts. Its conductors are 605,000 c.m. A.C.S.R. For the first ten years after it was built in 1913, it was operated at 150,000 volts. In 1923, the voltage was increased to 220,000, at which potential the line is still in regular operation.

The transmission map of California, like that of most other parts of the country, is largely a map of A.C.S.R. lines, with an enviable record of successful operation. Engineers have shown an overwhelming preference for A.C.S.R.; for the fundamental soundness of its construction; for its downright dependability in operation; for its economy.

So it is logical that the whole of the great unified project of building Boulder Dam and of supplying its power to the Colorado River Aqueduct pumping plants should depend on A.C.S.R. transmission lines.

★ ALCOA ALUMINUM ★ ALCOA

Section of San Bernardino-Boulder Dam Line near Victorville.

THE biggest customer for Boulder Dam power will be the mammoth pumping plants of the Colorado River Aqueduct. The transmission line for that power is now under construction. The conductors are 795,000 c.m. A.C.S.R., having a diameter of 1.1 inches. This line is 237 miles long, and will operate at 230,000 volts.

Boulder Dam itself was constructed with power supplied by the San Bernardino-Boulder Dam Line built in 1931. The conductors on this line are 4/0 A.C.S.R. It is 222 miles long and henceforth will transmit power away from Boulder Dam.

Added together, these comprise 459 miles of line in what is undoubtedly the most spectacular project of our time. The engineers have written for A.C.S.R. one more chapter in a continuing story of confidence.

The performance of A.C.S.R. is based on basic principles which time cannot change: it only proves! Among these fundamentals are: soundly logical use of materials for strength, conductivity and control of corona by diameter; longer spans with safe stresses; protection of the high strength core by the external strands; and a complete system of protection against vibration-fatigue. ALUMINUM COMPANY OF AMERICA, 2149 Gulf Building, Pittsburgh, Pennsylvania.

ALUMINUM

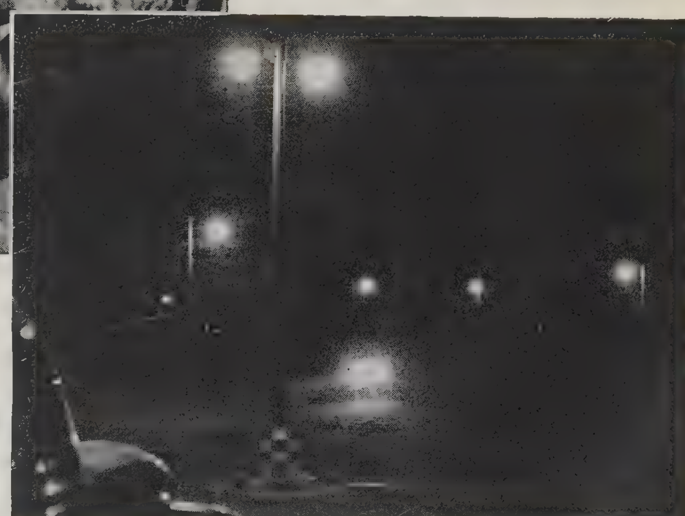
Out of the Depths!

OUT of the depths of the sea!

Out of the depths of many years of experience and operation comes that knowledge and background that enables us to produce the most reliable—most satisfactory Non-Metallic Trench Cables.

Hundreds of miles of Non-Leaded Submarine Cable have been installed and operated over the past 30 years. Each length is subjected to water—cold and warm, fresh and salt. Each length has its shore end buried in the ground. Each length is a forerunner of the more modern Triple "L" Non-Metallic Trench Cable.

American Steel & Wire Company Triple "L" Non-Metallic Trench Cables are available in single, twin and multiple conductor constructions and in all sizes for any voltage. The conductor insulation is our Amerite Rubber, having low water absorption—perfect aging characteristics and is in daily operation at voltages up to 13 KV. Fibrous fillers are eliminated and replaced by an integral sheath and fillers of acid, alkali and water resisting compound.



AMERICAN STEEL & WIRE COMPANY

208 S. La Salle Street, Chicago

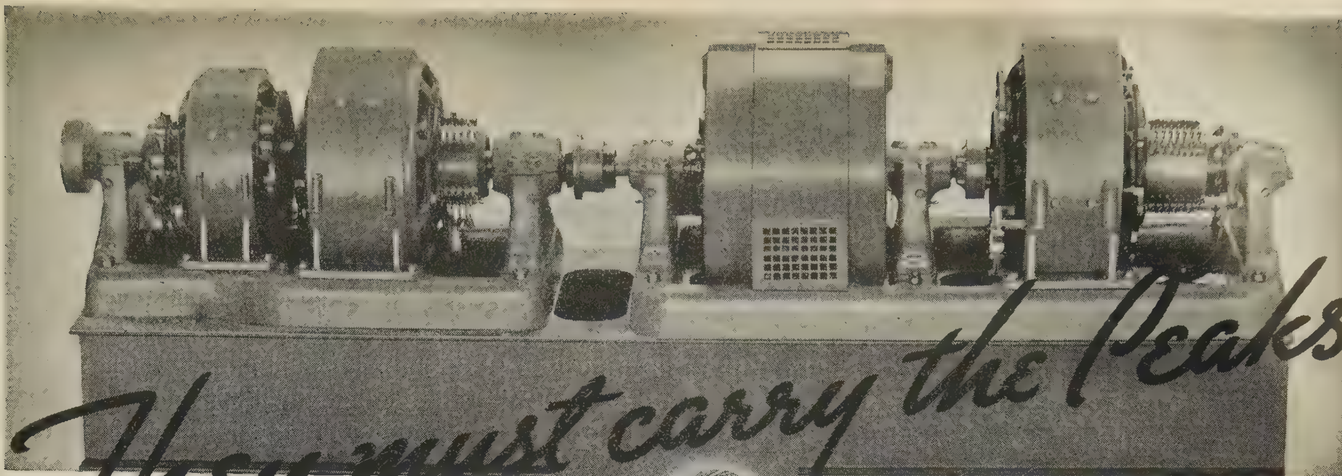
Columbia Steel Company, San Francisco, *Pacific Coast Distributors*

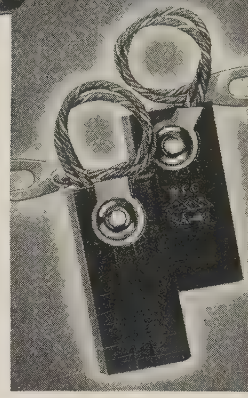
Empire State Building, New York

United States Steel Products Company, New York, *Export Distributors*

UNITED STATES STEEL

They must carry the Peaks



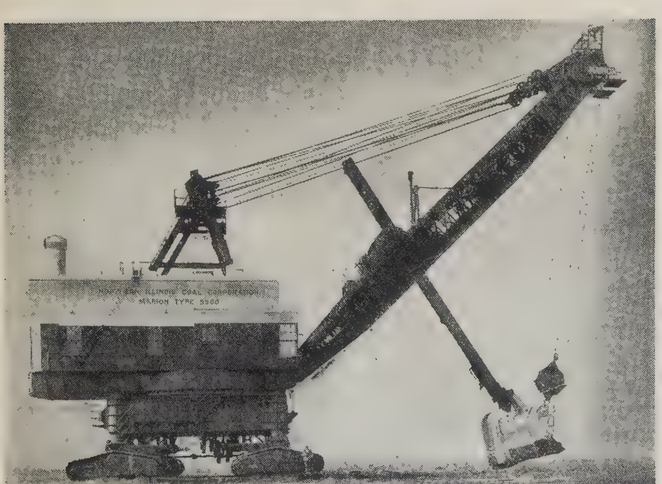


"SA" SERIES BRUSHES set a standard of performance on electric shovel generators never before attained. They carry extreme peaks day after day, reducing maintenance expense and eliminating interruption of service.

The generators on the huge electric shovels, used in the strip mining of coal, present one of the most difficult problems encountered in brush application. The loads are a succession of severe peaks, recurring at intervals of one minute or less, and often reaching the stalling point of the motors.

Besides meeting such extreme loads, the graduated performance characteristics of the "SA" Series of electro-graphitic grades make them applicable to almost every class of commutating type equipment, both A. C. and D. C. Other National Carbon grades take care of those applications not covered by the "SA" Series.

LET US DEMONSTRATE THE SUPERIORITY OF NATIONAL CARBON BRUSHES FOR YOUR SERVICE



NATIONAL CARBON COMPANY, INC.

Carbon Sales Division, Cleveland, Ohio

Unit of Union Carbide UCC and Carbon Corporation

BRANCH SALES OFFICES: NEW YORK PITTSBURGH CHICAGO SAN FRANCISCO

Employment Bulletin

Engineering Societies Employment Service

MAINTAINED for their members by the national societies of civil, mining, mechanical, and electrical engineers, in co-operation with other organizations. An inquiry to any of the three offices will bring full information.

A weekly bulletin of engineering positions open is available to members of the co-operating societies at a subscription of $3 per quarter or $10 per annum, payable in advance.

In the interest of effective service, it is essential that members using the employment service keep the bureau office serving them advised at reasonable intervals concerning their availability for employment, concerning any change in status, and immediately upon acceptance of any employment.

Employers interested in the following announcements should address replies to the key numbers indicated, and mail to the New York Office.

●

Men Available

E.E., B.S., George Washington Univ, 29, married; 7 yrs Gen Elec Co, State Pub Service Com and U.S. Govt. Util valuation, ry electrification, auditing exper. C-4583.

ENGR with 17 yrs exper in elec ry and auxiliary motor des, elec and mech, desires pos as des or eqpt engr. Has additional training in gen mech engg problems. D-4967.

ASSOC PROF OF E.E., age 36, married; 12 yrs teaching; also exper in indus. Subjects taught: AC machy and lab, elec des, com, illumination and electronics. Available in Sept. C-4113.

SENIOR, CCNY, 21, first quarter of class, desires employment for the summer; salary secondary to exper. Any sort of elec work, anywhere. Free after June 15. D-4839.

E.E. Grad, RPI, '35, Christian, 23. Desires pos with opportunity for devpmt and advancement in elec field. Salary and location secondary. D-4848.

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